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ADCSS-Labeling of Product Related Graphs

Research Article

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Abstract: A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. In this paper, we introduce the new concept, an absolute difference of cubic and square sum labeling of a graph. The graph for which every edge label is the absolute difference of the sum of the cubes of the end vertices and the sum of the squares of the end vertices. It is also observed that the weights of the edges are found to be multiples of 2. Here we characterize few graphs for cubic and square sum labeling.

Keywords: Graph labeling, sum square graph, square sum graphs, cubic graphs.

1. Introduction

All graphs in this paper are finite and undirected. The symbol $V(G)$ and $E(G)$ denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p, q) graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2], [3] and [4]. Some basic concepts are taken from Frank Harary [2]. We introduced the new concept, an absolute difference of cubic and square sum labeling of a graph [1]. In this paper we investigated some new results on ADCSS labeling of product related graphs.

Definition 1.1 ([1])**.** *Let $G = (V(G),\ E(G))$ be a graph. A graph G is said to be absolute difference of the sum of the cubes of the vertices and the sum of the squares of the vertices, if there exist a bijection $f : V(G) \rightarrow \{1, 2, \ldots, p\}$ such that the induced function $f^*_{adcss} : E(G) \rightarrow$ multiples of 2 is given by $f^*_{adcss}(uv) = |f(u)^3 + f(v)^3 - (f(u)^2 + f(v)^2)|$ is injective.*

Definition 1.2. *A graph in which every edge associates distinct values with multiples of 2 is called the sum of the cubes of the vertices and the sum of the squares of the vertices. Such a labeling is called an absolute difference of cubic and square sum labeling or an absolute difference css-labeling.*

2. Main Results

Definition 2.1. *Let G_1 and G_2 be any two graphs with n_1 and n_2 vertices respectively. Then the Cartesian product $G_1 \times G_2$ has $n_1 n_2$ vertices which are $\{(u,\ v)/u \in G_1,\ v \in G_2\}$. The edges are obtained as follows: (u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \times G_2$ if either $u_1 = u_2$ and v_1 and v_2 are adjacent in G_2 or u_1 and u_2 are adjacent in G_1 and $v_1 = v_2$.*

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Theorem 2.2. *The planar grid $P_n \times P_n$ is the absolute difference of the css-labeling.*

Proof. Let $G = P_n \times P_n$ and let $v_1, v_2, \ldots, v_{n2}$ are the vertices of G. Here $|V(G) = n^2|$ and $|E(G)| = 2n(n-1)$. Define a function $f : V \to \{1, 2, 3, \ldots, n^2\}$ by $f(v_i) = i,\ i = 1, 2, \ldots, n^2$. For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

Case (1): When n is odd

$$f^*_{adcss}[v_{(i-2)n+j}v_{in-j+1}] = \{(i-2)n+j\}^2\{(i-2)n+j-1\} + \{in-j+1\}^2\{in-j\},\ j = 1, \ldots, (n-1),\ i = 2, 4, \ldots, (n-1)$$
$$f^*_{adcss}[v_{in-j+1}v_{in+j}] = (in-j+1)^2(in-j) + (in+j)^2(in+j-1),\ j = 2, \ldots, n,\ i = 2, 4, \ldots, (n-1)$$
$$f^*_{adcss}[v_iv_{i+1}] = i^2(i-1) + (i+1)^2 i,\ i = 1, 2, \ldots, n^2-1.$$

Case (2): When n is even.

$$f^*_{adcss}[v_{(i-2)n+j}v_{in-j+1}] = \{(i-2)n+j\}^2\{(i-2)n+j-1\} + \{in-j+1\}^2\{in-j\},\ j = 1, 2, \ldots, (n-1),\ i = 2, 4, \ldots, n$$
$$f^*_{adcss}[v_{in-j+1}v_{in+j}] = (in-j+1)^2(in-j) + (in+j)^2(in+j-1),\ j = 2, \ldots, n,\ i = 2, 4, \ldots, (n-2)$$
$$f^*_{adcss}[v_iv_{i+1}] = i^2(i-1) + (i+1)^2 i,\ i = 1, 2, \ldots, n^2-1.$$

All edge values of G are distinct, which are multiples of 2. That is the edge values of G are in the form of an increasing order. Hence $P_n \times P_n$ admits absolute difference of css-labeling. $\square$

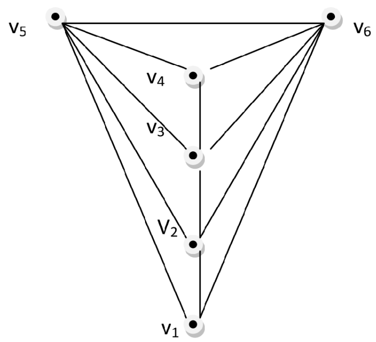
Theorem 2.3. *The graph PL_n is the absolute difference of the css-labeling.*

Proof. Let $G = PL_n$ and let $v_1, v_2, \ldots, v_n$ are the vertices of G. Here $|V(G)| = n$ and $|E(G)| = 3n-6$. Define a function $f : V \to \{1, 2, 3, \ldots, n\}$ by $f(v_i) = i,\ i = 1, 2, \ldots, n$. For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$$f^*_{adcss}(v_i\ v_{i+1}) = (i+1)^2 i + i^2(i-1),\ i = 1, 2, \ldots, n-1.$$
$$f^*_{adcss}(v_n\ v_{n-i}) = n^2(n-1) + (n-i)2(n-i-1),\ i = 2, \ldots, n-1.$$
$$f^*_{adcss}(v_{n-1}\ v_{n-i-1}) = (n-1)^2(n-2) + (n-i-1)^2(n-i-2),\ i = 2, \ldots, n-2.$$

All edge values of G are distinct, which are multiples of 2. That is the edge values of G are in the form of an increasing order. Hence PL_n admits absolute difference of css-labeling. $\square$

Example 2.4. $G = PL_6$



Theorem 2.5. *The graph $C_m \times P_n$ is the absolute difference of the css-labeling.*

Proof. Let $G = C_m \times P_n$ and let $v_1, v_2, \ldots, v_{mn}$ are the vertices of G. Here $|V(G)| = mn$ and $|E(G)| = 2mn - m$. Define a function $f : V \to \{1, 2, 3, \ldots, mn\}$ by $f(v_i) = i,\ i = 1, 2, \ldots, mn$. For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$$f^*_{adcss}(v_{mi-m+j}\ v_{mi-m+j+1}) = (mi-m+j)^2(mi-m+j-1) + (mi-m+j+1)^2(mi-m+j),\ j = 1, \ldots, m-1,\ i = 1, \ldots, n$$

$$f^*_{adcss}(v_{mi-m+j}\ v_{mi+j}) = (mi-m+j)^2(mi-m+j-1) + (mi+j)^2(mi+j-1),\ j = 1, \ldots, m,\ i = 1, \ldots, n-1$$

$$f^*_{adcss}(v_{mi-m+1}\ v_{mi}) = (mi-m+1)^2(mi-m) + (mi)^2(mi-1), i = 1, \ldots, n.$$

All edge values of G are distinct, which are multiples of 2. That is the edge values of G are in the form of an increasing order. Hence $C_m \times P_n$ admits absolute difference of css-labeling. □

Theorem 2.6. *The graph $Q_3 \times P_m$ is the absolute difference of the css-labeling.*

Proof. Let $G = Q_3 \times P_m$ and let $v_1, v_2, \ldots, v_{8m}$ are the vertices of G. Here $|V(G)| = 8m$ and $|E(G)| = 20m - 8$. Define a function $f : V \to \{1, 2, 3, \ldots, 8m\}$ by $f(v_i) = i,\ i = 1, 2, \ldots, 8m$. For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$$f^*_{adcss}(v_i\ v_{i+1}) = (i+1)^2 i + i^2(i-1)$$

$$i = 1, 2, 3, \ldots, n-2$$

$$i = n, n+1, \ldots, 2n-3$$

$$i = 2n-1, 2n, \ldots, 3n-4$$

$$i = 3n-2, 3n-1, \ldots, 4n-5$$

$$i = 4n-3, 4n-2, \ldots, 5n-6$$

$$i = 5n-4, 5n-3, \ldots, 6n-7$$

$$i = 6n-5, 6n-4, \ldots, 7n-8$$

$$i = 7n-6, 7n-5, \ldots, 8n-9$$

where $n = m + 1$

$$f^*_{adcss}(v_i\ v_{m+i}) = i^2(i-1) + (m+i)^2(m+i-1),\ i = 1, 2, \ldots, m$$

$$f^*_{adcss}(v_{2m+i}\ v_{3m+i}) = (2m+i)^2(2m+i-1) + (3m+i)^2(3m+i-1),\ i = 1, 2, \ldots, m$$

$$f^*_{adcss}(v_{4m+i}\ v_{5m+i}) = (4m+i)^2(4m+i-1) + (5m+i)^2(5m+i-1),\ i = 1, 2, \ldots, m$$

$$f^*_{adcss}(v_{6m+i}\ v_{7m+i}) = (6m+i)^2(6m+i-1) + (7m+i)^2(7m+i-1),\ i = 1, 2, \ldots, m$$

$$f^*_{adcss}(v_i\ v_{6m+i}) = i^2(i-1) + (6m+i)^2(6m+i-1),\ i = 1, 2, \ldots, m$$

$$f^*_{adcss}(v_{2m+i}\ v_{4m+i}) = (2m+i)^2(2m+i-1) + (4m+i)^2(4m+i-1),\ i = 1, 2, \ldots, m$$

$$f^*_{adcss}(v_{3m+i}\ v_{5m+i}) = (3m+i)^2(3m+i-1) + (5m+i)^2(5m+i-1),\ i = 1, 2, \ldots, m$$

$$f^*_{adcss}(v_{m+i}\ v_{7m+i}) = (m+i)^2(m+i-1) + (7m+i)^2(7m+i-1),\ i = 1, 2, \ldots, m$$

$$f^*_{adcss}(v_i\ v_{2m+i}) = (i)^2(i-1) + (2m+i)^2(2m+i-1),\ i = 1, 2, \ldots, m$$

$$f^*_{adcss}(v_{m+i}\ v_{3m+i}) = (m+i)^2(m+i-1) + (3m+i)^2(3m+i-1),\ i = 1, 2, \ldots, m$$

$$f^*_{adcss}(v_{4m+i}\ v_{6m+i}) = (4m+i)^2(4m+i-1) + (6m+i)^2(6m+i-1),\ i = 1, 2, \ldots, m$$

$$f^*_{adcss}(v_{5m+i}\ v_{7m+i}) = (5m+i)^2(5m+i-1) + (7m+i)^2(7m+i-1),\ i = 1,2,\ldots,m$$

All edge values of G are distinct, which are multiples of 2. That is the edge values of G are in the form of an increasing order. Hence $Q_3 \times P_m$ admits absolute difference of css-labeling. □

Definition 2.7. *Let u be a vertex of $P_m \times P_n$ such that $deg(u) = 2$. Introduce an edge between every pair of distinct vertices v, w with $deg(v)$, $deg(w) \neq 4$, if $d(u,v) = d(u,w)$, where $d(u,v)$ is the distance between u and v. The graph so obtained is defined as the level joined planar grids and it is denoted by $LJ_{m,n}$.*

Theorem 2.8. *The graph $LJ_{m,n}$ is the absolute difference of the css-labeling.*

Proof. Let $G = LJ_{m,n}$ and let v_1, v_2,..., v_{nm} are the vertices of G. Here $|V(G)| = nm$ and $|E(G)| = 2mn - 3$. Define a function $f : V \to \{1,2,3,\ldots,nm\}$ by $f(v_i) = i$, $i = 1,2,\ldots,nm$. For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$$\begin{aligned} f^*_{adcss}(v_i\ v_{i+1}) &= i^2(i-1) + (i+1)^2 i, \\ & \quad i = 2,3,\ldots,m-1 \\ & \quad = m+1, m+2,\ldots,2m-1 \\ & \quad = 2m+1, 2m+2,\ldots,3m-1 \\ & \quad = 3m+1,\ldots,4m-1 \\ & \quad \ldots\ldots\ldots\ldots \\ & \quad = (n-1)m+1,\ldots,nm-1 \end{aligned}$$

$$f^*_{adcss}(v_{mi-m+j}\ v_{mi+j}) = (mi-m+j)^2(mi-m+j-1) + (mi+j)^2(mi+j-1),\ j = 1,2,3,\ldots,m,\ i = 1,2,3,\ldots,n-1$$

$$f^*_{adcss}(v_{i+1}\ v_{mi+1}) = (i+1)^2 i + (mi+1)^2(mi),\ i = 1,2,\ldots,m-1$$

$$f^*_{adcss}(v_{mi+m}\ v_{(n-1)m+i}) = (mi+m)^2(mi+m-1) + \{(n-1)m+i\}^2\{(n-1)m+i-1\},\ i = 1,2,3,\ldots,n-2$$

All edge values of G are distinct, which are multiples of 2. That is the edge values of G are in the form of an increasing order. Hence $LJ_{m,n}$ admits absolute difference of css-labeling. □

Theorem 2.9. *The graph P_n^2, where P_n is a path is the absolute difference of the css-labeling.*

Proof. Let $G = P_n^2$ and let v_1, v_2, ..., v_n are the vertices of G. Here $|V(G)| = n$ and $|E(G)| = 2n - 3$. Define a function $f : V \to \{1,2,3,\ldots,n\}$ by $f(v_i) = i$, $i = 1,2,\ldots,n$. For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$$\begin{aligned} f^*_{adcss}(v_i\ v_{i+1}) &= i^2(i-1) + (i+1)^2 i,\ i = 1,2,\ldots,n-1. \\ f^*_{adcss}(v_{2i-1}\ v_{2i+1}) &= (2i-1)^2(2i-2) + (2i+1)^2(2i) \\ & \quad i = 1,2,3,\ldots,\frac{n-2}{2},\ \text{n is even} \\ & \quad i = 1,2,3,\ldots,\frac{n-1}{2},\ \text{n is odd} \\ f^*_{adcss}(v_{2i}\ v_{2i+2}) &= (2i)^2(2i-1) + (2i+2)^2(2i-1) \\ & \quad i = 1,2,3,\ldots,\frac{n-2}{2},\ \text{n is even} \\ & \quad i = 1,2,3,\ldots,\frac{n-3}{2},\ \text{n is odd} \end{aligned}$$

All edge values of G are distinct, which are multiples of 2. That is the edge values of G are in the form of an increasing order. Hence P_n^2 admits absolute difference of css-labeling. $\square$

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