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A Mathematical Model For Blood Flow Through Inclined Stenosed Artery

Research Article

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Abstract: A mathematical model has been developed for studying the blood flow characteristics in an inclined artery having radially non-symmetrical stenosis. Blood is considered as Newtonian fluid. Navier-Stokes equations governing the fluid flow are solved analytically using perturbation scheme. Analytic expressions are obtained for various flow parameters and their variations with several flow parameters, such as inclination angle, stenosis height, shape parameter, are presented through graphs. It is found that axial velocity and flow rate increases with the increase in inclination angle of the artery.

Keywords: Newtonian fluid, Blood, Inclined artery, Shape parameter, Stenosis.

1. Introduction

Cardiovascular disease (CVD) is an abnormal functioning of the heart or blood vessels. Atherosclerosis is a vascular pathology that has become a prominent disease in western society. The term comes from the Greek words athero (gruel or paste) and sclerosis (hardness) and is characterized by the progressive narrowing and the occlusion of blood vessels [1, 3]. When fatty substances, cholesterols, cellular waste products, calcium, and fibrin build up waste products, calcium, and fibrin build up in the inner lining of an artery, this causes the narrowing of the lumen of the vessel and also an increase in the wall stiffness or decrease in compliance of the vessel. The buildup that results is called plague. Some level of stiffening of the arteries and narrowing is a normal result of aging. Eventually the plague can block an artery and restrict flow through that vessel, resulting in heart attack if the vessel is blocked in one that supplied blood to the heart. [7] have found that, when an obstruction developed in an artery, blood flow to the particular vascular bed supplied by the artery is reduced due to increased resistance.

Quite a good number of analytical studies pertaining to the blood flow through stenosed arteries have been carried out by [4–6, 10, 16] in order to analyze the arterial constriction on the flow characteristics of blood. The study of blood flow through arteries with complex geometries (inclining, bending, tapering, bifurcation etc.) is quite important, as this obviously affects the hemodynamic factors. Several researchers [2, 8] have considered the blood flow through uniform circular tubes. It is well known that in physiological systems arteries are not all horizontal but have some inclination with axis. The gravitational force is accounted due to the consideration of inclined tube. Singh et al. have studied the blood flow through radially non-symmetric stenosed artery. Maruti Prasad and [9] have proposed a model of blood flow through an inclined tube with a stenosis.

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While modeling blood flow in a stenosed tube, it was initially assumed that, the flow obeys Newtonian hypothesis and the flow variables have been computed by using basic Navier-Stoke's equations. [15, 16] has investigated that when blood flows through larger arteries at high rate, blood behaves like a Newtonian fluid. [14] have presented several solutions to the problem of viscous flow of an incompressible Newtonian liquid in a converging tube. Their solutions include the pressure-flow relations for cases for pulsatile and steady flows in a rigid tube and, pulsatile flow in an elastic tube. [11] have proposed a mathematical model to see the effect of inclination of a stenosed artery using casson fluid with periodic body acceleration. In the present investigation we have considered the blood flow through inclined artery and seen the effect of inclination angle on various flow parameters.

2. Formulation of the Problem

In the present study, we have considered the one-dimensional, steady, laminar, and fully developed flow of blood through an inclined stenosed artery, as shown in Figure 1. It is consider that axially non-symmetric but radially symmetric stenosis is formed in the lumen of the artery and depends upon the axial distance z and the height of it's growth. Further, the length of the artery is assumed too large in comparison to the radius of the artery, so that we can neglect the entrance effects and special wall effects.

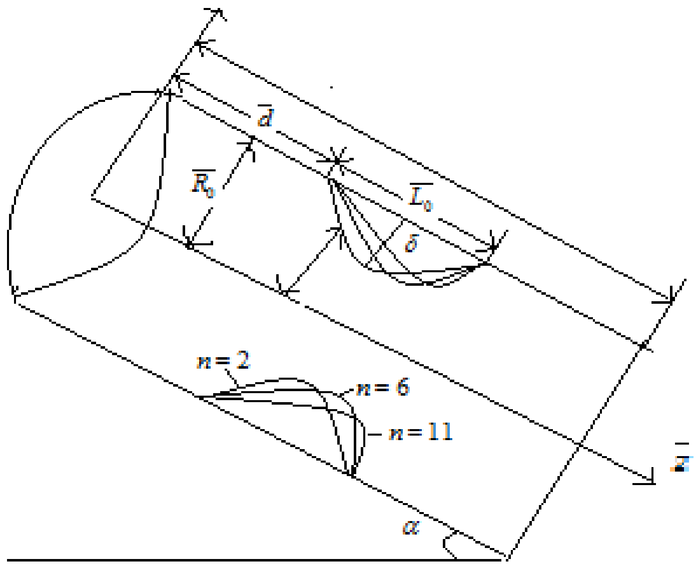

Figure 1. **Schematic diagram of blood flow in an inclined stenosed artery**

The geometry of the flow is shown in Fig.1 and is given by

$$\left.\begin{aligned} \frac{\overline{R}(\overline{z})}{R_0} &= 1-\overline{A}\left[\overline{L_0}^{(n-1)}\left(\overline{z}-\overline{d}\right)-\left(\overline{z}-\overline{d}\right)^n\right], \; \overline{d}\le\overline{z}\le\overline{d}+\overline{L_0} \\ &= 1, \qquad\qquad\qquad\qquad otherwise \end{aligned}\right\} \tag{1}$$

Where $\overline{R}\left(\overline{z}\right)$ and R_o is the radius of the artery with and without stenosis respectively. L_o is the length of the stenosis and d indicates it's location, $n \geq 2$ is the stenosis shape parameter and the parameter A is given by

$$A = \frac{\delta}{R_0 L_0^n}\frac{n^{n/(n-1)}}{(n-1)} \tag{2}$$

Where δ denotes the maximum height of the stenosis at $z = \frac{(d+L_o)}{n^{1/(n-1)}}$ such that $\delta/R_o < 1$. The Navier-Stokes equations governing the fluid flow is given by Equation (3). Here blood is assumed to be Newtonian fluid.

$$0 = -\left(\frac{\partial \overline{p}}{\partial \overline{z}}\right) - \frac{1}{\overline{r}}\frac{\partial}{\partial \overline{r}}\left(\overline{r}\overline{\tau}\right) + \overline{\rho g}\sin\alpha, \tag{3}$$

$$\frac{\partial \overline{p}}{\partial \overline{r}} = 0, \tag{4}$$

$$\frac{\partial \overline{p}}{\partial \overline{\theta}} = 0, \tag{5}$$

Equation of Newonian fluid is given as

$$\tau = -\mu\frac{\partial \overline{u}}{\partial \overline{r}} \tag{6}$$

μ is the shear viscosity of Newtonian fluid. Equation (3) with the help of Equation (6) can be written as,

$$\left(\frac{\partial \overline{p}}{\partial \overline{z}}\right) = \frac{\overline{\mu}}{\overline{r}}\frac{\partial}{\partial \overline{r}}\left(\overline{r}\frac{\partial \overline{u}}{\partial \overline{r}}\right) + \overline{\rho g}\sin\alpha, \tag{7}$$

Boundary Conditions: The boundary conditions are

$$\overline{u} = 0 \text{ at } \overline{r} = \overline{R}\left(\overline{z}\right), \tag{8a}$$

$$\frac{\partial \overline{u}}{\partial \overline{r}} = 0 \text{ at } \overline{r} = 0 \tag{8b}$$

By introducing the following non-dimensional variables

$$\begin{array}{l} u = \frac{\overline{u}}{\overline{U_0}}, z = \overline{z}/\overline{R_0},\ d = \overline{d}/\overline{R_0},\ R(z) = \overline{R}(\overline{z})/\overline{R_0}, \delta = \overline{\delta}/\overline{R_0}, \mu = \frac{\overline{\mu}}{\mu_0}, \\ (L, L_0) = \frac{\left(\overline{L}, \overline{L_0}\right)}{\overline{R_0}}, F = \frac{\overline{C_0}}{\overline{\rho g}}, A = \frac{\overline{A}}{\overline{R_0}^{n-1}}, \frac{dp}{dz} = \frac{dp/dz}{\overline{C_0}} \end{array} \tag{9}$$

Equations (1-8) in the non-dimensional form can be written as, Using non-dimensional quantities given in Equation (9), Equation (1) becomes

$$\left.\begin{array}{ll} R\left(z\right) = 1 - A\left[L_0^{(n-1)}\left(z-d\right) - \left(z-d\right)^n\right], & d \le z \le d + L_0 \\ \quad = 1, & otherwise \end{array}\right\} \tag{10}$$

The equation governing the fluid flow in dimension less form becomes,

$$4\frac{dp}{dz} = \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{4\sin\alpha}{F}, \tag{11}$$

Boundary conditions in dimension less form are becomes,

$$u = 0 \text{ at } r = R \tag{12a}$$

$$\frac{\partial u}{\partial r} = 0 \text{ at } r = 0 \tag{12b}$$

The non-dimensional volumetric flow rate is defined by

$$Q\left(z,t\right) = 4\int_0^{R(z)} r\, u\left(z,r,t\right) dr, \tag{13}$$

The non-dimensional form of the shear stress is given by

$$\tau = -\frac{1}{2}\mu \tag{14}$$

3. Solution of the Problem

The expression for axial velocity is obtained from Equation (11) using boundary conditions (12) is given by

$$u = \frac{1}{\mu}\left(P + \frac{\sin\alpha}{F}\right)(R^2 - r^2) \tag{15}$$

The flow rate is given by solving the Equations (13) and (15),

$$Q = \frac{R^4}{\mu}\left(P + \frac{\sin\alpha}{F}\right) \tag{16}$$

The pressure gradient is obtained from Equation (16) can be written as,

$$\frac{dp}{dz} = \frac{\sin\alpha}{F} - \frac{\mu Q}{R^4} \tag{17}$$

The wall shear stress at $r = R$ can be put in the form

$$\tau_R = R\left(P + \frac{\sin\alpha}{F}\right) \tag{18}$$

4. Results and Discussion

The present work is concerned with the modeling of blood flow through inclined stenosed artery. We have seen the effect of inclination angle on different flow parameters such as velocity, flow rate, wall shear stress etc. MATLAb software has been used to find the analytical solution of the problem.

In this analysis, the combined effect of several parameters have been seen in case of inclined and non-inclined artery. Computer codes have been developed for the numerical computation of the different flow quantities, for parameter values $\delta = 0 - 0.3$, $\alpha = 0°, 30°, 45°$, pressure gradient $P = 0.5,\ 1.0,\ 1.5$ [13], $F = 0.3$ [9].

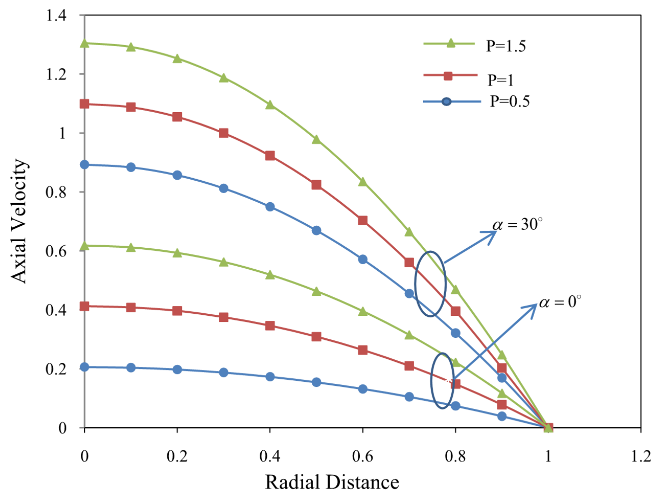

Figure 2. Variation of axial velocity with the radial distance for different values of pressure gradient P and inclination angles

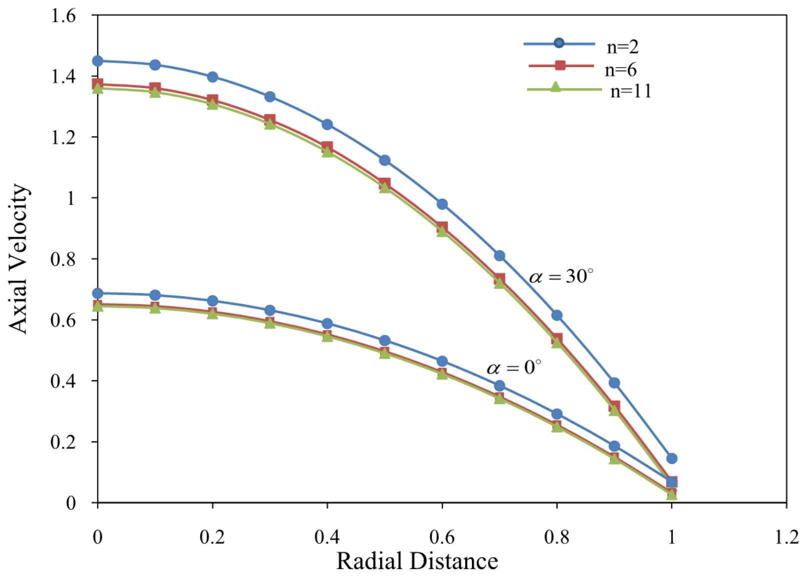

Figure 3. Variation of axial velocity with the radial distance for different values of shape parameter and inclination angles

It is noticed from the Fig. 2 that axial velocity decreases when the radial distance increases. It is also seen that magnitude of velocity is increases with the increase in pressure gradient P. velocity is more when the artery is inclined at some angle than that of non-inclined artery.

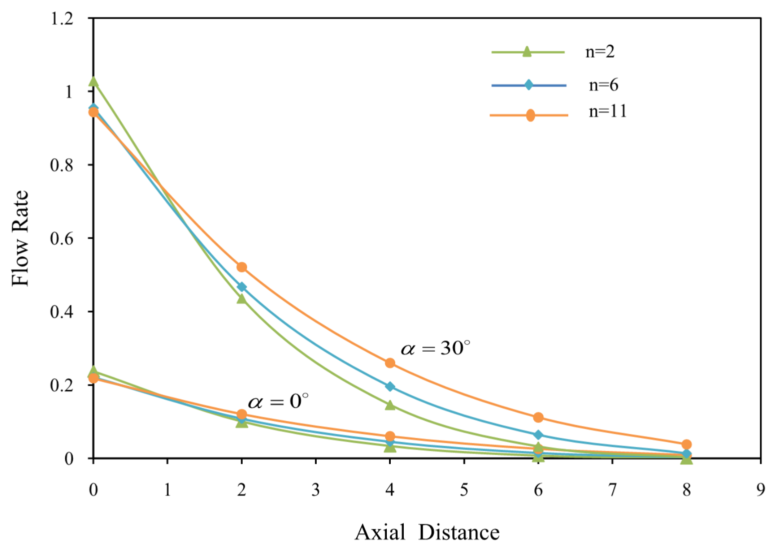

Figure 4. Variation of flow rate with the axial distance for different shape parameter

Fig.3 shows the variation of axial velocity with the radial distance r for different values of shape parameter and inclination angles. It is found that as the value of shape parameter 'n' increase velocity decreases in both the cases for inclined as well as non-inclined artery.

Fig. 4, 5 shows the variation of volumetric flow rate with the axial distance z for different shape parameter and pressure gradient P. It is depicted that flow rate decreases when the axial distance z increases. It further decreases with the increase in shape parameter for $n = 6, 11$. In the case of $\alpha = 0^\circ$ i.e. when the artery is non-inclined magnitude of flow rate is low as compared to the in case of inclined artery by angle $\alpha = 30°$. Fig. depicts that flow rate increases with the increase in pressure gradient.

Fig. 6 shows the variation of wall shear stress with stenosis height for different inclination angles. It increases with the stenosis height and with the inclination angle.

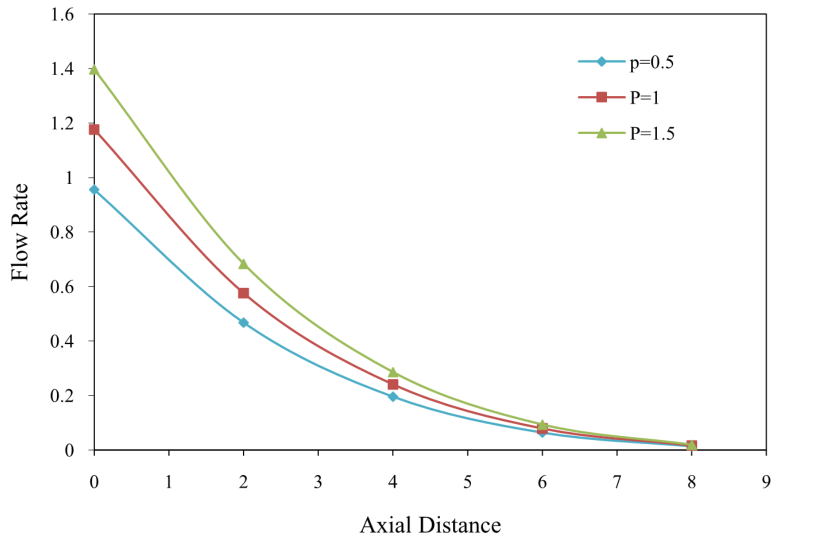

Figure 5. **Variation of flow rate with the axial distance for different values of pressure gradient**

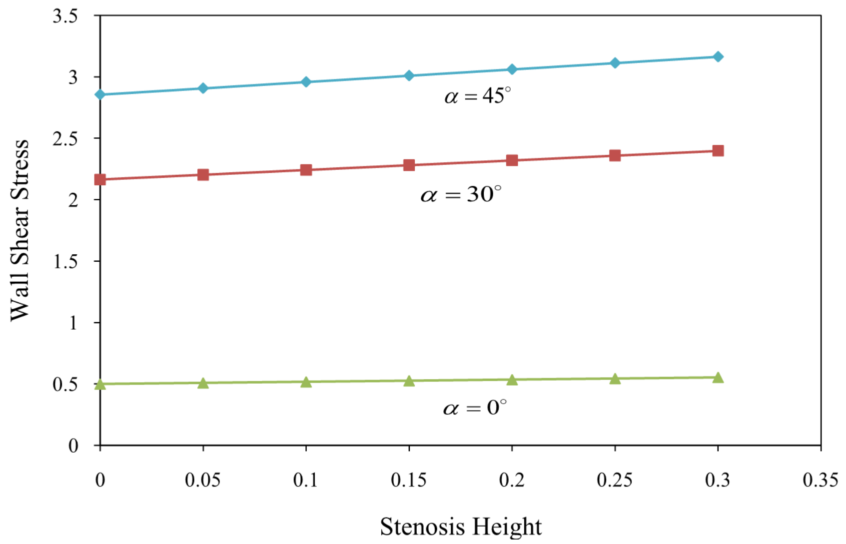

Figure 6. **Variation of wall shear stress with the stenosis height for different values of inclination angles**

5. Conclusion

In the present model we consider the Newtonian flow of blood through inclined artery with axially symmetric but radially non-symmetric mild stenosis. It is easily concluded that we get better result when consider inclined artery with asymmetric stenosis than that of non-inclined symmetric stenosed artery. It is also found that for increase in inclination, axial velocity, flow rate, wall shear stress increases. Hence from all the above discussions we can conclude that, it is quite important to get accurate measure in accordance with physiological situations in blood flow modeling and can be used in medical applications. Further we can extend this model analyze the effect of stenosis on various flow parameters in the presence of magnetic field.

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