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Similarity Solution of Non-Newtonian Three-dimensional Boundary-layer Through Porous Medium

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Abstract: An analysis is made to study a three-dimensional boundary-layer flow of non-newtonian fluid over a porous media. The governing partial differential equations of momentum are transformed into self-similar non-linear ordinary differential equations by using new form of suitable similarity transformations. The obtain equations are then should by using shooting method. The flow phenomena is characterized by the power-law index 'n', permeability of porous media 'ω' porosity parameter 'ϕ' effective viscosity 'λ', three-dimensionality parameters 'α'. The numerical results of the velocity profiles are obtained and displayed graphically.

Keywords: Porous medium, Power-law fluid, Effective viscosity, Porosity, Similar solution.

1. Introduction

In recent years, the evolution of the study in non-newtonian fluids have effectively attracted many researchers in the field of science and engineering by its challenging applications. An interesting model for non-newtonian fluids is from Lee and Ames [1] and Hansen and Na [5]. The non-newtonian model generalizes the visco-inelastic behaviour of some fluids as Power-law, Powel-Eyring, Prandtl, Eyring, Ellis and Reiner-Philippoff fluids. The special interest is towards the two models i.e, the Power law fluids and the Powel Eyring fluids. The power law fluids are the non-newtonian model used commonly and the Powel-Eyring fluids are brought up by the kinetic theory of liquids. In past decades, the work on the three dimensional boundary layer flow of the non-newtonian fluids is much less comparitivity with the two dimensional boundary layer flow of the non-newtonian fluids because its mathematical complexity of the problem, hence therefore such problems were generally avoided. There are large number of research articles (REF) based on the study of the Newtonian and non-newtonian boundary layer flows.

The work of Schowalter [11], Na and Hansen [5] and Timol and Kalathia [13], refers to two-dimensional flows of non-newtonian Power-law fluids. The laminar three-dimensional incompressible boundary layer equations of non-newtonian power-law fluids and obtained similarity solution of such equations was first analysed by Schowalter [11]. According to him, the similarity solution exists only if free stream velocities $V(x)$ and $W(x)$ in X-direction are constant. Further Na and Hansen [5] extended Schowalter [11] analysis and arrived a similarity solution for the system of laminar three-dimensional boundary-layer equations of power-law fluids. According to their work for similarity requirement, the free stream velocities

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$V(x)$ and $W(x)$ in X-direction must distinct by a multiplicative constant. This work generalized by the his conclusion which is somewhat more presice than that of Schowalter [11].

On improving the understanding the isothermal flow of non-newtonian fluids through porous medium have numerous work with regard to is diverse applications in petroleum reservoirs, naval architecture, polymer processing and lubrication in porous bearing. There are many other applications orienting with that of non-isothermal flows in some important engineering applications such as enhanced discovery of heavy oils by thermal methods, and polymer processing in packed beds.

The solution process is carried numerically for three dimensional boundary layer for stagnation point flows is given by Howarth [15] and also showed the nodal points attaches to the flow. He have also obtained the solution for two orthogonal Heimenz flows of non-axisymmetric stagnation point flow depending on a unique parameter $\alpha = \frac{V_\infty}{U_\infty}$ in his work. Rosenhead [10] in his book, considered the three dimensional boundary layer flow of a Newtonian fluid, which shows the solution range from 0 to 1. Davey [2] improved the solution procedure of Howarth from the range -1 to 1. Libby (1967), Scholiefed and Davey [12], Wang [18], Duck and Stow [4], Weidmman [19] recently Kudenatti and Kirsur [6] have studied importance of the same study for various other situations. The above researcher have discussed on Newtonian three-dimensional boundary-layer, which effectively extended the interest in analysing the three-dimensional boundary layer for the same situations.

The similar solutions of non-newtonian fluids in three-dimensional stagnation flow was obtained by Subba [16]. Yurusoy and Pakdemirli [17] explored on the Lie group analysis for unsteady three-dimensional boundary layer of non-newtonian fluids to generalize PDE to ODE. Timol and Patel [14] effectively investigated ten different models of the two- and three-dimensional stagnation point flow of non-newtonian fluid. Nadeem (2014) analyzed MHD effects in shear-thinning fluid (Casson fluid) on three-dimensional boundary-layer on the linearly stretching sheet with convective boundary condition.

In the present work, section 2 involves the model for the theory of three-dimensional flow over a porous medium of power-law fluid. The study of the non-newtonian effects which are PDE (partial differential equations) are reduced to ODE (ordinary differential equations) by using similarity transformation. The obtained equations are coupled and nonlinear in nature are solved numerically by the hybrid method i.e, the shooting technique. The procedure of the method and convergence condition to the problem is given in section 3. Section 4 infers the discussion on the solutions of the problem graphically. Results of velocity profiles, wall shear stress for various parameters are discussed in some detailed.

2. Flow Theory

Let us consider the steady, three-dimensional, laminar, incompressible, homogeneous non-newtonian fluid immersed in a porous media on stagnation surface. The mathematical model have co-ordinates axes i.e. (x,y,z), where x and y direction are measured as co-ordinates parallel to the stagnation surface and z is normal to it towards the main stream. The equations governs the above flow model are,

$$\nabla \cdot \overrightarrow{q} = 0 \tag{1}$$

$$\rho\left(\overrightarrow{q}.\nabla\right)\overrightarrow{q} = -\nabla P + \nabla\tau - \mu/k\overrightarrow{q}_d \tag{2}$$

Where velocity vector is define as $\overrightarrow{q} = \overrightarrow{q}(u,v,w)$ and (u,v,w) are velocity components of x, y and z-direction respectively, ρ is fluid density, P is hydrodynamic pressure and τ is the Cauchy's stress tensor which is related to the fluid motion non-linearly, μ fluid viscosity of a non-newtonian, K is a permeability and $\overrightarrow{q}_d$ Darcy velocity vector.

For an incompressible non-newtonian fluid the non-linear relation between stress tensor and the rate of deformation tensor is define as constitutive relation

$$\tau = -\mu\nu \tag{3}$$

where ν is the rate of deformation tensor which is defined as $\nu = (\nabla \overrightarrow{q} + \nabla \overrightarrow{q}^T)$. To balance momentum equation in porous medium (Nield and Bejan 2013) by using Dupuit-Forchchiemen relation $\phi \overrightarrow{q} = \overrightarrow{q}_d$. Therefore the equation of motion (2), is

$$\rho(\overrightarrow{q}_d.\nabla)\overrightarrow{q}_d = -\nabla P + \left(-\mu\left(\nabla \overrightarrow{q}_d + \nabla \overrightarrow{q}_d{}^T\right)\right) - \mu/k\overrightarrow{q}_d \tag{4}$$

The non-newtonian fluid viscosity is considered as scalar function of a deformation tensor ν which is given as $\mu = \mu(\nu)$. From power-law analysis μ is described as,

$$\frac{1}{\rho}\mu = -K(\nabla\vec{q} = +\nabla\vec{q}^T)^{\frac{n-1}{2}} \tag{5}$$

here K is fluid consistency and n is fluid index.

The modified governing equations of (1) and (4) by choosing usual boundary layer approximations along with (5), the governing momentum equations with continuity equations are,

$$\frac{1}{\phi^2}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \frac{-1}{\rho}\frac{\partial p}{\partial x} + K\frac{\partial}{\partial z}\left(\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right]^{\frac{n-1}{2}}\frac{\partial u}{\partial z}\right) - \frac{\nu}{k}u \tag{6}$$

$$\frac{1}{\phi^2}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \frac{-1}{\rho}\frac{\partial p}{\partial y} + K\frac{\partial}{\partial z}\left(\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right]^{\frac{n-1}{2}}\frac{\partial v}{\partial z}\right) - \frac{\nu}{k}v \tag{7}$$

$$\frac{\partial p}{\partial z} = 0 \tag{8}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{9}$$

From Bernoulli's theory the pressure gradient in equation (6) and (7) are reduces as

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{1}{\phi^2}\left(U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y}\right) \tag{10}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} = \frac{1}{\phi^2}\left(U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y}\right) \tag{11}$$

But in equation (8) the pressure gradient is zero i.e, pressure is assumed along z-direction (flow region), where *U(x,y)*, *V(x,y)* are the main-stream momentums in x & y directions. Therefore the system (6-7) becomes.

$$\frac{1}{\phi^2}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \frac{1}{\phi^2}\left(U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y}\right) + K\frac{\partial}{\partial z}\left(\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right]^{\frac{n-1}{2}}\frac{\partial u}{\partial z}\right) - \frac{\nu}{k}u \tag{12}$$

$$\frac{1}{\phi^2}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \frac{1}{\phi^2}\left(U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y}\right) + K\frac{\partial}{\partial z}\left(\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right]^{\frac{n-1}{2}}\frac{\partial v}{\partial z}\right) - \frac{\nu}{k}v \tag{13}$$

with the corresponding relevant BCs are,

$$at \quad z = 0, \quad u = U_w(x,y), \quad v = V_w(x,y), \quad w = 0, \tag{14}$$

as $z \to \infty$, $u \to U(x,y)$, $v \to V(x,y)$, here, $U_w(x,y)$, $V_w(x,y)$ are the wall velocities along $x-$ & $y-$ directions. Since the three-dimensional flow has two outer stagnation point flows i.e. $U(x,y) = U_\infty(x+y)$ and $V(x,y) = V_\infty(x+y)$, where U_∞ is known as strain rate and V_∞ is known as shear rate constants. The above partial different equations (12-13) are transformed in to non-linear ODEs by introducing stream functions ψ_1, ψ_2 and similarity transformation.

Whereas stream function denotes $u = \dfrac{\partial \psi_1}{\partial z}$, $v = \dfrac{\partial \psi_2}{\partial z}$ and $w = -\left(\frac{\partial \psi_1}{\partial x} + \frac{\partial \psi_2}{\partial y}\right)$ which is define as

$$\psi_1 = U f(\eta)\left(\frac{n(n+1)\nu x}{1+(2n-1)U^{2-n}}\right)^{\frac{1}{n+1}}, \tag{15}$$

$$\psi_2 = V g(\eta)\left(\frac{n(n+1)\nu x}{1+(2n-1)U^{2-n}}\right)^{\frac{1}{n+1}}, \tag{16}$$

where $f(\eta)$ and $g(\eta)$ are velocities on x and y directions, η is similarity variable, which is expressed as

$$\eta = z\left(\frac{1+(2n-1)U^{2-n}}{n(n+1)\nu x}\right)^{\frac{1}{n+1}} \tag{17}$$

by substituting in to system (12-13) we obtain a non-newtonian stagnation boundary-layer equations are.

$$\begin{aligned}\frac{\lambda}{\phi^{n-3}}\frac{2}{n+1}\left(f''^2+\alpha^2 g''^2\right)^{\frac{n-1}{2}} f''' + \frac{2\,(n-1)}{n+1} f''\left(f''^2+\alpha^2 g''^2\right)^{\frac{n-3}{2}}\left(f'' f''' + \alpha^2 g'' g'''\right)\\ + \frac{2n}{n+1} f''\,(f+\alpha g) + \left(1-f'^2\right) + \alpha\left(1-f'g'\right) + \omega\phi^2(f'-1) = 0.\end{aligned} \tag{18}$$

$$\begin{aligned}\frac{\lambda}{\phi^{n-3}}\frac{2}{n+1}\left(f''^2+\alpha^2 g''^2\right)^{\frac{n-1}{2}} g''' + \frac{2\,(n-1)}{n+1} g''\left(f''^2+\alpha^2 g''^2\right)^{\frac{n-3}{2}}\left(f'' f''' + \alpha^2 g'' g'''\right)\\ + \frac{2n}{n+1} f''\,(f+\alpha g) + \alpha\left(1-g'^2\right) + \left(1-f'g'\right) - \omega\phi^2(g'-1) = 0.\end{aligned} \tag{19}$$

with boundary conditions.

$$\begin{aligned}(f,g)\,(0) &= 0,\\ (f',g')(0) &= (\lambda_1,\lambda_2),\\ (f',g')\,(\infty) &= 1\end{aligned} \tag{20}$$

where prime represent derivation w.r.t η similarity variable and $f(\eta)$, $g(\eta)$ are non-dimensional functions. The parameter $\lambda = \frac{\mu_{eff}}{\mu}$: viscosity ratio or effective viscosity, which is define as ratio of porous viscosity to fluid viscosity, ϕ: is a porosity of porous medium, n is power-law index, for $n = 1$ it's a case of Newtonian behavior. When $n < 1$ and $n > 1$, the fluid is shear thinning and shear thickening case, $\alpha = \frac{V_\infty}{U_\infty}$: ratio of shear to strain constant, hence it is known as shear-to-strain-rate parameter also it is called as three-dimensionality parameter because, based on this parameter, the above system (18-19) is classified as for $\alpha = 0$ the system reduce to classical two-dimensional boundary-layer equation, for $\alpha = 1$, the system (18-19) corresponds to the axisymmetric flow and $\omega = \frac{\nu}{k\,U_\infty}$: is permeability of porous media. Further, $\lambda_1 = \frac{U_w}{U_\infty}$ and $\lambda_2 = \frac{V_w}{V_\infty}$ are the ratios of main-stream velocities to surface velocities. Note that $\lambda_{1,2} = 0$ is the case for flow over on fixed surface, whereas $\lambda_{1,2} > 0$ and $\lambda_{1,2} < 0$ is the case for moving surface in the same and reverse direction to the flow direction. In present studies we discussed both the case on the system. The system (18-19) represent, steady three-dimensional boundary-layer stagnation point flow of a non-newtonian fluid by a power-law model on porous media with important physical parameter. These boundary-value system along with end conditions (20) are solved numerically by the shooting technique with a suitable initial guess for $f''(0)$ and $g''(0)$ to match the corresponding end boundary conditions at $f'(\infty)$ and $g'(\infty)$ respectively. The detailed procedure of numerical method is described in the further section.

3. Numerical Procedure

Analytical results are difficult to obtain for the above model because of highly non-linearity and coupled boundary value problem. Thus, we use numerical method to solve the above system. There are several methods in numerical techniques to solve BVP. In the present problem we use shooting analysis to solve the above equations. On applying shooting technique we obtain results for the all value of physical parameters. In this technique we reduce the higher order differential equations into system of first order equations, by assuming addition variable. Here the solution of non-linear problem is the combination of the solution to six initial value problems. We approximate the solution of the BVP by using the results to a sequence of IVP.

In the shooting technique, the unspecified beginning conditions are guessed to the set of differential equations with the initial conditions integrated numerically as an IVP (which is solved by Runge-Kutta fourth order method) to the mentioned terminal point. The accuracy of the considered beginning conditions verified by analogizing the evaluated value of the unknown variable at the point with the mentioned value. If variation exists, then improvise the values of the initial state by the repeating the process by replacing the obtained updated value. The computational code of shooting method is done by Matlab. A step size of $\Delta\eta = 0.001$ was selected to be satisfactory for convergence criterion of 10^{-8}. From the process of numerical computations the local skin-friction co-efficient $f''(0)$, $g''(0)$ are calculated and the numerical values of velocity profile are presented through graphically.

The solution procedure of the above system with boundary condition is described in detailed below. Coupled non-linear differential equations along with the boundary conditions are solved numerically using a Runge-Kutta method with shooting. Since the systems are heavy non-linear, a suitable treatment can be from numerical technique. Although the problem is a BVP, it is converted in to an IVP. In this technique the higher order differential equations (DE) are converted by introducing some additional unknown functions in to system of first order DE. The BVP in (18-19) with extreme conditions (20) is reconsider in the form of first order system of DE, which are taken the form of,

$$f' = E: \quad f^{''} = E' = F, \quad f^{'''} = E^{''} = F', \tag{21}$$

$$g' = G: \quad g^{''} = G' = H, \quad g^{'''} = G^{''} = H', \tag{22}$$

from equation (18) we get F' as

$$F' = \frac{-\frac{2(n-1)}{n+1}\alpha^2 FHH'(F^2+\alpha^2H^2)^{\frac{n-3}{2}} - RHS1}{[\frac{\lambda}{\phi^{n-3}}\frac{2}{n+1}(F^2+\alpha^2H^2)^{\frac{n-1}{2}} + \frac{2(n-1)}{n+1}F^2(F^2+\alpha^2H^2)^{\frac{n-3}{2}}]} \tag{23}$$

Similarity H' from (19)

$$H' = \frac{-\frac{2(n-1)}{n+1}HFF'(F^2+\alpha^2H^2)^{\frac{n-3}{2}} - RHS2}{[\frac{\lambda}{\phi^{n-3}}\frac{2}{n+1}(F^2+\alpha^2H^2)^{\frac{n-1}{2}} + \frac{2(n-1)}{n+1}\alpha^2H^2(F^2+\alpha^2H^2)^{\frac{n-3}{2}}]} \tag{24}$$

Sub (18) in (19) and simplify for F', we get

$$F' = \frac{\frac{2(n-1)}{n+1}\alpha^2 FH(F^2+\alpha^2H^2)^{\frac{n-3}{2}}RHS2 - RHS1[\frac{\lambda}{\phi^{n-3}}\frac{2}{n+1}(F^2+\alpha^2H^2)^{\frac{n-1}{2}} + \frac{2(n-1)}{n+1}\alpha^2H^2(F^2+\alpha^2H^2)^{\frac{n-3}{2}}]}{\frac{\lambda}{\phi^{n-3}}\frac{2}{n+1}(F^2+\alpha^2H^2)^{n-1}} \tag{25}$$

Again by re-substituting F' in (24) and simplify for H', which given as

$$H' = \frac{\frac{2(n-1)}{n+1}FH(F^2+\alpha^2H^2)^{\frac{n-3}{2}}RHS1 - RHS2[\frac{\lambda}{\phi^{n-3}}\frac{2}{n+1}(F^2+\alpha^2H^2)^{\frac{n-1}{2}} + \frac{2(n-1)}{n+1}F^2(F^2+\alpha^2H^2)^{\frac{n-3}{2}}]}{\frac{\lambda}{\phi^{n-3}}\frac{2}{n+1}(F^2+\alpha^2H^2)^{n-1}} \tag{26}$$

Where

$$RHS1 = \frac{2n}{n+1}f^{''}(f+\alpha g) + \left(1-E^2\right) + \alpha\left(1-GE\right) + \omega\phi^2(1-E) \tag{27}$$

$$RHS2 = \frac{2n}{n+1} g^{''}\left(f + \alpha g\right) + \left(1 - GE\right) + \alpha\left(1 - G^2\right) + \omega\phi^2(1 - G) \tag{28}$$

Therefore all six first order IVP with initial conditions are

$$\begin{aligned} at \quad f(0) = 0, \quad E(0) = \lambda_1, \quad g(0) = 0, \\ G(0) = \lambda_2, \quad F(0) = P_1, \quad H(0) = P_2 \end{aligned} \tag{29}$$

where P_1 and P_2 are assumed initial conditions, to solve the above system of ODE, we apply Runnge-Kutta fourth order method for solution. Since it is solution of IVP but our problem is BVP. To obtain solution for BVP, we match the value approximate initial guess with corresponding boundary-condition at $f'(\eta)$ and $g'(\eta)$. Since the end boundary condition is unbounded (i.e. η at ∞), but computation should be bounded hence we apply the artificial η at a limited value and it has been chosen at each time, such that the velocity profiles approach asymptotically to one. With the guessed initial condition, the outer edge condition is not satisfy then update at each correction until the solution convergence and at each iteration the correction of initial condition is done by secant method. Some of the results of the system are discussed for ample scopes of parameter are stated in further segment.

4. Discussion of the Flow

We treated the boundary layer equations of non-newtonian fluid over a moving surface in three-dimensional flow with a permeable medium. By using Ostwald-de-waele model we find the special coupled partial differential equations from the conservation of momentum. We can reduce the equations to an ordinary differential systems via new set of stream function and similarity transformations. Therefore, the above system solve numerically by shooting technique using R-K method. This method gives solution of ODE's in term of IVP.

In figure 1, Numerical results for various non-newtonian Co-efficient 'n' are plotted for $f'(\eta)$ and $g'(\eta)$. Both velocities are plotted for $n = 0.8, 1.0, 1.2, 1.4, 1.6, 1.8$, when $n = 1$ corresponds to Newtonian flow and $n = 0.8$ is for shear thinning $n = 1.2, 1.4, 1.6, 1.8$ are corresponds to shear thickening. An increase in 'n' represents an increase in the non-newtonian behavior, figure 1 conclude that the boundary layer thickness increases as the non-newtonian effects increase. Actually, with an increase in non-newtonian parameter it produces resistance in the fluid flow.

Figure 2a & 2b shows the influence of three-dimensional parameter on both velocities. In figure 2a the dimensionless velocity components at the wall reduces with increase in Shear-to-Strain parameters and hence there will be a decrease in the boundary-layer thickness. The velocity profiles damped out for the high amount of 'α' because of an interception which exhibits among them.

Figure 3& figure 4 depict the effect of the wall velocity parameter $\lambda_{1,2}$ on velocity. Actually when $\lambda_2 = 0$ & $\lambda_1 = 0$ the figure 3b & figure 4a shows not much variation on boundary-layer. Because when $\lambda_2 = 0$ & λ_1 varies the x-direction surface move with the speed of $\lambda_1 = \frac{U_w}{U_\infty}$ only a mild effect applied on y-direction surface, so the figure 3b i.e, velocity profiles $g'(\eta)$ shows no variations. Similarly, when λ_2 vary & $\lambda_1 = 0$ figure 4a also shows very less variation on different λ_2, but compare to figure 3b, figure 4a i.e, velocity graph of $f'(\eta)$ is influced more by y-direction surface velocity. For both variations of λ_1 & λ_2, the velocities f' & g' is found to be maximized closer to the boundary-layer & minimized the thickness of layer i.e, the flow is accelerated with increase wall velocities.

Further, the figure 5,6,7 depict the porous medium effects on velocities of boundary-layer. In consistency with the earlier graphs described same variation for velocities evolutions, figure 5 also described same variation of velocities. But in figure 6 & figure 7 i.e, effects of effective velocity and porosity shows entirely operative nature. The effects of the 'w' (i.e, permeability of porous media) on the profiles $f'(\eta)$ & $g'(\eta)$ are presented in fig 5a & 5b. The fare-field boundary-conditions are satisfied asymptotically, which supports the numerical results obtained. As 'w' increases it accelerate the fluid flow & boundary layer gets thicker.

The effects of porosity (π) & effective viscosity ratio (λ) on the primitive flow variables of velocity f' &g' is shown in fig 6ure & 7, effective viscosity signifies the ratio of porous velocity to fluid velocity . An increase in 'λ' from 1 through 2,3,4,5 strongly depresses velocities in the region. For $\lambda = 1$ the porous viscosity is equal to fluid viscosity i.e, the velocity boundary–layer thickness, with increasing 'n' values, velocity as shown in figure 6a & 6b is markedly reduced throughout the boundary-layer because it react the fluid flow.

Similarly ,fig 7 shows same behavior of profiles inches the effects of porosity. As porosity increases it pressure the velocity in the medium that resist the fluid to flow in the in the region. For increase the thickness of layer.

In this study, numerical solution have been presented for flow on both $x-$ & $y-$ direction of non-newtonian fluid in a permeable medium with moving boundary-layer wall. The effects of the governing parameter on the flow have been analyzed through graphically & presented for each parameter in this section.

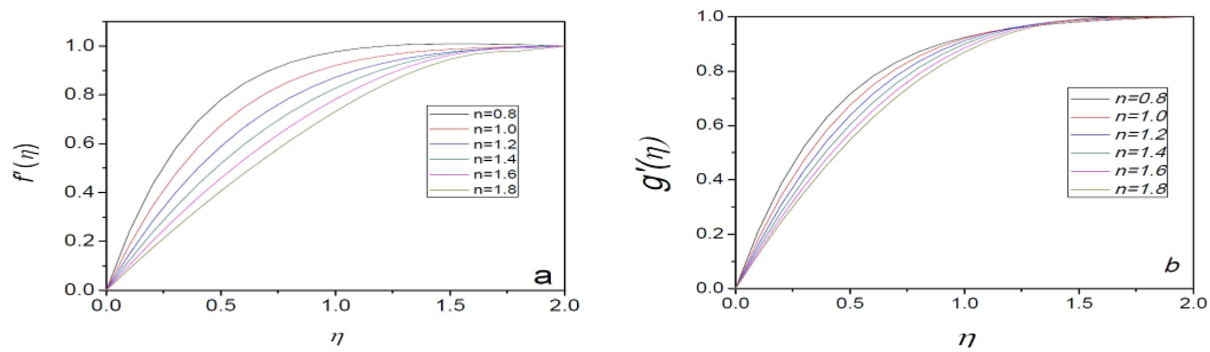

Figure 1. **Plot of different velocity profile with respect to η on modification of n**

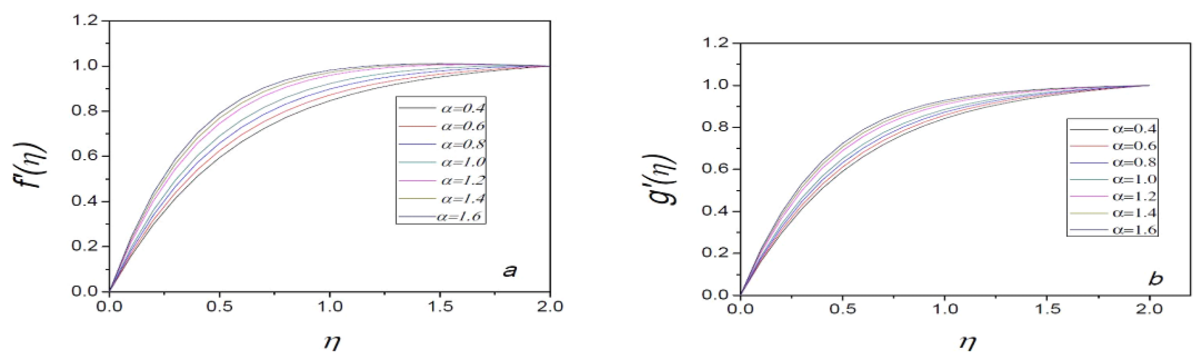

Figure 2. **Plot of different velocity profile with respect to η on modification of α**

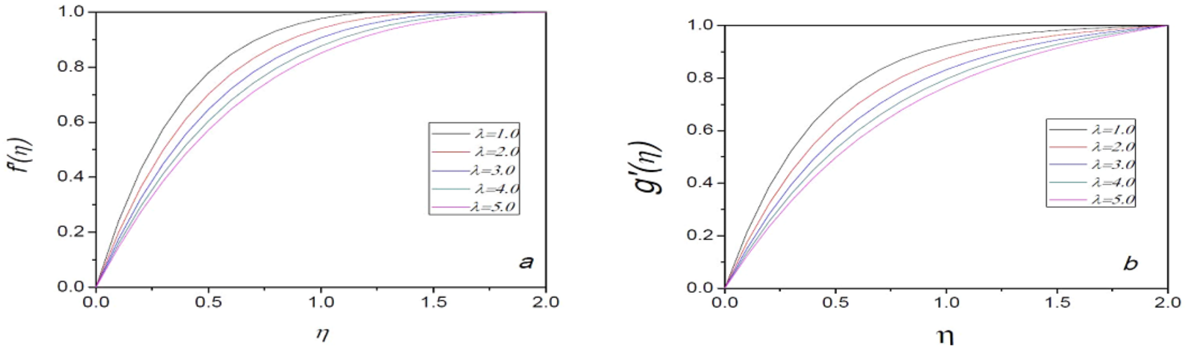

Figure 3. **Plot of different velocity profile with respect to η on modification of λ**

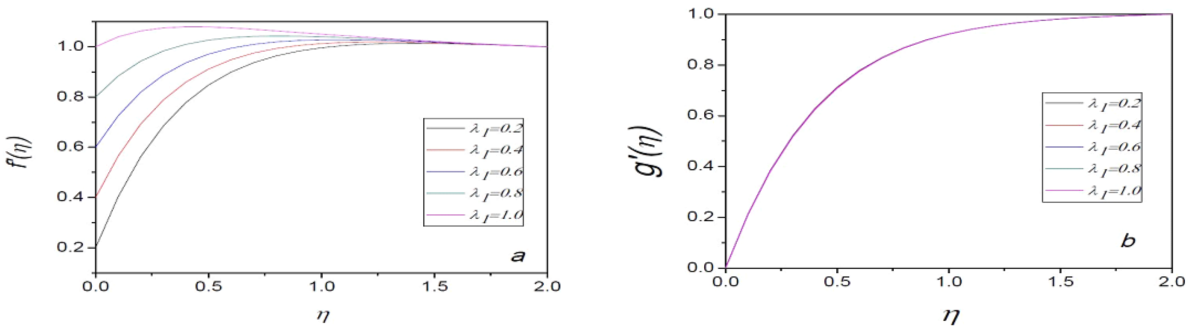

Figure 4. **Plot of different velocity profile with respect to η on modification of λ_1 fixed $\lambda_2 = 0.0$**

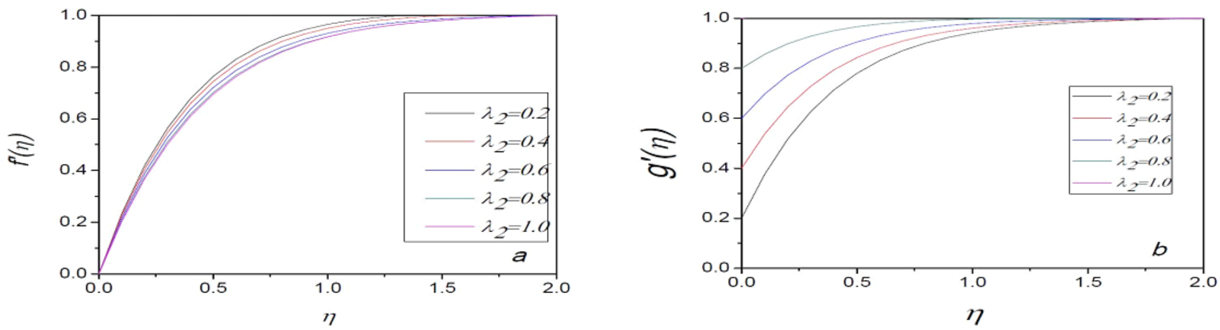

Figure 5. **Plot of different velocity profile with respect to η on modification of λ_2 fixed $\lambda_1 = 0.0$**

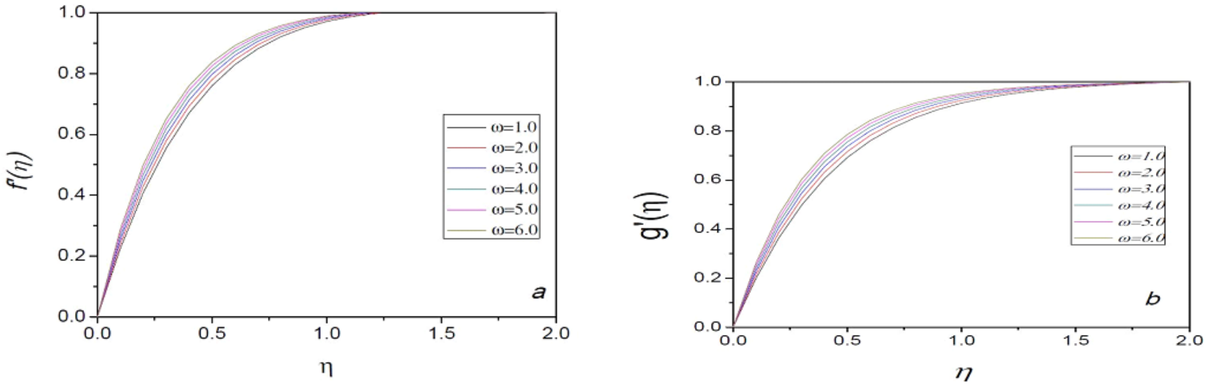

Figure 6. **Plot of different velocity profile with respect to η on modification of ω**

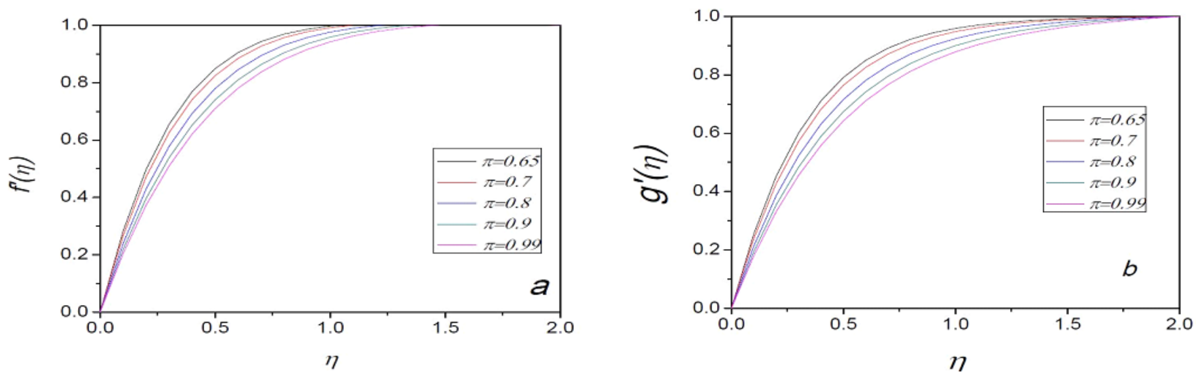

Figure 7. **Plot of different velocity profile with respect to η on modification of π**

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