International Journal of Mathematics And its Applications
Volume 4, Issue 2–D (2016), 125–130.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

# Edge Domination in Shadow Distance Graphs

Research Article

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**Abstract:** Let $G$ be a simple connected and undirected graph. The shadow graph of $G$, denoted $D_2(G)$ is the graph constructed from $G$ by taking two copies of $G$ namely $G$ itself and $G^{'}$ and by joining each vertex $u$ in $G$ to the neighbors of the corresponding vertex $u^{'}$ in $G^{'}$. Let $D$ be the set of all distances between distinct pairs of vertices in $G$ and let $D_s$ (called the distance set) be a subset of $D$. The distance graph of $G$ denoted by $D(G, D_s)$ is the graph having the same vertex set as that of $G$ and two vertices $u$ and $v$ are adjacent in $D(G, D_s)$ whenever $d(u, v) \in D_s$. In this paper, we define a new graph called the shadow distance graph and determine the edge domination number of the shadow distance graph of the path graph, the cycle graph and the sunlet graph with specified distance sets.

**MSC:** 05C69.

**Keywords:** Dominating set, Edge domination number, Minimal edge dominating set, Shadow distance graph.



## 1. Introduction

The theory of domination in graphs is of vast interest to researchers in graph theory due to its many and varied applications in fields such as optimization, linear algebra, design and analysis of communication networks, military surveillance and social sciences.

By a graph $G = (V, E)$ we mean a finite undirected graph without loops and multiple edges. Throughout this paper, $P_n$ and $C_n$ will denote the path graph and cycle graph with $n$ vertices respectively. A subset $S$ of $V$ is called a dominating set of $G$ if every vertex not in $S$ is adjacent to some vertex in $S$. The domination number of $G$ denoted by $\gamma(G)$ is the minimal cardinality taken over all dominating sets of $G$. A subset $F$ of $E$ is called an edge dominating set if each edge in $E$ is either in $F$ or is adjacent to an edge in $F$. An edge dominating set $F$ is called minimal if no proper subset of $F$ is an edge dominating set. The edge domination number of $G$ denoted by $\gamma^{'}(G)$ is the minimum cardinality taken over all edge dominating sets of $G$.

The concept of edge domination in graphs was introduced by Mitchell and Hedetniemi [2]. Arumugam and Velammal[3] have characterized connected graphs for which $\gamma^{'}(G) = \lfloor \frac{V}{2} \rfloor$ and in [4], Armugam and Jerry have studied edge domination and fractional edge domination in graphs. In [2], Vaidya and Pandit have discussed edge domination in some path and cycle related graphs and have determined the edge domination number for shadow graphs, middle graph, and total graphs of paths and cycles.

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The open neighbourhood of an edge $e \in E$ denoted by $N(e)$ is the set of all edges adjacent to $e$ in $G$. If $e = (u, v)$ is an edge in $G$, the degree of $e$ denoted by $deg(e)$ is defined as $deg(e) = deg(u) + deg(v) - 2$. The maximum degree of an edge in $G$ is denoted by $\triangle^{'}(G)$. The shadow graph of $G$, denoted by $D_2(G)$ is the graph constructed from $G$ by taking two copies of $G$ namely $G$ itself and $G^{'}$ and by joining each vertex $u$ in $G$ to the neighbors of the corresponding vertex $u^{'}$ in $G^{'}$. Let $D$ be the set of all distances between distinct pairs of vertices in $G$ and let $D_s$ (called the distance set) be a subset of $D$. The distance graph of $G$ denoted by $D(G, D_s)$ is the graph having the same vertex set as that of $G$ and two vertices $u$ and $v$ are adjacent in $D(G, D_s)$ whenever $d(u, v) \in D_s$. The shadow distance graph of $G$, denoted by $D_{sd}(G, D_s)$ is constructed from $G$ with the following conditions:

i) consider two copies of $G$ say $G$ itself and $G^{'}$

ii) if $u \in V(G)$ (first copy) then we denote the corresponding vertex as $u^{'} \in V(G^{'})$ (second copy)

iii) the vertex set of $D_{sd}(G, D_s)$ is $V(G) \cup V(G^{'})$

iv) the edge set of $D_{sd}(G, D_s)$ is $E(G) \cup E(G^{'}) \cup E_{ds}$ where $E_{ds}$ is the set of all edges between two distinct vertices $u \in V(G)$ and $v^{'} \in V(G^{'})$ that satisfy the condition $d(u, v) \in D_s$ in $G$.

The shadow distance graphs $D_{sd}(P_5, \{2\})$ and $D_{sd}(C_6, \{3\})$ are illustrated in figure 1 and figure 2 respectively.

The n-sunlet graph denoted by $S_n$ is the graph on $2n$ vertices obtained by attaching $n-$pendant edges to each of the

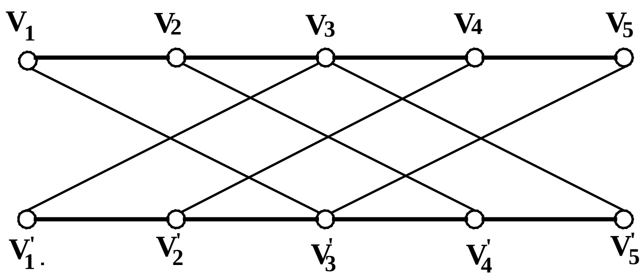


**Figure 1.** **The graph** $D_{sd}(P_5, \{2\})$

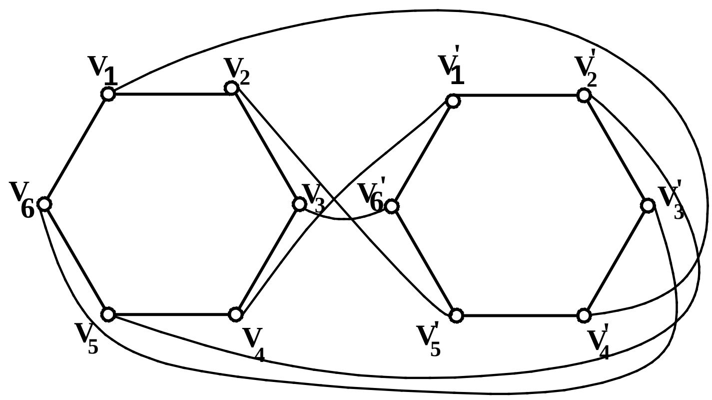


**Figure 2.** **The graph** $D_{sd}(C_6, \{3\})$

vertices of the cycle graph $C_n$. In the next section we determine the edge domination number of the shadow distance graph of the path graph, the cycle graph and the sunlet graph with $D_s = \{2\} and \{3\}$ respectively.

## 2. Main Results

We recall the following results related to the edge domination number of a graph.

**Theorem 2.1** ([5]). *An edge dominating set $F$ is minimal if and only if for each edge $e \in G$, one of the following two conditions holds:*

*i)* $N(e) \cap F = \phi$

*ii) there exists an edge* $e \in E - F$ *such that* $N(e) \cap F = \{e\}$.

**Theorem 2.2** ([6]). $\gamma^{'}(C_n) = \lceil \frac{n}{3} \rceil$ *for* $n \geq 3$.

We begin our results with the distance shadow graph associated with the path $P_n$.

**Theorem 2.3.** *For* $n \geq 3$, $\gamma^{'}(D_{sd}\{P_n, \{2\}\}) = 2\lceil \frac{n-2}{2} \rceil$.

*Proof.* Consider two copies of $P_n$, one $P_n$ itself and the other denoted by $P_n^{'}$. Let $v_1, v_2, \ldots, v_n$ be the vertices of $P_n$ and let $v_1^{'}, v_2^{'}, \ldots, v_n^{'}$ be the vertices of $P_n^{'}$. Let $e_1, e_2, \ldots, e_{n-1}$ be the edges of the first copy $P_n$ and $e_1^{'}, e_2^{'}, \ldots, e_{n-1}^{'}$ be the edges of the second copy $P_n^{'}$, where $e_i = (v_i, v_{i+1})$, $e_i^{'} = (v_i^{'}, v_{i+1}^{'})$ for $i = 1, 2, \ldots, n-1$. Let $G = (D_{sd}\{P_n, \{2\}\})$. Then $|V(G)| = 2n$, $|E(G)| = 4n - 6$ and $E(G) = \{e_i\} \cup \{e_i^{'}\} \cup \{e_{(j),(j+2)'}\} \cup \{e_{(k-2)',(k)}\}$ where $1 \leqslant i \leqslant n-1$, $1 \leqslant j \leqslant n-2$, $3 \leqslant k \leqslant n$.

For $n = 3$, since $G \cong C_6$ it follows that $\gamma^{'}(G) = 2$ (Theorem 2.2). For $n = 4$ and 5, it is easy to verify that the sets $F = \{e_2, e_2^{'}\}$ and $F = \{e_2, e_2^{'}, e_4, e_4^{'}\}$ are the respective minimal edge dominating sets with minimum cardinality and hence $\gamma^{'}(G) = 2 = 2\lceil \frac{n-2}{2} \rceil$ for $n = 4$ and $\gamma^{'}(G) = 4 = 2\lceil \frac{n-2}{2} \rceil$ for $n = 5$ respectively.

Let $n \geqslant 6$. Consider the set $F = \{e_2, e_2^{'}, e_4, e_4^{'}, ...., e_{2i+2}, e_{2i+2}^{'}\}$ for each $i$ such that $0 \leqslant i \leqslant \lceil \dfrac{n-4}{2} \rceil$. Then, clearly, $|F| = 2\lceil \frac{n-2}{2} \rceil$.

Now, each edge of $G$ is either an element of $F$ or is adjacent to some element in $F$. Hence $F$ is an edge dominating set. Also, this set $F$ is a minimal edge dominating set since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in $G$ and moreover,

$$N(e_{n-1}) = \{e_{n-2}, e_{(n-3)'(n-1)}, e_{(n-2)'(n)}\}$$
$$N(e_{n-1}^{'}) = \{e_{n-2}^{'}, e_{(n-3)(n-1)'}^{'}, e_{(n-2)(n)'}^{'}\}$$

and for all other $e_i$ such that $e_i \in F$,

$$N(e_i) = \{e_{i-1}, e_{i+1}, e_{(i-2)'(i)}, e_{(i+2)'(i)}, e_{(i+1)(i-1)'}, e_{(i+1)(i+3)'}\}$$
$$N(e_i^{'}) = \{e_{i-1}^{'}, e_{i+1}^{'}, e_{(i-2)(i)'}^{'}, e_{(i+2)(i)'}^{'}, e_{(i-1)(i+1)'}^{'}, e_{(i+3)(i+1)'}^{'}\}$$

Hence, $N(e_i) \cap F = \phi$. Further, $\triangle^{'}(G) = 6$ which implies that an edge of $G$ can dominate atmost seven distinct edges including itself. But the graph $G$ is such that there are 4 edges of degree 3, 4 edges of degree 4, 8 edges of degree 5 and $2(2n - 11)$ edges of degree 6. Hence atmost $2(2n - 11)$ distinct edges of $G$ can dominate seven distinct edges including itself and each of the remaining edges can dominate less than 6 edges of $G$. Hence, any set containing the edges less that in $F$ cannot be an edge dominating set of $G$.

This implies that the set $F$ described above is of minimum cardinality and since $|F| = 2\lceil \frac{n-2}{2} \rceil$, it follows that $\gamma^{'}(G) = 2\lceil \frac{n-2}{2} \rceil$. □

**Theorem 2.4.**

$$\gamma^{'}(D_{sd}\{P_n, \{3\}\}) = \begin{cases} 3, & n = 4 \\ 2\lceil \frac{n-2}{2} \rceil, & n \geq 5 \end{cases}$$

*Proof.* Let $G = (D_{sd}\{P_n, \{3\}\})$. We consider the vertex set of $G$ as in Theorem 2.3 and $E(G) = \{e_i\} \cup \{e_i^{'}\} \cup \{e_{(j),(j+3)'}\} \cup \{e_{(k-3)',(k)}\}$ where $1 \leqslant i \leqslant n-1$, $1 \leqslant j \leqslant n-3$, $4 \leqslant k \leqslant n$. Clearly, $|V(G)| = 2n$, $|E(G)| = 4n-8$.

For $n = 4$, since $G \cong C_8$, it follows that $\gamma^{'}(G) = 3$ (Theorem 2.2). For $n = 5, 6$ and $n = 7, 8$ the sets $F = \{e_{2i+2}, e_{2i+2}^{'}\}$, for $0 \leqslant i \leqslant 1$ and $0 \leqslant i \leqslant 2$ respectively are minimal edge dominating sets with minimum cardinality and hence $\gamma^{'}(G) = 2\lceil \frac{n-2}{2} \rceil$.

Let $n \geqslant 9$. Consider the set $F = \{e_2, e_2^{'}, e_4, e_4^{'}, ...., e_{2i+2}, e_{2i+2}^{'}\}$ for each $i$ such that $0 \leqslant i \leqslant \lceil \dfrac{n-4}{2} \rceil$. Then, clearly, $|F| = 2\lceil \frac{n-2}{2} \rceil$.

Now, each edge of $G$ is either an element of $F$ or is adjacent to some element in $F$. Hence $F$ is an edge dominating set. Also this set $F$ is a minimal edge dominating set since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in $G$ and, moreover,

$$N(e_{n-1}) = \{e_{n-2}, e_{(n-4)'(n-1)}, e_{(n-3)'(n)}\}$$
$$N(e_{n-1}^{'}) = \{e_{n-2}^{'}, e_{(n-4)(n-1)'}^{'}, e_{(n-3)(n)'}^{'}\}$$

and for all other $e_i$ such that $e_i \in F$

$$N(e_i) = \{e_{i-1}, e_{i+1}, e_{(i-3)'(i)}, e_{(i+3)'(i)}, e_{(i+1)(i-2)'}, e_{(i+1)(i+4)'}\}$$
$$N(e_i^{'}) = \{e_{i-1}^{'}, e_{i+1}^{'}, e_{(i-3)(i)'}^{'}, e_{(i+3)(i)'}^{'}, e_{(i-2)(i+1)'}^{'}, e_{(i+4)(i+1)'}^{'}\}$$

Hence, $N(e_i) \cap F = \phi$. Further, $\triangle^{'}(G) = 6$ which implies that an edge of $G$ can dominate at most seven distinct edges including itself. But the graph $G$ is such that there are 4 edges of degree 3, 8 edges of degree 4, 12 edges of degree 5 and $2(2n-16)$ edges of degree 6. Hence atmost $2(2n-16)$ distinct edges of $G$ can dominate seven distinct edges including itself and each of the remaining edges can dominate less than 6 edges of $G$. Hence, any set containing the edges less that in $F$ cannot be an edge dominating set of $G$. This implies that the set $F$ described above is of minimum cardinality and since $|F| = 2\lceil \frac{n-2}{2} \rceil$, it follows that $\gamma^{'}(G) = 2\lceil \frac{n-2}{2} \rceil$. □

For the cycle graph $C_n$, we have the following results.

**Theorem 2.5.** *For $n \geqq 4$,*

$$\gamma^{'}(D_{sd}\{c_n, \{2\}\}) = \begin{cases} 2\lceil \frac{n-1}{3} \rceil, & \textit{if } n \equiv 0 \textit{ or } 2 \ (mod3) \\ 2\lceil \frac{n+1}{3} \rceil, & \textit{otherwise} \end{cases}$$

*Proof.* Consider two copies of $C_n$, one $C_n$ itself and the other denoted by $C_n^{'}$. Let $v_1, v_2, \ldots, v_n$ be the vertices of $C_n$ and $v_1^{'}, v_2^{'}, \ldots, v_n^{'}$ be the vertices of $C_n^{'}$. Let $e_1, e_2, \ldots, e_n$ be the edges of the first copy $C_n$ and $e_1^{'}, e_2^{'}, \ldots, e_n^{'}$ be the edges of the second copy $C_n^{'}$. Where $e_i = (v_i, v_{i+1})$ and $e_i^{'} = (v_i^{'}, v_{i+1}^{'})$ for $i = 1, 2, \ldots, n$, where computation is under modulo $n$. Let $G = (D_{sd}\{C_n, \{2\}\})$. Then, for $n = 4$, $|V(G)| = 8$ and $|E(G)| = 12$ and the set $F = \{e_2, e_4, e_2^{'}, e_4^{'}\}$ is the minimal dominating set with minimum cardinality and hence $\gamma'(G) = 4$.

Let $n \geqq 5$. Then $|V(G)| = 2n$, $|E(G)| = 4n$ and $E(G) = \{e_i\} \cup \{e_i^{'}\} \cup \{e_{(j),(j+2)'}\} \cup \{e_{(j)',(j+2)}\} \cup \{e_{(k),(k+n-2)'}\} \cup \{e_{(k+n-2),(k)'}\}$ where $1 \leqslant i \leqslant n$, $1 \leqslant j \leqslant n-2$, $1 \leqslant k \leqslant 2$. Consider the set

$$F = \begin{cases} \{e_2, e_5, \ldots, e_{3i+2}, e'_2, e'_5, \ldots, e'_{3i+2}\}, & \text{if } n \equiv 0 \text{ or } 2 \ (mod3) \\ \{e_2, e_5, \ldots, e_{3i+2}, e'_2, e'_5, \ldots, e'_{3i+2}\} \cup \{e_n, e'_n\}, & \text{otherwise.} \end{cases}$$

where $0 \leqslant i \leqslant \lfloor \frac{n-2}{3} \rfloor$. Clearly, $|F| = 2\lceil \frac{n-1}{3} \rceil$ for $n \equiv 0$ or 2 $(mod3)$ and $|F| = 2\lceil \frac{n+1}{3} \rceil$ otherwise.
This set $F$ is a minimal edge dominating set with minimum cardinality since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in $G$. Hence, any set containing edges less than that of $F$ cannot be a dominating set of $G$. Also $G$ is regular of degree 4 and each edge of $G$ is of degree 6 and an edge of $G$ can dominate atmost seven distinct edges of $G$ including itself. □

**Theorem 2.6.** *For $n \geqq 6$, $\gamma^{'}(D_{sd}\{C_n, \{3\}\}) = 2\lceil \frac{n-1}{2} \rceil$.*

*Proof.* Let $G = (D_{sd}\{C_n, \{3\}\})$. The vertex set of $G$ is as in Theorem 2.5. For $n = 6$, $|V(G)| = 12$ and $|E(G)| = 18$ and the edge set is $E(G) = \{e_i\} \cup \{e_i^{'}\} \cup \{e_{(j),(j+3)'}\} \cup \{e_{(j)',(j+3)}\}$ where $1 \leqslant i \leqslant n$, $1 \leqslant j \leqslant 3$ and the set $F = \{e_2, e_4, e_6, e_2^{'}, e_4^{'}, e_6^{'}\}$ is a minimal edge dominating set with minimum cardinality. Hence $\gamma^{'}(G) = 6$.
For $n = 7$, $|V(G)| = 14$ and $|E(G)| = 28$ and the edge set is $E(G) = \{e_i\} \cup \{e_i^{'}\} \cup \{e_{(j),(j+3))'}\} \cup \{e_{(k),(k+(n-3)))'}\} \cup \{e_{(j)',(j+3)}\} \cup \{e_{(k)',(k+(n-3))}\}$ where $1 \leqslant i \leqslant n$, $1 \leqslant j \leqslant 4$, $1 \leqslant k \leqslant 3$ and the set $F = \{e_2, e_5, e_7, e_2^{'}, e_5^{'}, e_7^{'}\}$ is a minimal edge dominating set with minimum cardinality. Hence $\gamma^{'}(G) = 6$.
Let $n \geqq 8$. Then $|V(G)| = 2n$ and $|E(G)| = 4n$ and the edge set is $E(G) = \{e_i\} \cup \{e_i^{'}\} \cup \{e_{(j),(j+3))'}\} \cup \{e_{(k),(k+n-3))'}\} \cup \{e_{(j)',(j+3)}\} \cup \{e_{(k)',(k+n-3)}\}$, where $1 \leqslant i \leqslant n$, $1 \leqslant j \leqslant n-3$, $1 \leqslant k \leqslant 3$. Consider the set

$$F = \begin{cases} \{e_2, e_4, ....., e_{2i}, e_2^{'}, e_4^{'}, ....., e_{2i}^{'}\}, & 0 \leqslant i \leqslant \frac{n}{2} \text{ (n is even)} \\ \{e_2\} \cup \{e_2^{'}\} \cup \{e_5, e_7, e_9....., e_{2j+3}, e_5^{'}, e_7^{'}, ....., e_{2j+3}^{'}\} \cup \{e_n, e_n^{'}\}, & 0 \leqslant i \leqslant \lceil \frac{n-1}{3} \rceil \text{ (n is odd).} \end{cases}$$

Clearly $|F| = 2\lceil \frac{n-1}{2} \rceil$. This set $F$ is a minimal edge dominating set since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in $G$. Hence any set containing edges less than that of $F$ cannot be a dominating set of $G$. Also $G$ is regular of degree 4 and each of its edges has degree 6 and therefore an edge of $G$ can dominate atmost seven distinct edges of $G$ including itself. Hence the above set $F$ is of minimum cardinality and therefore $\gamma^{'}(G) = 2\lceil \frac{n-1}{2} \rceil$. □

For the sunlet graph $S_n$, we have the following results.

**Theorem 2.7.** *For $n \geqq 3$, $\gamma^{'}(D_{sd}\{S_n, \{2\}\}) = 2\lfloor \frac{n+1}{2} \rfloor$.*

*Proof.* Consider two copies of $S_n$ namely $S_n$ itself and $S_n{'}$. In the first copy $S_n$, let $(v_1)_1, (v_2)_1, \ldots, (v_n)_1$ be the vertices of the cycle, $(v_1)_1^{'}, (v_2)_1^{'}, \ldots, (v_n)_1^{'}$ be the pendant vertices, let the edges of the cycle be $e_i = ((v_1)_i, (v_1)_{i+1})$, $i = 1, 2, \ldots, n$ where computation is under modulo $n$ and let the pendant edges be $e_{p_i} = ((v_i)_1, (v_i)_1^{'})$ where $i = 1, 2, \ldots, n$. In the second copy, let $(v_1)_2, (v_2)_2, \ldots, (v_n)_2$ be the vertices of the cycle, $(v_1)_2^{'}, (v_2)_2^{'}, \ldots, (v_n)_2^{'}$ be the pendant vertices, let the edges of the cycle be $e_i^{'} = ((v_2)_i^{'}, (v_2)_{i+1}^{'})$, $i = 1, 2, \ldots, n$ where computation is under modulo $n$ and let the pendant edges be $e_{p_i}^{'} = ((v_i)_2^{'}, (v_i)_2^{'})$ where $i = 1, 2, \ldots, n$. Let $G = (D_{sd}\{S_n, \{2\}\})$.
For $n = 3$, the set $F = \{e_1, e_{p_3}, e_1^{'}, e_{p_3}^{'}\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G) = 4$.
Let $n \geq 4$. Consider the set $F = \{e_1, e_3, e_5, \ldots, e_{2i+1}, e_1^{'}, e_3^{'}, e_5^{'}, \ldots, e_{2i+1}^{'}\}$ Where $0 \leq i \leq \lfloor \frac{n-1}{2} \rfloor$.
This set $F$ is a Minimal edge dominating set since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in $G$. Hence any set containing edges less than that of $F$ cannot be a dominating set of $G$. Further, $\triangle^{'}(G) = 12$ which implies that an edge of $G$ can dominate atmost 13 distinct edges including itself. But the graph $G$ is such that there are $6n$ edges of degree 8 and $4n$ edges of degree 12. Hence atmost $4n$ distinct edges of $G$ can dominate 13 distinct edges including itself and each of the remaining edges can dominate less than 12 edges of $G$. Therefore, any set containing the edges less that in $F$ can not be an edge dominating set of $G$. This implies that the set $F$ described above is of minimum cardinality and therefore $\gamma^{'}(D_{sd}\{S_n, \{2\}\}) = 2\lfloor \frac{n+1}{2} \rfloor$. □

**Theorem 2.8.** *For $n \geqq 3$, $\gamma^{'}(D_{sd}\{S_n, \{3\}\}) = 2\lceil \frac{2n-1}{2} \rceil$.*

*Proof.* Let $G$=($D_{sd}\{S_n, \{3\}\}$),The vertex set of $G$ is as in Theorem 2.7. For $n$= 3, the set $F = \{e_{p_1}, e_{p_2}, e_{p_3}, e^{'}_{p_1}, e^{'}_{p_2}, e^{'}_{p_3}\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G) = 6$.

For $n$= 4, the set $F = \{e_{p_1}, e_{p_2}, e_{p_3}, e_{p_4}, e^{'}_{p_1}, e^{'}_{p_2}, e^{'}_{p_3}, e^{'}_{p_4}\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G) = 8$. For $n$= 5, the set $F = \{e_{p_1}, e_{p_2}, e_{p_3}, e_{p_4}, e_{p_5}, e^{'}_{p_1}, e^{'}_{p_2}, e^{'}_{p_3}, e^{'}_{p_4}, e^{'}_{p_5}\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G) = 10$. For $n$= 6, the set $F = \{e_{p_1}, e_{p_2}, e_{p_3}, e_{p_4}, e_{p_5}, , e_{p_6}, e^{'}_{p_1}, e^{'}_{p_2}, e^{'}_{p_3}, e^{'}_{p_4}, e^{'}_{p_5}, e^{'}_{p_6}\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G) = 12$.

Let $n \geq 7$. Consider the set $F$ =$\{e_{p_1}, e_{p_2}, e_{p_3}......, e_i, e^{'}_{p_1}, e^{'}_{p_2}, e^{'}_{p_3}....., e^{'}_i\}$.where $1 \leq i \leq n$. This set $F$ is a minimal edge dominating set since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in $G$. Hence any set containing edges less than that of $F$ cannot be a dominating set of $G$. Further, $\triangle^{'}(G) = 12$ which implies that an edge of $G$ can dominate atmost 13 distinct edges including itself. But the graph $G$ is such that there are $2n$ edges of degree 8 , $6n$ edges of degree 10 and $4n$ edges of degree 12. Hence atmost $4n$ distinct edges of $G$ can dominate 13 distinct edges including itself and each of the remaining edges can dominate less than 12 edges of $G$. Therefore, any set containing the edges less that in $F$ cannot be an edge dominating set of $G$. This implies that the set $F$ described above is of minimum cardinality and therefore $\gamma^{'}(D_{sd}\{S_n, \{3\}\}) = 2\lceil \frac{2n-1}{2} \rceil$. □

# Acknowledgement

The first author is thankful to the Management and staff of the School of Physical Science and Computer Applications, REVA University, Bengaluru. The authors are also thankful to the Management and Research centre, Department of Mathematics, Dr. Ambedkar Institute of Technology, Bengaluru.

References

[1] S.K.Vaidya and R.M.Pandit, *Edge domination in some path and cycle related graphs*, ISRN Discrete Mathematics, 2014(2014), 1-5.

[2] S.T.Hedetniemi and R.C.Laskar, *Bibliography on domination in graphs and some basic definitions of domination parameters*, Discrete Mathematics, 6(13)(1990), 257277.

[3] S.Arumugam and S.Velammal, *Edge domination in graphs*, Taiwanese Journal of Mathematics, 2(2)(1998), 173179.

[4] S.Arumugam and S.Jerry, *Fractional edge domination in graphs*, Applicable Analysis and Discrete Mathematics, 3(2)(2009), 359370.

[5] V.R.Kulli, *Theory of domination in graphs*, Vishwa International Publications, (2013).

[6] S.R.Jayaram, *Line domination in graphs*, Graphs Combin., 3(1987), 357-363.