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# Models For Non - Newtonian Blood Flow Through An Elastic Artery

**Research Article**

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**Abstract:** Hydrodynamics of the arterial system, arterial wall elasticity are important factors for study of natural velocity of the blood (Mc Donald, 1974). Here we study effect of non-Newtonian nature of blood through an Elastic artery by considering power-law model and Herschel-Bulkley model. The elasticity of arterial wall on longitudinal velocity of blood is investigated for both the models of blood mentioned above. Based on our investigation, we discovered that the power law model's velocity profiles in the elastic artery outperform the Herchel-Bulkley model for fixed values of power index n and other parameters, and that the velocity of both fluids decreases as power index increases. Furthermore, we see that under linear transmular pressure, blood velocity drops as elasticity increases up to a transition point of unit value z; beyond this point, the outcome reverses. Every vessel's elasticity value causes the fluid's velocity to decrease downstream; nevertheless, if the vessel's elasticity is low, the downstream velocity from the transition point.

**Keywords:** Artery, Elasticity, Pulsatile flow, Non-Newtonian.


## 1. Introduction

An artery is elastic tube whose diameter changes in response to pulsating pressure. It also propagates pressure waves and flow waves caused by the heart's evacuation of blood at a specific velocity that is mostly regulated by the wall's elastic properties. Understanding the arterial wall's elastic constants, which allow one to calculate the "natural" velocity, is crucial for knowing the hydrodynamics of the artery system [1]. Numerous research pertaining to blood flow analysis have examined the behavior of an elastic tube filled with a viscous fluid subjected to pulsatile flow. The study of non-Newtonian fluids flow through elastic tubes reveals many information to understand bio-fluid dynamics proximal to real physiological conditions. The pulsaltile blood flow with stenosis artery was investigated by [2]. The pulsatile blood flow in an initially stressed anisotropic elastic tube was examined by [3]. They also talked about how pressure waves spread. Johnson [4] discuss a model of pulsatile flow in a uniform deformable vessel. The laminar non-Newtonian power-law fluid flow evolution in a pipe studied by [5]. The effects of the fluid's permeability factor and yield stress on the distribution of shear stress, wall shear stress, plug flow radius, flow rates, and frictional resistance were investigated by [6] in study of Casson fluid flow in a pipe filled with a homogeneous porous media. Pedrizzetti [7] investigated the pulsatile flow within a moderately elastic artery and found that wall elasticity affected both the flow and the erratic shear stress on the wall. Ahmad [8] investigated the Casson's fluid flow in an elastic tube that resembled an artery to determine how the elastic tube's character was affected by the shear stress and velocity field. By solving a generalized equation in conjunction with the Casson constitutive equation, they were able to determine the velocity distribution.

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A new coefficient of elasticity connected to the elastic state of the blood vessels was proposed by [9]. The non-Newtonian blood flow in the human right cornary arteries was investigated by [10]. They came to the conclusion that, although the Newtonian model of blood viscosity is a decent approximation in areas of mid-range to high shear, a better estimate of wall shear stress at low shear is best achieved by using the generalized power law model. Non-Newtonian and non-linear blood flow through a stenosed artery were covered by [11]. The generalized power law model characterizes the non-Newtonian rheology of the flowing blood. By simulating blood as Herschel Bulkley fluid and analyzing the pulsatile flow of blood through catheterized arteries, [12]. By simulating blood as Herschel Bulkley fluid and the catheter and artery as rigid coaxial circular cylinders, [12] investigated the pulsatile flow of blood through catheterized arteries. With all parameters held constant, they found that when yield stress increases, wall shear stress and longitudinal impedance rise. In contrast, velocity and flow rate decrease. In order to replicate arterial blood flow, [13] created an experimental setup for the investigation of pulsatile flow properties in elastic tubes. Sankar [14] examined the pulsatile flow of blood via a mildly stenosed artery treating the blood as Herschel-Bulkley fluid. They noticed that as yield increased, the plug core radius, pressure drop, and wall shear stress all increased. They noticed that when the stenosis height or yield stress increased, so did the plug core radius, pressure drop, and wall shear stress. Mallik [16] investigated a non-Newtonian fluid model for blood flow utilizing power law via an atherosclerotic arterial segment having slip velocity. Nadeem [15] examined the Power law fluid model for blood flow through a tapering artery with a stenosis. Diwakar [17] Acquired a power law fluid mathematical model applied to the flow of blood via a stenotic artery. The current study's objective is to examine how blood's non-Newtonion character affects blood flow via an artery. The power-law and Herschel-Bulkley models take into account blood and the artery as elastic.

## 2. Main Results

**Problem Formulation:** Non-Newtonian blood flow through an elastic artery is taken into consideration in this particular instance. Blood is thought to be inside the vessel and encircled by fluid while it is at rest. Measured along the axis of z is the vessel's axis. Arteries are treated as long, cylindrical tubes for mathematical ease, avoiding entrance and end effects.

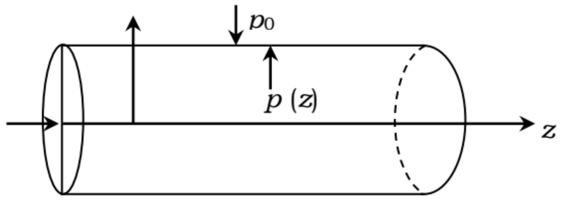


**Figure 1.** **Physical model**

The transmural pressure difference [18],

$$Th = r\left[p\left(z\right) - p_0\right] \tag{1}$$

Here h - wall thickness, r - radius, $p_0$ - exterior pressure and p - interior Blood pressure, $p - p_0$ - transmural pressure difference, T - tension per unit length per unit thickness. Now from Hooke's law tension T is

$$T = E\frac{(r - r_0)}{r_0} \tag{2}$$

If T is zero $r = r_0$ is equilibrium position, E - Young's modules. From the relation (1) and (2), we have

$$r = \frac{r_o}{1 - \frac{r_o}{Eh}(p(z) - p_o)} \tag{3}$$

**Equation of Motion:** For $(r^*, \theta^*, z^*)$ cylindrical coordinates and by considering fluid as Newtonian.

$$\mu\left[\frac{1}{r^*}\frac{d}{dr^*}\left(r^*\frac{du^*}{dr^*}\right)\right] = \frac{dp^*}{dz^*}, \quad 0 \le r^* \le r_0 \tag{4}$$

be the equation of motion $\mu$ - viscosity, $u^*$ - axial velocity, $p^*$ - pressure and $r_0$ - radius of the tube. Using Newton's law of viscosity

$$\tau^* = -\mu\frac{du^*}{dr^*} \tag{5}$$

From equation (4), we have

$$\tau^* = -\frac{1}{2}r^*\frac{dp^*}{dz^*}, \quad 0 \le r^* \le r_0 \ \ and \ \ \tau^* = 0 \ \ at \ \ r^* = 0 \tag{6}$$

**Non-Newtonian Power-Law Equation**

$$\tau^* = \mu_\infty\left(\dot{\gamma}\right)^n \quad or \quad \tau^* = \mu_\infty\left(-\frac{du^*}{dr^*}\right)^n \tag{7}$$

$\tau^*$ - shear stress, $\dot{\gamma}$ - shear strain rate and $\mu_\infty$ - apparent viscosity and for Herschel - Bulkley model

$$\begin{aligned}\tau^* &= \mu_\infty\left(\dot{\gamma}\right)^n + \tau_y; \quad \tau^* \ge \tau_y\\ \dot{\gamma} &= 0; \quad \tau^* \le \tau_y\end{aligned} \tag{8}$$

$\tau_y$ - yield stress and $\mu_\infty$ - apparent viscosity. For core region

$$\dot{\gamma} = 0 \ \ i.e. \ \ \frac{du^*}{dr^*} = 0 \tag{9}$$

Boundary Condition $u^* = 0$ at $r^* = r_0$

**Method of Solution:** Following non-dimensional variables

$$r = \frac{r^*}{r_0}, u = \frac{u^*}{u_0}, \tau = \frac{\tau^*}{\tau_0}, \quad where \ \ u_0 = \frac{r_0^2}{2\mu_\infty}\frac{dp^*}{dz^*}, \tau_0 = \frac{\mu_\infty}{r_0}u_0$$

are characteristic values of velocity and shear stress respectively.

**Velocity for Power Law Model**

$$u = \frac{A_1 n}{n+1}\left(-1 + \left(\frac{r_o}{1 - \frac{r_o}{Eh}(p(z) - p_o)}\right)^{\frac{1}{n}+1}\right), \quad where, \quad A_1 = -\left(\frac{\mu_\infty}{\tau_o}\right)^{\frac{n-1}{n}}$$

Volume flow rate for power law model:

$$Q = \frac{4n}{3n+1}\left(\frac{\mu_\infty}{\tau_o}\right)^{\frac{n-1}{n}}$$

Velocity for Hershel bulkley model

$$u = \frac{a_1}{Eh}\left(-1 + \frac{a_3}{\frac{Eh}{r_o} - (p(z) - p_o)}\right)^{\frac{1}{n}} \times \left(\frac{na_4}{1+n}\right)\left(\frac{1}{a_3} - \frac{1}{\frac{Eh}{r_o} - (p(z) - p_o)}\right) - a_1\left(-1 + \frac{1}{\hat{\tau}}\right)^{\frac{1}{n}}\left(\frac{n}{n+1}\right)(\hat{\tau} - 1),$$

where

$$a_1 = \left(\frac{\mu_\infty}{\tau_0}\right)^{\frac{n-1}{n}}(\hat{\tau})^{\frac{1}{n}}, \quad a_3 = \frac{Eh}{\hat{\tau}}, \quad a_4 = (Eh)^2$$

Volume Flow Rate For Hershel Bulkley Model:

$$Q = 8\int_0^1 urdr = \left[8u\frac{r^2}{2}\right]_0^1 - 8\int_0^1 \frac{du}{dr}\frac{r^2}{2}dr$$

Now using no-slip condition $u = 0$, at $r = 1$ and $\tau = r$ for $0 \le r \le 1$, we have

$$Q = 4\left(\frac{\mu_\infty}{\tau_0}\right)^{\frac{n-1}{n}}\frac{n}{(n+1)}\left[(1-\hat{\tau})^{\frac{1}{n}+1} - \frac{2n}{(2n+1)} \times \left((1-\hat{\tau})^{\frac{1}{n}+2} - \frac{n}{(3n+1)}(1-\hat{\tau})^{\frac{1}{n}+3}\right)\right.$$
$$\left. - r_p^2(r_p - \hat{\tau})^{\frac{1}{n}+1} + \frac{2n}{(2n+1)}\left(r_p(r_p - \hat{\tau})^{\frac{1}{n}+2}\right) - \frac{n}{3n+1}(r_p - \hat{\tau})^{\frac{1}{n}+3}\right]$$

# 3. Conclusions

The difference in flow profiles between power-law model and Harchel-Bulkely model of blood can be visualize in figure 1. We see that, in comparison to the Herchel-Bulkley model for fixed values of power index n and other parameters, the velocity profiles for the power law model in the elastic artery is advanced. Furthermore, when the power index rises for both types of fluid, the fluid's velocity falls. Figure 2, shows that, with linear transmular pressure, blood velocity declines with increasing elasticity up to a transition point of unit value z; beyond this point, the outcome reverses. Each time a vessel's elasticity value is greater, the fluid's downstream velocity reduces; however, when a vessel's elasticity value is smaller, the downstream velocity from the transition point lowers significantly. The impact of elasticity on the Herschel-Bulkley fluid's flow velocity can be viewed in figure 3. The fluid velocity is found to decrease as the vessel's elastic value increases. This phenomenon is comparable to power law fluid flow, with the transition point in the flow occurring at $z = 1$, where the elasticity's influence on the velocity is reversed. The sudden change in velocity downstream to the transition point is caused by the small value of $Eh = 100$. Figure 4, shows that, when the power index n increases, the Herschel-Bulkley fluid velocity in the elastic vessel, where the transmulor pressure varies linearly with z, falls. The volume flow rate for the power law and Herchel Bulkley fluid is shown in Figures 5 and 6. Also figure 7, shows that the volume flow rate increases as the power index n increases and reaches its maximum near the axial line. Figure 8, shows how an increase in yield stress causes a decrease in the Herschel-Bukley fluid's flow velocity in the elastic vessel. The increase in yield stress above 0.5 forced into the back flow while for small value of $\tau^*$ flow profile tends to the parabolic with axis as axis of the vessel. Figure 9 illustrates how the shear stress for a power law fluid grows as the power index n increases. However, in the case of the Herschel-Bulkley fluid, the shear stress at the center line drops, and $r = 0.2$ becomes the transition point; above this point, the shear stress increases as the power index, n, increases. In both cases, the wall shear then increases as the power index n increases. Figure 10 shows that shear stress increases with increase of yield stress.

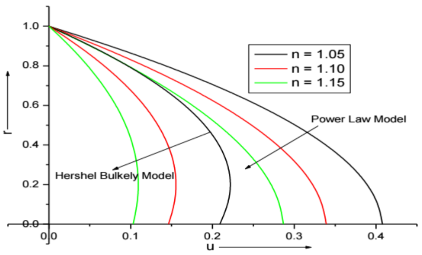

fig. 1 velocity profiles for Power Law model and Herschel Bulkley model for different value of power index(n) at Yield stress=0.2

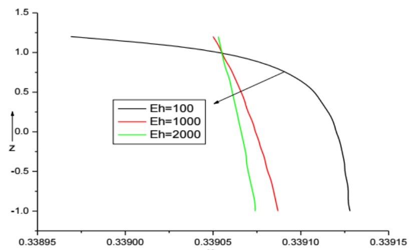

fig. 2 effect of elasticity on velocity for Power Law model at different location along the axis at n=0.75

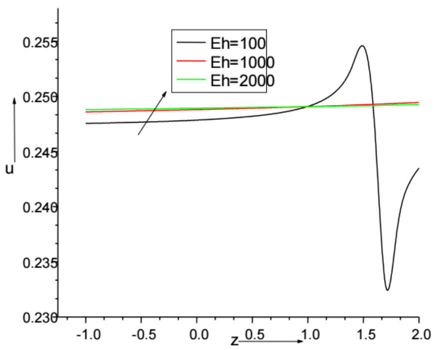

fig. 3 effect of elasticity on velocity for Herschel Bulkley model at different location along the axis (At n=1.1,Yield stress=0.2)

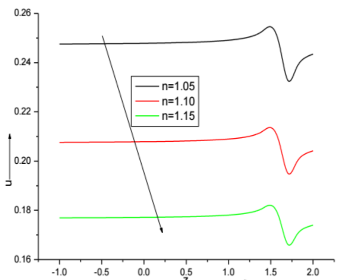

fig. 4 effect of power index(n) versus velocity profile for Herschel Bulkley model (At Eh=100,Yield stress=0.2)

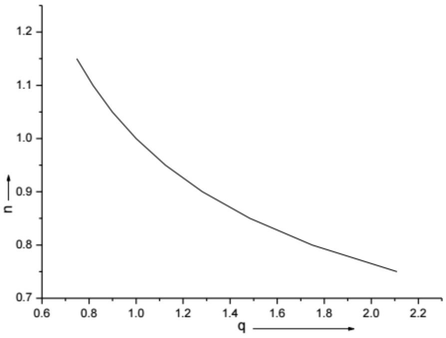

fig. 5 volume flow rate versus power index(n) for Power Law model

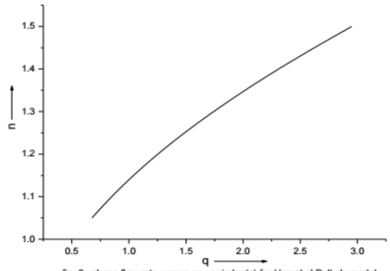

fig. 6 volume flow rate versus power index(n) for Herschel Bulkely model (At $r_p$=0.75,Eh=100, Yield stress=0.2)

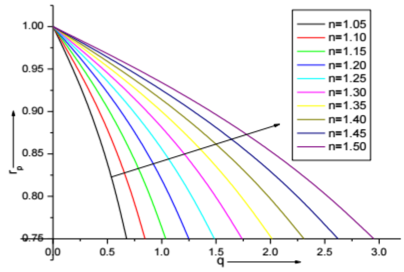

fig. 7 radial variation of volume flow rate of Herschel Bulkley model at Yield stress=0.2

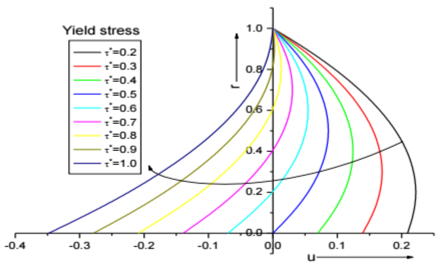

fig. 8 effect of yield stress on the fluid velocity for Herschel Bulkley model at n=1.1

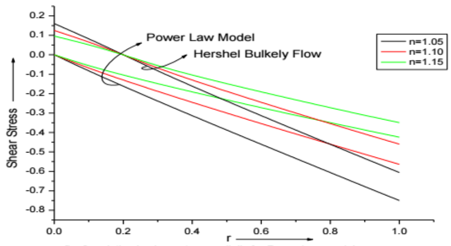

fig. 9 variation in shear stress radially for Power Law model and herschel bulkley model at Yield stress=0.2

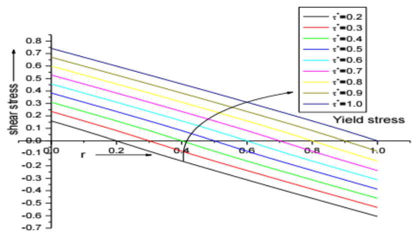

fig. 10 effect of yield stress on the shear stress for Herschel Bulkley model at n=1.1

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