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A Note on Connected Ordered Topological Spaces

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Abstract

Some properties of Connected Ordered Topological Spaces (COTS) are obtained. We prove that if a connected space X has at most two non-cut points and an R(i) subset, the closure of which contains all closed points of X, then X is a COTS with endpoints. The concept of locally cut point convex topological space is introduced. It is proved that in a connected and locally cut point convex space, for a, b in X, $S[a, b]$ is compact whenever it is closed.

Keywords: connected space; closed point; R(i) subset; COTS; cut point convex subset.
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1. Introduction

The concept of connected ordered topological space (COTS), introduced by Khalimsky, Kopperman and Meyer in [8] have been studied by Kamboj and Kumar from the view point of non-cut points in [2-7]. A topological space is assumed to contain at least three points to study properties of COTS. In notation and terminology we will follow [7]. In this paper, we study some properties of COTS and also establish a relationship between $S[a, b]$ and locally cut point convex spaces. The main results of the paper appear in Sections 2, 3 and 4. In Section 2, we prove that if X is a COTS, then there is no proper non-empty open subset of X containing all closed points of X. We show that every COTS is c-simple. It is proved that if X is a connected space such that $X - \{x\}$ is a COTS for every $x \in X$, then every two-point disconnected set of X leaves X disconnected. Finally we show that if in a connected space X with $ctX = \phi$, where ctX denotes the set of all non-cut points of X, every two-point disconnected set of X leaves X disconnected, then for every two-point disconnected set $\{a, b\}$ of X, there are exactly two COTS with endpoints a and b. Thus Lemma 5.2(c) (except the last assertion of this lemma) of [8] and Theorem 16 of [9] are strengthened. In Section 3, we prove that if a connected space X has an $R(i)$ subset H such that $cd(X) \cap ctX \subset cl_X(H)$, then there is no proper cut point convex subset of X containing all non-cut points of X; and thus such a space has at least two non-cut points. It is shown

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that if a connected and locally connected space X has at most two non-cut points and an $R(i)$ subset H such that $cd(X) \cap ctX \subset cl_X(H)$, then X is a compact COTS with endpoints. In Section 4, It is proved that in a connected and locally cut point convex space, for a, b in $X, S[a,b]$ is compact whenever it is closed; and thus it a strengthened version of Theorem 3.4 of [3].

2. Connected Ordered Topological Spaces

Theorem 2.1. *Let X be a COTS. Then there exists no proper non-empty open subset of X containing all closed points of X.*

Proof. We assume that X is a non-indiscrete space having at least two points. In view of Proposition 2.9 of [8], X is $T_{1/2}$ as X is non-indiscrete. So, X has a closed point as X is a connected space. Let G be an open subset of X containing $cd(X)$. We prove that $G = X$. Suppose not. Then there exists some open point x of X such that $x \notin G$ as X is $T_{1/2}$. Now x has either a successor or a predecessor. Assume that x has a successor (the other case is similar). Then x has an immediate successor say x^+ in view of Lemma 2.8(b) of [8]. x^+ is a closed point by Lemma 2.8(c) of [8] such that $x^+ \in cl_X(\{x\})$. Since G is open and $x \notin G$, $G \cap cl_X(\{x\}) = \phi$. Therefore $x^+ \notin G$ which is a contradiction to given condition. This completes the proof. □

Corollary 2.2. *Let X be a COTS. If the set of all closed points of X is connected, then every cut point of X is closed.*

A topological space X is called c-simple ([1]) if for every nonempty connected subset A of X, A is open in $cl_X(A)$. In following result, we prove that every COTS is c-simple. For this, we need the following. Let X be a COTS be such that $<$ is an order of the COTS. For $x \in ctX$, let $L(x) = \{y \in X : y < x\}$ and $R(x) = \{y \in X : x < y\}$. In view of Theorem 2.7 of [8], $L(x)$ and $R(x)$ are the separating sets of $X - \{x\}$.

Theorem 2.3. *Let X be a COTS. Then X is c-simple.*

Proof. We suppose that X is non-indiscrete. In view of Proposition 2.9 of [8], X is $T_{1/2}$. Let A be a non-empty connected subset of X. To prove that A is open in $cl_X(A)$, let $x \in A$ be a closed point of X. If $A = \{x\}$, the result is clear. Let $y \in A - \{x\}$. We can assume that $x < y$ in an order $<$ of the COTS X in view of Theorem 2.7 of [8].

Case (i): $A \subset (R(x))^{+x}$. Since x is a closed point, $L(x)$ is open in X. Now it follows that $(L(x)) \cap cl_X(A) = \phi$. So $((L(y))^{+y}) \cap cl_X(A) \subset ((L(y))^{+y}) \cap (R(x))^{+x} \subset S[x,y]$ using Lemma 3.19 of [8]. Therefore using Proposition 3.1(i) of [6], we have $(L(y))^{+y} \cap cl_X(A) \subset A$ as A is connected. Since $x < y$, it follows that $x \in int_X((L(y))^{+y}) \cap cl_X(A)$ as $L(y) \subset int_X((L(y))^{+y})$ by Lemma 3.1 of [2].

Case (ii): $A \not\subset (R(x))^{+x}$. Then there is some $z \in A$ such that $z < x$. By Proposition 3.1(i) of [6], $S[z,y] \subset A$ as A is connected. Using Lemma 3.19 of [7], $(R(z))^{+z} \cap (L(y))^{+y} \subset S[z,y]$. So $(R(z))^{+z} \cap (L(y))^{+y} \subset$

A. Since $z < x < y$, it follows that $x \in int_X((R(z))^{+z}) \cap int_X((L(y))^{+y})$ as $R(z) \subset int_X((R(z))^{+z})$ and $L(y) \subset int_X((L(y))^{+y})$ by Lemma 3.1 of [2].
Thus both cases shows that A is open in $cl_X(A)$. This completes the proof. □

Theorem 2.4. *Let X be a connected space such that $X - \{x\}$ is a COTS for every $x \in X$. Then every two-point disconnected set leaves the space disconnected.*

Proof. Let $\{a, b\}$ be a two-point disconnected subset of X. Since X is connected, there is some $p \in X - \{a, b\}$. Then $a, b \in X - \{p\}$. By Theorem 3.1(ii) and (iii) of [6], $S_{(X-\{p\})}[a, b]$ is connected since $X - \{p\}$ is a COTS. Since $\{a, b\}$ is disconnected, there is some $q \in S_{(X-\{p\})}(a, b)$. So there is some separation $H|K$ of $X - \{p, q\}$ such that $a \in H$ and $b \in K$ or conversely. Assume that $a \in H$ and $b \in K$. Consider the three-point set $\{p, q, a\}$ of the COTS $X - \{b\}$. Since $X - \{p\}$ and $X - \{q\}$ are connected by given condition, so $H \cup \{p\}$ and $H \cup \{q\}$ are connected.
If $X - \{b, p\}$ is disconnected, then $H \cup \{q\} \subset A_{\{b,p\}}(a)$; so p is not a separating point between a and q in $X - \{b\}$. If $X - \{b, q\}$ is disconnected, then $H \cup \{p\} \subset A_{\{b,q\}}(a)$; so q is not a separating point between a and p in $X - \{b\}$. Therefore $a \in S_{(X-\{b\})}(p, q)$ since $X - \{b\}$ is a COTS. This completes the proof. □

The following lemma strengthens Lemma 12 of [9] as there is no need of the assumption that every two-point set of the space under consideration leaves the space disconnected.

Lemma 2.5. *Let X be a connected space and $ctX = \phi$. Let D be a subset of X, having at least two points. Let $H|K$ be a separation of $X - D$. If $x \in H$ and $y \in K$ such that $X - \{x, y\}$ is disconnected, then $D \cap A_{\{x,y\}} \neq \phi$ and $D \cap B_{\{x,y\}} \neq \phi$.*

Proof. We prove the result by contradiction. Suppose not; either $D \cap A_{\{x,y\}} = \phi$ or $D \cap B_{\{x,y\}} = \phi$. Suppose $D \cap A_{\{x,y\}} = \phi$. Then $(A_{\{x,y\}})^{+\{x,y\}} \subset X - D$. So $(A_{\{x,y\}})^{+\{x,y\}} \subset H$ or $(A_{\{x,y\}})^{+\{x,y\}} \subset K$ as $(A_{\{x,y\}})^{+\{x,y\}}$ is connected by Lemma 3.3 of [4]. Therefore, we arrive at a contradiction to the given condition. This completes the proof. □

Corollary 2.6. *Let X be a connected space with $ctX = \phi$ such that every two-point disconnected set leaves the space disconnected. If for $a, b \in X$, $X - \{a, b\}$ is disconnected, then $a \notin cl_X(\{b\})$ and $b \notin cl_X(\{a\})$.*

Proof. Let $x \in A_{\{a,b\}}$ and $y \in B_{\{a,b\}}$. Then $\{x, y\}$ is disconnected using Lemma 4 of [9]. So $X - \{x, y\}$ is disconnected by given condition. Therefore applying Lemma 2.5 to $D = \{a, b\}$, either $a \in A_{\{x,y\}}$ and $b \in B_{\{x,y\}}$ or conversely. This implies that $a \notin cl_X(\{b\})$ and $b \notin cl_X(\{a\})$ using Lemma 4 of [9]. □

In view of Theorem 2.4, the following theorem strengthens Lemma 5.2(c) except the last assertion of it of [8], and Theorem 16 of [9] as it is sufficient to assume that every two-point disconnected set of a connected space X with $ctX = \phi$ leaves X disconnected.

Theorem 2.7. *Let X be a connected space with* $ctX = \phi$ *such that every two-point disconnected set leaves the space disconnected. If for* $a, b \in X$, $X - \{a, b\}$ *is disconnected, then (i) each one of* $(A_{\{a,b\}})^{+\{a,b\}}$ *and* $(B_{\{a,b\}})^{+\{a,b\}}$ *is a COTS with endpoints a and b. (ii) there are exactly two COTS with endpoints a and b.*

Proof. (i) By Lemma 3.3 of [4], $(A_{\{a,b\}})^{+\{a,b\}}$ is connected such that a and b are non-cut points of $(A_{\{a,b\}})^{+\{a,b\}}$. Let $x \in A_{\{a,b\}}$ and $y \in B_{\{a,b\}}$. Then $\{x, y\}$ is disconnected using Lemma 4 of [9]. So, by given condition $X - \{x, y\}$ is disconnected. Now applying Lemma 2.5 to $D = \{a, b\}$, we have either $a \in A_{\{x,y\}}$ and $b \in B_{\{x,y\}}$ or conversely. Since $((A_{\{a,b\}})^{+\{a,b\}}) - \{x\} \subset X - \{x, y\}$, so $(((A_{\{a,b\}})^{+\{a,b\}}) \cap A_{\{x,y\}}) | (((A_{\{a,b\}})^{+\{a,b\}}) \cap B_{\{x,y\}})$ is a separation of $((A_{\{a,b\}})^{+\{a,b\}}) - \{x\}$. This proves that x is a separating point of $(A_{\{a,b\}})^{+\{a,b\}}$ between a and b in $(A_{\{a,b\}})^{+\{a,b\}}$. Thus $(A_{\{a,b\}})^{+\{a,b\}}$ is a connected space with endpoints a and b. Similarly $(B_{\{a,b\}})^{+\{a,b\}}$ is a connected space with endpoints a and b. Now (i) follows by Theorem 3.2 of [3]. (ii) If $K \subset X$ is a COTS with endpoints a and b, then by Lemma 3.8(i) of [5], $K - \{a, b\}$ is connected. This implies that either $K \subset (A_{\{a,b\}})^{+\{a,b\}}$ or $K \subset (B_{\{a,b\}})^{+\{a,b\}}$. Therefore using (i), it follows from Theorem 3.2(b) of [8] that $K \in \{(A_{\{a,b\}})^{+\{a,b\}}, (B_{\{a,b\}})^{+\{a,b\}}\}$. This completes the proof of (ii). □

3. R(i) Subset and COTS

Theorem 4.5 of [7] is proved for connected spaces having an $R(i)$ subset which contains $cd(X) \cap ctX$. Following result is for connected spaces having an $R(i)$ subset, the closure of which contains $cd(X) \cap ctX$ and thus is a strengthened version of Theorem 4.5 of [7].

Theorem 3.1. *If a connected space X has an* $R(i)$ *subset H such that* $cd(X) \cap ctX \subset cl_X(H)$, *then there is no proper cut point convex subset of X containing all non-cut points of X.*

Proof. Let Y be a non-empty cut point convex set of X such that $X - Y \subset ctX$. Then by Theorem 3.7 of [7], we have an infinite chain α of proper connected sets of the form $(A_x(Y))^{+x}$, where $x \in cd(X) \cap (X - Y)$, covering X. For $(A_x(Y))^{+x} \in \alpha$, let B_x be the other separating set of $X - \{x\}$ corresponding to A_x. Now by Theorem 4.3(i) of [7], $B_x \cap cl_X(H) \neq \phi$ for each B_x corresponding to $(A_x(Y))^{+x} \in \alpha$. This implies that $B_x \cap H \neq \phi$ for each B_x corresponding to $(A_x(Y))^{+x} \in \alpha$ as $x \in cd(X)$. But then, by Lemma 4.1, H is non-$R(i)$, which is a contradiction to given condition. The proof is complete. □

The following result is the non-cut point existence theorem for a subclass of connected spaces.

Theorem 3.2. *If a connected space X has an* $R(i)$ *subset H such that* $cd(X) \cap ctX \subset cl_X(H)$, *then X has at least two non-cut points.*

Proof. Suppose to the contrary. Then $X - ctX$ has at most one element. Thus there exists some $x \in X$ such that $X - ctX \subset \{x\}$. But this is a contradiction in view of Theorem 3.1. □

Theorem 3.3. *If a connected space X has at most two non-cut points and an* $R(i)$ *subset H such that* $cd(X) \cap ctX \subset cl_X(H)$, *then X is a COTS with endpoints.*

Proof. By Theorem 3.2 and the given condition, X has exactly two non-cut points, say, a and b. Let $x \in X - \{a, b\}$. Then $x \in ctX$. By Proposition 3.1(i) of [6], each of $(A_x)^{+x}$ and $(B_x)^{+x}$ is a cut point convex set. By Theorem 3.1, there is no proper cut point convex subset of X containing $X - ctX$. Therefore, $a \in A_x$ and $b \in B_x$, or conversely, because $X - ctX = \{a, b\}$. This implies that $x \in S(a, b)$. Hence $X = S[a, b]$. Now by Theorem 3.2 of [3], X is a COTS with endpoints a and b. □

Theorem 3.4. *If a connected space X has an $R(i)$ subset H such that $cd(X) \cap ctX \subset cl_X(H)$, then for each $x \in ctX$, A_x contains a non-cut point of X.*

Theorem 3.5. *If a connected and locally connected space X has at most two non-cut points and an $R(i)$ subset H such that $cd(X) \cap ctX \subset cl_X(H)$, then X is a compact COTS with endpoints.*

Proof. By Theorem 3.3, X is a COTS with endpoints. Now the theorem follows by Theorem 4.4 of [3]. □

Theorem 3.6. *If a T_1 separable, connected and locally connected space X has at most two non-cut points and an $R(i)$ subset H such that $cd(X) \cap ctX \subset cl_X(H)$, then X is homeomorphic to the closed unit interval.*

Proof. By Theorem 3.3, X is a COTS with endpoints. Now the theorem follows by Corollary 6.2(i) of [4]. □

4. Locally Cut Point Convex Space

Call a topological space X locally cut point convex at a point $x \in X$ if for any neighbourhood W of x there exists a cut point convex neighbourhood V of x such that $V \subset W$. A topological space is locally cut point convex if it is locally cut point convex at each of its points. In view of Proposition 3.1(iii) of [6], for a COTS X, if X is locally cut point convex, then X is locally connected; thus, the two concepts locally connected and locally cut point convex coincide in a COTS.

Lemma 4.1. *Let X be a connected and $x \in ctX$. Let $p \in X$ be such that $p \notin int_X((A_x)^{+x})$. Then $x \in V$ for every cut point convex subset V of X, containing p such that $V \cap int_X((A_x)^{+x}) \neq \phi$.*

Proof. Suppose $x \notin V$ for some connected subset V of X containing p such that $V \cap int_X((A_x)^{+x}) \neq \phi$. Since by Lemma 3.1 of [2], $A_x \subset int_X((A_x)^{+x}) \subset (A_x)^{+x}$, $V \cap A_x \neq \phi$. Therefore $V \subset A_x$ using Lemma 2.5 of [7]. Thus $p \in A_x$. By Lemma 3.1 of [2], $p \in int_X((A_x)^{+x})$, which is a contradiction to the given condition. This contradiction proves the Lemma. □

For a subset T of a topological space X, $d(T)$ is used to denote the derived set of T.

Lemma 4.2. *Let X be a connected and locally cut point convex space. Let $K = S[a, b]$ be closed in X. Let M be a non-empty subset of $S(a, b)$. Let $\Omega = \{A : A = int_X((A_x(a))^{+x})$ for some separating set $A_x(a) of X - \{x\}$, $x \in M\}$. Let $T = \cup\{A : A \in \Omega\}$. Then* *(i)* $(X - T) \cap d(T)$ *is a non-empty subset of K.*

(ii) For $p \in (X - T) \cap d(T)$ *and an open covering* ς *of K in X, there exist* $x \in M$*, a member H of* ς*, and a cut point convex subset V of X containing both p and x such that* $V \subset H$*.*

Proof. (i) As elements of a separation, $b \notin A$ for every $A \in \Omega$, so $b \notin T$. Thus, T is not closed in X as X is connected and T is open. Therefore, there exists $p \in X - T$ such that p is a limit point of T. This proves that $(X - T) \cap d(T)$ is non-empty. To show $(X - T) \cap d(T) \subset K$, let $p \in (X - T) \cap d(T)$. If $p \notin K$, p is not a limit point of K as K is closed. Since X is a cut point convex space, there exists a cut point convex neighbourhood G of p such that $G \cap K = \phi$. Now $G \cap int_X((A_y(a))^{+y}) \neq \phi$ for some separating set $A_y(a)$, for some $y \in M$ as p is a limit point of T. Since $p \in X - T$, $p \notin int_X((A_y(a))^{+y})$. Using Lemma 4.1, $y \in G$, which is not possible as $G \cap K = \phi$, showing that $(X - T) \cap d(T) \subset K$.

(ii) Let $p \in (X - T) \cap d(T)$ and ς be an open covering of K in X. By (i), $p \in K$. ς being an open covering of K in X, we get a member of H of ς containing p. Since X is locally cut point convex, there exists a cut point convex neighbourhood V of p such that $V \subset H$. Since $p \in X - T$ and p is a limit point of T, $V \cap int_X((A_x(a))^{+x}) \neq \phi$ for some separating set $A_x(a)$ of $X - \{x\}$, $x \in M$. By Lemma 4.1, $x \in V$, and the proof is complete.

□

G. T. Whyburn in [10] proved that in a T_1 connected and locally connected space, for a and b in X, $S[a, b]$ which is closed in X, is compact. Kamboj and Kumar in [3] proved this result without assuming any separation axiom. The following result is for connected and locally cut point convex spaces and thus is a strengthened version Theorem 4.4 of [3].

Theorem 4.3. *Let X be a connected and locally cut point convex space. Then for* a, b *in X,* $S[a, b]$ *is compact whenever it is closed.*

Proof. We assume that $S(a, b)$ is non-empty. To prove, $K = S[a, b]$ is compact, let ς be an open covering of K in X. Let $K' = \{x \in K : S[a, x]$ is covered by a finite subcollection of $\varsigma\}$. We prove that $b \in K'$. Suppose to the contrary.
We claim that $K' = S[a, p]$ for some $p \in K'$. If $K' = \{a\}$, then $K' = S[a, a]$. So we suppose that $K' \neq \{a\}$. Then $K' - \{a\}$ is non-empty. Taking $M = K' - \{a\}$ in Lemma 4.2, $T = \cup\{int_X((A_x(a))^{+x}); x \in K' - \{a\}\}$, and there exists $p \in (X - T) \cap K$, $x \in K' - \{a\}$, a member H of ς and a cut point convex subset V containing p and x such that $V \subset H$. $S[a, x]$ is covered by a finite subcollection, say ς^1, of ς. Let $\varsigma^2 = \varsigma^1 \cup \{H\}$. We prove that $p \in K'$. Let $y \in S[a, p] - (S[a, x] \cup \{p\})$. $X - \{y\} = A_y(a) \cup B_y(p)$. Since $y \in S(a, p) - S[a, x]$ and $a \in A_y(a)$, by Lemma 2.1(I)(ii) of [3], $x \in A_y(a)$. Therefore $V \cap A_y(a) \neq \phi$; also $V \cap B_y(p) \neq \phi$. If $y \notin V$, $V \subset A_y(a)$ or $V \subset B_y(p)$ using Lemma 2.5 of [7]. Thus $y \in V$ and so $y \in H$. This proves that $S[a, p]$ is covered by members of ς^2. Therefore $p \in K'$. Now using Lemma 2.1(II) of

[3], we see that $S[a,p] \subset K'$. Let $x \in K' - \{a,p\}$. As $p \notin T$, so $p \notin int_X((A_x(a))^{+x})$. Using Lemma 3.1 of [2], $p \notin A_x(a)$. So $p \in B_x(b)$. This shows that $x \in S[a,p]$. Thus $K' = S[a,p]$. This proves our claim. Now $K - K' \neq \{b\}$ because then $K = K' \cup \{b\}$ is covered by finitely many members of ς. Let $T^\wedge = \cup\{B : B = int_X((B_x(b))^{+x})$ for some separating set $B_x(b) of X - \{x\}, x \in K - (K' \cup \{b\})\}$. Using Lemma 4.2, there exists $q \in (X - T^\wedge) \cap K$, $y \in K - (K' \cup \{b\})$, a member N of ς and a cut point convex subset W containing q and y such that $W \subset N$. Suppose $S[a,q]$ is covered by a finite subcollection of ς. Let $z \in S[a,y] - (S[a,q] \cup \{y\})$. $X - \{z\} = A_z(a) \cup B_z(y)$. Since $z \in S(a,y) - S[a,q]$ and $a \in A_z(a)$, by Lemma 2.1(I)(ii) of [3], $q \in A_z(a)$. Therefore $W \cap A_z(a) \neq \phi$, also $W \cap B_z(y) \neq \phi$. If $z \notin W$, $W \subset A_z(a)$ or $W \subset B_z(p)$ using Lemma 2.5 of [7]. Thus $z \in W$ and so $y \in N$. This proves that $S[a,y]$ is covered by a finite subcollection of ς, which is a contradiction as $y \in K - (K' \cup \{b\})$. Thus $S[a,q]$ is not covered by any finite subcollection of ς and so $q \in K - (K' \cup \{b\})$. As $q \in X - T^\wedge$, therefore for every $x \in K - (K' \cup \{b\})$, we have $q \notin int_X((B_x(b))^{+x})$ for any separating set $B_x(b)$ of $X - \{x\}$. This implies using Lemma 4.1 of [2] that for every $x \in K - (K' \cup \{b\})$, $q \notin B_x(b)$ for any separating set $B_x(b)$ of $X - \{x\}$. Therefore for every $x \in K - (K' \cup \{b,q\}$, $q \in A_x(a)$ for every separating set $A_x(a)$ of $X - \{x\}$. This implies that $S[a,q] \subset (K' \cup \{b,q\})$. Thus $S[a,q]$ is covered by a finite subcollection of ς. Thus $q \in K'$ which is a contradiction as $q \in K - (K' \cup \{b\})$. The contradiction proves that $b \in K'$. Hence K is compact. □

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