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Fixed Point Contractive Mapping of Wardoski Type and its Application

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Abstract: In the paper, we introduce a new concept of soft contraction of Wardoski type which is generalization of Banach contractive condition and prove a soft fixed point theorem which generalizes Banach contraction principle in a different ways. We also give some examples which shows the validity of our results.

MSC: 74H10, 54H25.

Keywords: Fixed point, metric space, F-contraction.

1. Introduction

Recently, Wardowski [18] introduced a new type of contraction called F–contraction and proved a fixed point result in complete metric spaces which in turn generalizes the Banach contraction principle is the Wardowski fixed point theorem [18]. Before providing the Wardowski fixed point theorem, we recall that a self-map T on a metric space (X, d) is said to be an F-contraction if there exist $F \in \mathcal{F}$ and $\tau \in (0, \infty)$ such that

$$\forall x, y \in X, \quad \big[d(Tx, Ty) > 0 \Rightarrow \tau + F\big(d(Tx, Ty)\big) \leq F\big(d(x, y)\big)\big], \tag{1}$$

where $\mathcal{F}$ is the family of all functions $F : (0, \infty) \to \mathbb{R}$ such that

(F1). F is strictly increasing, i.e. for all $x, y \in \mathbb{R}_+$ such that $x < y$, $F(x) < F(y)$;

(F2). for each sequence $\{\alpha_n\}_{n=1}^{\infty}$ of positive numbers,

$$\lim_{n\to\infty} \alpha_n = 0$$

if and only if

$$\lim_{n\to\infty} F(\alpha_n) = -\infty,$$

(F3). there exists $k \in (0, 1)$ such that $\lim_{\alpha\to 0^+} \alpha^k F(\alpha) = 0$.

Obviously every F-contraction is necessarily continuous. The Wardowski fixed point theorem is given by the following theorem.

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Theorem 1.1. *Let (X, d) be a complete metric space and let $T : X \to X$ be an F-contraction. Then T has a unique fixed point $x^* \in X$ and for every $x \in X$ the sequence $\{T^n x\}_{n\in\mathbb{N}}$ converges to x^*.*

Later, Wardowski and Van Dung [19] have introduced the notion of an F-weak contraction and prove a fixed point theorem for F-weak contractions, which generalizes some results known from the literature. They introduced the concept of an F-weak contraction as follows.

Definition 1.2. *Let (X, d) be a metric space. A mapping $T : X \to X$ is said to be an F-weak contraction on (X, d) if there exist $F \in \mathcal{F}$ and $\tau > 0$ such that, for all $x, y \in X$, $d(Tx, Ty) > 0 \quad \Rightarrow \quad \tau + F\big(d(Tx, Ty)\big) \le F\big(M(x, y)\big)$, where*

$$M(x,y) = \max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(y,Tx)}{2}\right\}. \tag{2}$$

By using the notion of F-weak contraction, Wardowski and Van Dung [19] have proved a fixed point theorem which generalizes the result of Wardowski as follows.

Theorem 1.3. *Let (X, d) be a complete metric space and let $T : X \to X$ be an F-weak contraction. If T or F is continuous, then T has a unique fixed point $x^* \in X$ and for every $x \in X$ the sequence $\{T^n x\}_{n\in\mathbb{N}}$ converges to x^*.*

Recently, by adding values $d(T^2x, x)$, $d(T^2x, Tx)$, $d(T^2x, y)$, $d(T^2x, Ty)$ to 2, Dung and Hang [8] introduced the notion of a modified generalized F-contraction and proved a fixed point theorem for such maps. They generalized an F-weak contraction to a generalized F-contraction as follows.

Definition 1.4. *Let (X, d) be a metric space. A mapping $T : X \to X$ is said to be a generalized F-contraction on (X, d) if there exist $F \in \mathcal{F}$ and $\tau > 0$ such that $\forall \;\; x, y \in X, \quad \big[d(Tx, Ty) > 0 \Rightarrow \tau + F\big(d(Tx, Ty)\big) \le F\big(N(x, y)\big)\big]$, where*

$$N(x,y) = \max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(y,Tx)}{2}, \frac{d(T^2x,x)+d(T^2x,Ty)}{2}, d\big(T^2x,Tx\big), d\big(T^2x,y\big), d\big(T^2x,Ty\big)\right\}.$$

By using the notion of a generalized F-contraction, Dung and Hang have proved the following fixed point theorem, which generalizes the result of Wardowski and Van Dung [19].

Theorem 1.5. *Let (X, d) be a complete metric space and let $T : X \to X$ be a generalized F-contraction. If T or F is continuous, then T has a unique fixed point $x^* \in X$ and for every $x \in X$ the sequence $\{T^n x\}_{n\in\mathbb{N}}$ converges to x^*.*

Very recently, Piri and Kumam [15] described a large class of functions by replacing the condition (F3) in the definition of F-contraction introduced by Wardowski with the following one:

(F3'): F is continuous on $(0, \infty)$.

They denote by $\mathfrak{F}$ the family of all functions $F : \mathbb{R}_+ \to \mathbb{R}$ which satisfy conditions (F1), (F2), and (F3'). Under this new set-up, Piri and Kumam proved some Wardowski and Suzuki type fixed point results in metric spaces as follows.

Theorem 1.6. *Let T be a self-mapping of a complete metric space X into itself. Suppose there exist $F \in \mathfrak{F}$ and $\tau > 0$ such that $\forall \;\; x, y \in X, \quad \big[d(Tx, Ty) > 0 \Rightarrow \tau + F\big(d(Tx, Ty)\big) \le F\big(d(x, y)\big)\big]$. Then T has a unique fixed point $x^* \in X$ and for every $x_0 \in X$ the sequence $\{T^n x_0\}_{n=1}^{\infty}$ converges to x^*.*

Theorem 1.7. *Let T be a self-mapping of a complete metric space X into itself. Suppose there exist $F \in \mathfrak{F}$ and $\tau > 0$ such that $\forall \;\; x, y \in X, \quad \left[\frac{1}{2}d(x, Tx) < d(x, y) \Rightarrow \tau + F\big(d(Tx, Ty)\big) \le F\big(d(x, y)\big)\right]$. Then T has a unique fixed point $x^* \in X$ and for every $x_0 \in X$ the sequence $\{T^n x_0\}_{n=1}^{\infty}$ converges to x^*.*

Beside this, the concept of soft theory as new mathematical tool for dealing with uncertainties is discussed in 1999 by Molodtsov [13]. A soft set is a collection of approximate descriptions of an object this theory has rich potential applications. On soft set theory many structures contributed by many researchers (see [5, 9, 11]). Shabir and Naz [17] were studied about soft topological spaces. In these studies, the concept of soft point is explained by different techniques. Later a different concept of soft point introduced by Das and Samanta ([6, 7]) using a different notion of soft metric space and investigated some basic properties of these spaces. Now we recall some definition which are required for the proof of our results.

Definition 1.8. *Let X be an initial universe set and E be a set of parameters. A pair (F, E) is called a soft set over X if and only if F is a mapping from E into the set of all subsets of the set X, i.e., $F : E \to P(X)$ where $P(X)$ is the power set of X.*

Definition 1.9. *Let $VisionRes.$, $\tilde{s}$ be two soft real numbers. Then the following statements hold:*

(1). $VisionRes. \leq \tilde{s}$ if $VisionRes.\ (e) \leq \tilde{s}(e)$ for all $e \in E$,

(2). $VisionRes. \geq \tilde{s}$ if $VisionRes.\ (e) \geq \tilde{s}(e)$ for all $e \in E$,

(3). $VisionRes. < \tilde{s}$ if $VisionRes.\ (e) < \tilde{s}(e)$ for all $e \in E$,

(4). $VisionRes. > \tilde{s}$ if $VisionRes.\ (e) > \tilde{s}(e)$ for all $e \in E$.

Definition 1.10. *A soft set (F, E) over X is said to be a soft point, denoted by $\tilde{x}_e$, if for the element $e \in E$, $F(e) = \{x\}$ and $F(\tilde{e}) = \phi$, for all $\tilde{e} \in E \setminus \{e\}$.*

Definition 1.11. *Two soft points $\tilde{x}_e, \tilde{y}_e$ are said to be equal if $e = \tilde{e}$ and $x = y$. Thus $\tilde{x}_e \neq \tilde{y}_e$ or $e \neq \tilde{e}$.*

Definition 1.12. *A mapping $\tilde{d} : SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(E)^*$ is said to be a soft metric on the soft set $\tilde{X}$ if $\tilde{d}$ satisfies the following conditions:*

(M1). $\tilde{d}(\tilde{x}_e, \tilde{y_{e'}}) \tilde{\geq} \tilde{0}$ for all $\tilde{x}_e, \tilde{y_{e'}} \tilde{\in} \tilde{X}$,

(M2). $\tilde{d}(\tilde{x}_e, \tilde{y_{e'}}) \tilde{\geq} \tilde{0}$ if and only if $\tilde{x}_e = \tilde{y_{e'}}$,

(M3). $\tilde{d}(\tilde{x}_e, \tilde{y_{e'}}) = \tilde{d}(\tilde{y_{e'}}, \tilde{x}_e)$ for all $\tilde{x}_e, \tilde{y_{e'}} \tilde{\in} \tilde{X}$,

(M4). For all $\tilde{x}_e, \tilde{y_{e'}}, \tilde{z_{e''}} \tilde{\in} \tilde{X}$, $vd(\tilde{x}_e, \tilde{z_{e''}}) \tilde{\leq} \tilde{d}(\tilde{x}_e, \tilde{y_{e'}}) + \tilde{d}(\tilde{y_{e'}}, \tilde{z_{e''}})$.

The soft set $\tilde{X}$ with a soft metric $\tilde{d}$ on $\tilde{X}$ is called a soft metric space and denoted by $(\tilde{X}, \tilde{d}, E)$.

Definition 1.13. *Let $\{x^{\tilde{n}}_{e_n}\}$ be a sequence of soft points in a soft metric space $(\tilde{X}, \tilde{d}, E)$. Then the sequence $\{x^{\tilde{n}}_{e_n}\}$ is said to be convergent in $(\tilde{X}, \tilde{d}, E)$ if there is a soft point $x^{\tilde{0}}_{e_0} \tilde{\in} \tilde{X}$ such that $\tilde{d}(x^{\tilde{n}}_{e_n}, x^{\tilde{0}}_{e_0}) \to \tilde{0}$ as $n \to \infty$. This means for every $\tilde{\epsilon} \tilde{>} \tilde{0}$, chosen arbitrarily, there is a natural number $N = N(\tilde{\epsilon})$ such that $\tilde{0} \tilde{<} \tilde{d}(x^{\tilde{n}}_{e_n}, x^{\tilde{0}}_{e_0}) \tilde{\leq} \tilde{\epsilon}$ whenever $n > N$.*

Definition 1.14. *Limit of a sequence in a soft metric space, if exist, is unique.*

Definition 1.15 (Cauchy Sequence)**.** *The sequence $\{x^{\tilde{n}}_{e_n}\}$ of soft points in $(\tilde{X}, \tilde{d}, E)$ is called a Cauchy sequence in $\tilde{X}$ if corresponding to every $\tilde{\epsilon} \tilde{>} \tilde{0}$, there is a $m \in N$ such that $\tilde{d}(x^{\tilde{i}}_{e_i}, x^{\tilde{j}}_{e_j}) \tilde{\leq} \tilde{\epsilon}$ for all $i, j \geq m$ i.e. $\tilde{d}(x^{\tilde{i}}_{e_i}, x^{\tilde{j}}_{e_j}) \to \tilde{0}$ as $i, j \to \infty$.*

Definition 1.16 (Complete Metric Space)**.** *The soft metric space $(\tilde{X}, \tilde{d}, E)$ is called complete if every Cauchy Sequence in $\tilde{X}$ converges to some point of $\tilde{X}$. The soft metric space $(\tilde{X}, \tilde{d}, E)$ is called incomplete if it is not complete.*

The aim of this paper is to introduce the modified generalized F-contractions, by combining the ideas of Dung and Hang [8], Piri and Kumam [15], Wardowski [18] and Wardowski and Van Dung [19] and give some soft fixed point result for these type mappings on complete soft metric space.

2. Main results

Let $\mathfrak{F}_G$ denote the family of all functions $F:\mathbb{R}_+\to\mathbb{R}$ which satisfy conditions (F1) and (F3) and $\mathcal{F}_G$ denote the family of all functions $F:\mathbb{R}_+\to\mathbb{R}$ which satisfy conditions (F1) and (F3).

Definition 2.1. *Let $(\tilde{X},\tilde{d},E)$ be a soft metric space and $(T,\varphi):(\tilde{X},\tilde{d},E)\to(\tilde{X},\tilde{d},E)$ be a mapping. (T,φ) is said to be modified generalized F-contraction of type (A) if there exist $F\in\mathfrak{F}_G$ and $\tau>0$ such that*

$$\tilde{x}_\lambda,\tilde{y}_\mu\in\tilde{X},\quad \left[\tilde{d}((T,\varphi)\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)>0\to\tau+F\big(\tilde{d}((T,\varphi)\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)\big)\tilde{\leq}F\big(M_{(T,\varphi)}(\tilde{x}_\lambda,\tilde{y}_\mu)\big)\right],\tag{3}$$

where

$$\begin{aligned}M_{(T,\varphi)}(\tilde{x}_\lambda,\tilde{y}_\mu)=\max\Big\{&\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu),\frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{x}_\lambda)}{2},\frac{\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,\tilde{x}_\lambda)+\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)}{2},\\&\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda\big),\big((T,\varphi)^2\tilde{x}_\lambda,\tilde{y}_\mu\big),\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu\big)+\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda),\\&\tilde{d}((T,\varphi)\tilde{x}_\lambda,\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu)\Big\}.\end{aligned}$$

Remark 2.2. *Note that $\mathfrak{F}\subseteq\mathfrak{F}_W$. Since, for $\beta\in(0,\infty)$, the function $F(\alpha)=\frac{-1}{\alpha+\beta}$ satisfies the conditions (F1) and (F3) but it does not satisfy (F2), we have $\mathfrak{F}\subset\mathfrak{F}_W$ but $\mathcal{F}\neq\mathcal{F}_W$.*

Definition 2.3. *Let $(\tilde{X},\tilde{d},E)$ be a soft metric space and $(T,\varphi):(\tilde{X},\tilde{d},E)\to(\tilde{X},\tilde{d},E)$ be a mapping. (T,φ) is said to be modified generalized F-contraction of type (B) if there exist $F\in\mathcal{F}_G$ and $\tau>0$ such that*

$$\forall\tilde{x}_\lambda,\tilde{y}_\mu\in\tilde{X},\quad \left[\tilde{d}((T,\varphi)\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)>0\Rightarrow\tau+F\big(\tilde{d}((T,\varphi)\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)\big)\tilde{\leq}F\big(M_{(T,\varphi)}(\tilde{x}_\lambda,\tilde{y}_\mu)\big)\right].$$

Remark 2.4. *Note that $\mathcal{F}\subseteq\mathcal{F}_W$. Since, for $\beta\in(0,\infty)$, the function $F(\alpha)=\ln(\alpha+\beta)$ satisfies the conditions (F1) and (F3) but it does not satisfy (F2), we have $\mathcal{F}\subset\mathcal{F}_W$ but $\mathcal{F}\neq\mathcal{F}_W$.*

Remark 2.5.

(1). Every F-contraction is a modified generalized F-contraction.

(2). Let (T,φ) be a modified generalized F-contraction. From 3 for all $\tilde{x}_\lambda,\tilde{y}_\mu\in\tilde{X}$ with $(T,\varphi)\tilde{x}_\lambda\neq(T,\varphi)\tilde{y}_\mu$, we have

$$\begin{aligned}F\big(\tilde{d}((T,\varphi)\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)\big)<&\tau+F\big(\tilde{d}((T,\varphi)\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)\big)\\ &\tilde{\leq}F\bigg(\max\Big\{\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu),\frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{x}_\lambda)}{2},\frac{\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,\tilde{x}_\lambda)+\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)}{2},\\&\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda\big),\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,\tilde{y}_\mu\big),\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu\big)+\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda),\\&\tilde{d}((T,\varphi)\tilde{x}_\lambda,\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu)\Big\}\bigg).\end{aligned}$$

Then, by (F1), we get

$$\begin{aligned}\tilde{d}((T,\varphi)\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)<\max\Big\{&\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu),\frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{x}_\lambda)}{2},\frac{\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,\tilde{x}_\lambda)+\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)}{2},\\&\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda\big),\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,\tilde{y}_\mu\big),\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu\big)+\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda),\\&\tilde{d}((T,\varphi)\tilde{x}_\lambda,\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu)\Big\},\end{aligned}$$

for all $\tilde{x}_\lambda,\tilde{y}_\mu\in\tilde{X}$, $(T,\varphi)\tilde{x}_\lambda\neq(T,\varphi)\tilde{y}_\mu$.

The following examples show that the inverse implication of Remark 2.5(1) does not hold.

Example 2.6. *Let $X = [0,2]$ and $E = [1,\infty)$ and define a soft metric d on X by $d(x,y) =\mid x-y \mid$ and $d_1(x,y) = \min\{|x-y|,1\}$ then*

$$\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu) = \frac{1}{2}[d_1(\lambda,\mu)+d(x,y)]$$

is a soft metric space. Suppose $\varphi:\mathbb{R}^+\to\mathbb{R}^+$ and let $T:X\to X$ be given by

$$\varphi(t) = \frac{1}{2}t$$

and

$$Tx = \begin{cases} 1, & x\in[0,2), \\ \frac{1}{2}, & x=2. \end{cases}$$

Obviously, $(\tilde{X},\tilde{d},E)$ is complete soft metric space. Since (T,φ) is not continuous, (T,φ) is not an F-contraction. For $x\in[0,2)$ and $y=2$, we have

$$d((T,\varphi)\tilde{x}_\lambda,(T,\varphi)2) = d\left(1,\frac{1}{2}\right) = \frac{1}{2} > 0$$

and

$$\begin{aligned}
&\max\Big\{\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu), \frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{x}_\lambda)}{2}, \frac{\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,\tilde{x}_\lambda)+\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)}{2},\\
&\quad \tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda\big), \tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,\tilde{y}_\mu\big), \tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu\big)+\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda),\\
&\quad \tilde{d}((T,\varphi)\tilde{x}_\lambda,\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu)\Big\}\\
&\quad \geq \tilde{d}((T,\varphi)\tilde{x}_\lambda,\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu)\\
&\quad = \tilde{d}(1,2)+\tilde{d}\left(2,\frac{1}{2}\right)\\
&\quad = \frac{5}{2}.
\end{aligned}$$

Therefore

$$\begin{aligned}
d((T,\varphi)\tilde{x}^0_{\lambda_0},(T,\varphi)2) \leq \frac{1}{5}\max\Big\{& d(\tilde{x}_\lambda,\tilde{y}_\mu), \frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{x}_\lambda)}{2}, \frac{\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,\tilde{x}_\lambda)+\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)}{2},\\
& \tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda\big), \tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,\tilde{y}_\mu\big), \tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu\big)+\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda),\\
& \tilde{d}((T,\varphi)\tilde{x}_\lambda,\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu)\Big\}.
\end{aligned}$$

So, by choosing $F(\alpha)=\ln(\alpha)$ and $\tau=\ln\frac{1}{5}$ we see that (T,φ) is modified generalized F-contraction of type (A) and type (B).

Example 2.7. *Let $X=\{-2,-1,0,1,2\}$ and define a soft metric $\tilde{d}$ on X by*

$$d(x,y) = \begin{cases} 0, & \text{if } x=y,\\ 2, & \text{if } (x,y)\in\{(2,-2),(-2,2)\},\\ 1, & \text{otherwise.} \end{cases}$$

Then $(\tilde{X},\tilde{d},E)$ is a complete soft metric space. Let $(T,\varphi):(\tilde{X},\tilde{d},E)\to(\tilde{X},\tilde{d},E)$ be defined by

$$(T,\varphi)(-2) = (T,\varphi)(-1) = T0 = -2, \qquad (T,\varphi)1 = -1, \qquad (T,\varphi)2 = 0.$$

First observe that

$$\tilde{d}((T,\varphi)\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu) > 0 \quad \Leftrightarrow \quad \big[\big(\tilde{x}_\lambda \in \{-2,-1,0\} \wedge \tilde{y}_\mu = 1\big) \vee \big(\tilde{x}_\lambda \in \{-2,-1,0\} \wedge \tilde{y}_\mu = 2\big) \vee (\tilde{x}_\lambda = 1, \tilde{y}_\mu = 2)\big].$$

Now we consider the following cases:

Case 1. Let $\tilde{x}_\lambda \in \{-2,-1,0\} \wedge \tilde{y}_\mu = 1$, *then*

$$\tilde{d}((T,\varphi)\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu) = \tilde{d}(-2,-1) = 1, \qquad \tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu) = \tilde{d}(\tilde{x}_\lambda,1) = 1, \qquad \tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda) = \tilde{d}(\tilde{x}_\lambda,-2) = 0 \vee 1,$$

$$\tilde{d}(y,(T,\varphi)\tilde{y}_\mu) = \tilde{d}(1,-1) = 1, \qquad \frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu) + \tilde{d}((T,\varphi)\tilde{x}_\lambda,\tilde{y}_\mu)}{2} = \frac{\tilde{d}(\tilde{x}_\lambda,-1)+\tilde{d}(-2,1)}{2} = \frac{1}{2} \vee 1,$$

$$\frac{\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,\tilde{x}_\lambda) + \tilde{d}((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)}{2} = \frac{\tilde{d}(-2,\tilde{x}_\lambda)+\tilde{d}(-2,-1)}{2} = \frac{1}{2} \vee 1,$$

$$d\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda\big) = \tilde{d}(-2,-2) = 0, \qquad d\big((T,\varphi)^2\tilde{x}_\lambda,\tilde{y}_\mu\big) = \tilde{d}(-2,1) = 1,$$

$$\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu\big) = \tilde{d}(-2,-1) = 1,$$

$$d\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu\big) + \tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda) = \tilde{d}(-2,-1) + \tilde{d}(\tilde{x}_\lambda,-2) = 1 \vee 2,$$

$$\tilde{d}((T,\varphi)\tilde{x}_\lambda,\tilde{y}_\mu) + \tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu) = \tilde{d}(-2,1) + \tilde{d}(1,-1) = 2.$$

Case 2. Let $\tilde{x}_\lambda \in \{-2,-1,0\} \wedge \tilde{y}_\mu = 2$, *then*

$$\tilde{d}((T,\varphi)\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu) = \tilde{d}(-2,0) = 1, \qquad \tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu) = \tilde{d}(\tilde{x}_\lambda,2) = 1 \vee 2, \qquad \tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda) = \tilde{d}(\tilde{x}_\lambda,-2) = 0 \vee 1,$$

$$\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu) = \tilde{d}(2,0) = 1, \qquad \frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu) + \tilde{d}((T,\varphi)\tilde{x}_\lambda,\tilde{y}_\mu)}{2} = \frac{\tilde{d}(\tilde{x}_\lambda,0)+\tilde{d}(-2,2)}{2} = 1 \vee \frac{3}{2},$$

$$\frac{\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,\tilde{x}_\lambda) + \tilde{d}((T,\varphi)^2x,(T,\varphi)\tilde{y}_\mu)}{2} = \frac{\tilde{d}(-2,\tilde{x}_\lambda)+\tilde{d}(-2,0)}{2} = \frac{1}{2} \vee 1,$$

$$d\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda\big) = \tilde{d}(-2,-2) = 0, \qquad d\big((T,\varphi)^2\tilde{x}_\lambda,\tilde{y}_\mu\big) = \tilde{d}(-2,2) = 2,$$

$$d\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu\big) = \tilde{d}(-2,0) = 1,$$

$$d\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu\big) + \tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda) = \tilde{d}(-2,0) + \tilde{d}(\tilde{x}_\lambda,-2) = 1 \vee 2,$$

$$\tilde{d}((T,\varphi)\tilde{x}_\lambda,\tilde{y}_\mu) + \tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu) = \tilde{d}(-2,2) + \tilde{d}(2,0) = 3.$$

Case 3. Let $\tilde{x}_\lambda = 1 \wedge \tilde{y}_\mu = 2$, *then*

$$\tilde{d}((T,\varphi)\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu) = \tilde{d}(-1,0) = 1, \qquad \tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu) = \tilde{d}(1,2) = 1, \qquad \tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda) = \tilde{d}(1,-1) = 1,$$

$$\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu) = \tilde{d}(2,0) = 1, \qquad \frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu) + \tilde{d}((T,\varphi)\tilde{x}_\lambda,\tilde{y}_\mu)}{2} = \frac{\tilde{d}(1,0)+\tilde{d}(-1,2)}{2} = 1,$$

$$\frac{\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,\tilde{x}_\lambda) + \tilde{d}((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)}{2} = \frac{\tilde{d}(-2,1)+\tilde{d}(-2,0)}{2} = 1,$$

$$d\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda\big) = \tilde{d}(-2,-1) = 1, \qquad d\big((T,\varphi)^2\tilde{x}_\lambda,\tilde{y}_\mu\big) = \tilde{d}(-2,2) = 2,$$

$$d\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu\big) = \tilde{d}(-2,0) = 1, \qquad d\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu\big) + \tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda) = \tilde{d}(-2,0) + \tilde{d}(1,-1) = 2,$$

$$\tilde{d}((T,\varphi)\tilde{x}_\lambda,\tilde{y}_\mu) + \tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu) = \tilde{d}(-1,2) + \tilde{d}(2,0) = 2.$$

In Case 1, we have

$$\begin{aligned}
\tilde{d}((T,\varphi)\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu) &= \max\Big\{\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu), \tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda), \tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu), \frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu) + \tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{x}_\lambda)}{2}\Big\} \\
&= \max\Big\{\frac{\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,\tilde{x}_\lambda) + \tilde{d}((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)}{2}, d\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda\big), \\
&\qquad d\big((T,\varphi)^2\tilde{x}_\lambda,\tilde{y}_\mu\big), d\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu\big)\Big\} \\
&= 1.
\end{aligned}$$

This proves that for all $F \in \mathcal{F} \cup \mathfrak{F}$, (T,φ) is not an F-weak contraction and generalized F-contraction. Since every F-contraction is an F-weak contraction and a generalized F-contraction, (T,φ) is not an F-contraction. However, we see that

$$\tilde{d}((T,\varphi)\tilde{x}_\lambda,(T,\varphi)2) \leq \frac{1}{2}\max\Big\{\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu),\frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{x}_\lambda)}{2},\frac{\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,\tilde{x}_\lambda)+\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)}{2},$$
$$\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda\big),\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,\tilde{y}_\mu\big),\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu\big)+\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda),$$
$$\tilde{d}((T,\varphi)\tilde{x}_\lambda,\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu)\Big\}.$$

Hence, by choosing $F(\alpha)=\ln(\alpha)$ and $\tau=\ln\frac{1}{2}$ we see that T is modified generalized F-contraction of type (A) and type (B).

Theorem 2.8. *Let $(\tilde{X},\tilde{d},E)$ be a complete soft metric space and $(T,\varphi):(\tilde{X},\tilde{d},E)\to(\tilde{X},\tilde{d},E)$ be a modified generalized F-contraction of type (A).Then (T,φ) has a unique soft fixed point $\tilde{x}^*_\lambda\in\tilde{X}$ and for every $\tilde{x}^0_{\lambda_0}\in\tilde{X}$ the sequence $\{T^n\tilde{x}^0_{\lambda_0}\}_{n\in\mathbb{N}}$ converges to $\tilde{x}^*_{\lambda_*}$.*

Proof. Let $\tilde{x}^0_{\lambda_0}\in X$. Put $\tilde{x}^{n+1}_{\lambda_{n+1}}=T^n\tilde{x}^0_{\lambda_0}$ for all $n\in\mathbb{N}$. If, there exists $n\in\mathbb{N}$ such that $\tilde{x}^{n+1}_{\lambda_{n+1}}=\tilde{x}^n_{\lambda_n}$, then $(T,\varphi)\tilde{x}^n_{\lambda_n}=\tilde{x}^n_{\lambda_n}$. That is, $\tilde{x}^n_{\lambda_n}$ is a soft fixed point of (T,φ). Now, we suppose that $\tilde{x}^{n+1}_{\lambda_{n+1}}\neq\tilde{x}^n_{\lambda_n}$ for all $n\in\mathbb{N}$. Then $\tilde{d}(\tilde{x}^{n+1}_{\lambda_{n+1}},\tilde{x}^n_{\lambda_n})>0$ for all $n\in\mathbb{N}$. It follows from (3) that, for all $n\in\mathbb{N}$,

$$\begin{aligned}\tau+F\big(\tilde{d}((T,\varphi)\tilde{x}^{n-1}_{\lambda_{n-1}},(T,\varphi)\tilde{x}^n_{\lambda_n})\big)\tilde{\leq}F\Bigg(&\max\Big\{\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^n_{\lambda_n}),\frac{\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}},(T,\varphi)\tilde{x}^n_{\lambda_n})+\tilde{d}(\tilde{x}^n_{\lambda_n},(T,\varphi)\tilde{x}^{n-1}_{\lambda_{n-1}})}{2},\\
&\frac{\tilde{d}((T,\varphi)^2\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^{n-1}_{\lambda_{n-1}})+\tilde{d}((T,\varphi)^2\tilde{x}^{n-1}_{\lambda_{n-1}},T\tilde{x}^n_{\lambda_n})}{2},\\
&\tilde{d}\big((T,\varphi)^2\tilde{x}^{n-1}_{\lambda_{n-1}},(T,\varphi)\tilde{x}^{n-1}_{\lambda_{n-1}}\big),\tilde{d}\big((T,\varphi)^2\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^n_{\lambda_n}\big),\\
&\tilde{d}\big((T,\varphi)^2\tilde{x}^{n-1}_{\lambda_{n-1}},(T,\varphi)\tilde{x}^n_{\lambda_n}\big)+\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}},(T,\varphi)\tilde{x}^{n-1}_{\lambda_{n-1}}),\\
&\tilde{d}((T,\varphi)\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^n_{\lambda_n})+\tilde{d}(\tilde{x}^n_{\lambda_n},(T,\varphi)\tilde{x}^n_{\lambda_n})\Big\}\Bigg)\\
=F\Bigg(&\max\Big\{\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^n_{\lambda_n}),\frac{\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^{n+1}_{\lambda_{n+1}})+\tilde{d}(\tilde{x}^n_{\lambda_n},\tilde{x}^n_{\lambda_n})}{2},\\
&\frac{\tilde{d}(\tilde{x}^{n+1}_{\lambda_{n+1}},\tilde{x}^{n-1}_{\lambda_{n-1}})+\tilde{d}(\tilde{x}^{n+1}_{\lambda_{n+1}},\tilde{x}^{n+1}_{\lambda_{n+1}})}{2},\tilde{d}(\tilde{x}^{n+1}_{\lambda_{n+1}},\tilde{x}^{n+1}_{\lambda_{n+1}}),\tilde{d}(\tilde{x}^{n+1}_{\lambda_{n+1}},\tilde{x}^n_{\lambda_n}),\\
&\tilde{d}(\tilde{x}^{n+1}_{\lambda_{n+1}},\tilde{x}^{n+1}_{\lambda_{n+1}})+\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^n_{\lambda_n}),\tilde{d}(\tilde{x}^n_{\lambda_n},\tilde{x}^n_{\lambda_n})+\tilde{d}(\tilde{x}^n_{\lambda_n},\tilde{x}^{n+1}_{\lambda_{n+1}})\Big\}\Bigg)\\
=F\big(&\max\{\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^n_{\lambda_n}),\tilde{d}(\tilde{x}^n_{\lambda_n},\tilde{x}^{n+1}_{\lambda_{n+1}})\}\big).\end{aligned}\tag{4}$$

If there exists $n\in\mathbb{N}$ such that

$$\max\{\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^n_{\lambda_n}),\tilde{d}(\tilde{x}^n_{\lambda_n},\tilde{x}^{n+1}_{\lambda_{n+1}})\}=\tilde{d}(\tilde{x}^n_{\lambda_n},\tilde{x}^{n+1}_{\lambda_{n+1}})$$

then 4 becomes

$$\tau+F\big(\tilde{d}(\tilde{x}^n_{\lambda_n},\tilde{x}^{n+1}_{\lambda_{n+1}})\big)\tilde{\leq}F\big(\tilde{d}(\tilde{x}^n_{\lambda_n},\tilde{x}^{n+1}_{\lambda_{n+1}})\big).$$

Since $\tau>0$, we get a contradiction. Therefore

$$\max\big\{\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^n_{\lambda_n}),\tilde{d}(\tilde{x}^n_{\lambda_n},\tilde{x}^{n+1}_{\lambda_{n+1}})\big\}=\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^n_{\lambda_n}),\quad\forall n\in\mathbb{N}.$$

Thus, from 4, we have

$$\begin{aligned}F\big(\tilde{d}(\tilde{x}^n_{\lambda_n},\tilde{x}^{n+1}_{\lambda_{n+1}})\big)&=F\big(\tilde{d}((T,\varphi)\tilde{x}^{n-1}_{\lambda_{n-1}},(T,\varphi)\tilde{x}^n_{\lambda_n})\big)\tilde{\leq}F\big(\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^n_{\lambda_n})\big)-\tau\\
&<F\big(\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}},\tilde{x}^n_{\lambda_n})\big).\end{aligned}\tag{5}$$

It follows from 5 and (F1) that

$$\tilde{d}(\tilde{x}^{n}_{\lambda_n}, \tilde{x}^{n+1}_{\lambda_{n+1}}) < \tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}}, \tilde{x}^{n}_{\lambda_n}), \quad \forall n \in \mathbb{N}.$$

Therefore $\{\tilde{d}(\tilde{x}^{n+1}_{\lambda_{n+1}}, \tilde{x}^{n}_{\lambda_n})\}_{n\in\mathbb{N}}$ is a nonnegative decreasing sequence of real numbers, and hence

$$\lim_{n\to\infty} \tilde{d}(\tilde{x}^{n+1}_{\lambda_{n+1}}, \tilde{x}^{n}_{\lambda_n}) = \gamma \geq 0.$$

Now, we claim that $\gamma = 0$. Arguing by contradiction, we assume that $\gamma > 0$. Since $\{\tilde{d}(\tilde{x}^{n+1}_{\lambda_{n+1}}, \tilde{x}^{n}_{\lambda_n})\}_{n\in\mathbb{N}}$ is a nonnegative decreasing sequence, for every $n \in \mathbb{N}$, we have

$$\tilde{d}(\tilde{x}^{n+1}_{\lambda_{n+1}}, \tilde{x}^{n}_{\lambda_n}) \geq \gamma. \tag{6}$$

From 6 and (F1), we get

$$\begin{aligned} F(\gamma) &\leq F\big(\tilde{d}(\tilde{x}^{n+1}_{\lambda_{n+1}}, \tilde{x}^{n}_{\lambda_n})\big) \tilde{\leq} F\big(\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}}, \tilde{x}^{n}_{\lambda_n})\big) - \tau \\ &\leq F\big(\tilde{d}(\tilde{x}^{n-2}_{\lambda_{n-2}}, \tilde{x}^{n-1}_{\lambda_{n-1}})\big) - 2\tau \\ &\ \ \vdots \\ &\leq F\big(\tilde{d}(\tilde{x}^{0}_{\lambda_0}, \tilde{x}^{1}_{\lambda_1})\big) - n\tau, \end{aligned} \tag{7}$$

for all $n \in \mathbb{N}$. Since $F(\gamma) \in \mathbb{R}$ and $\lim_{n\to\infty}[F(\tilde{d}(\tilde{x}^{0}_{\lambda_0}, \tilde{x}^{1}_{\lambda_1})) - n\tau] = -\infty$, there exists $n_1 \in \mathbb{N}$ such that

$$F\big(\tilde{d}(\tilde{x}^{0}_{\lambda_0}, \tilde{x}^{1}_{\lambda_1})\big) - n\tau < F(\gamma), \quad \forall n > n_1. \tag{8}$$

It follows from 7 and 8 that

$$F(\gamma) \tilde{\leq} F\big(\tilde{d}(\tilde{x}^{0}_{\lambda_0}, \tilde{x}^{1}_{\lambda_1})\big) - n\tau < F(\gamma), \quad \forall n > n_1.$$

It is a contradiction. Therefore, we have

$$\lim_{n\to\infty} \tilde{d}(\tilde{x}^{n}_{\lambda_n}, (T,\varphi)\tilde{x}^{n}_{\lambda_n}) = \lim_{n\to\infty} \tilde{d}(\tilde{x}^{n}_{\lambda_n}, \tilde{x}^{n+1}_{\lambda_{n+1}}) = 0. \tag{9}$$

Simply, we can prove that $\{\tilde{x}^{n}_{\lambda_n}\}_{n=1}^{\infty}$ is a Cauchy sequence. So by completeness of $(\tilde{X}, \tilde{d}, E)$, $\{\tilde{x}^{n}_{\lambda_n}\}_{n=1}^{\infty}$ converges to some point $\tilde{x}^{*}_{\lambda_*}$ in $\tilde{X}$. Therefore,

$$\lim_{n\to\infty} \tilde{d}\big(\tilde{x}^{n}_{\lambda_n}, \tilde{x}^{*}_{\lambda_*}\big) = 0. \tag{10}$$

Finally, we will show that $\tilde{x}^{*}_{\lambda_*} = (T,\varphi)\tilde{x}^{*}_{\lambda_*}$. We only have the following two cases:

(I) $\forall n \in \mathbb{N}$, $\exists i_n \in \mathbb{N}$, $i_n > i_{n-1}$, $i_0 = 1$ and $\tilde{x}^{i_n+1}_{\lambda_{i_n+1}} = (T,\varphi)\tilde{x}^{*}_{\lambda_*}$,

(II) $\exists n_3 \in \mathbb{N}$, $\forall n \geq n_3$, $\tilde{d}((T,\varphi)\tilde{x}^{n}_{\lambda_n}, (T,\varphi)\tilde{x}^{*}_{\lambda_*}) > 0$.

In the first case, we have

$$\tilde{x}^{*}_{\lambda_*} = \lim_{n\to\infty} x_{i_{n+1}} = \lim_{n\to\infty} (T,\varphi)\tilde{x}^{*}_{\lambda_*} = (T,\varphi)\tilde{x}^{*}_{\lambda_*}.$$

In the second case from the assumption of Theorem 2.8, for all $n \geq n_3$, we have

$$\begin{aligned}\tau + F\big(\tilde{d}(\tilde{x}^{n+1}_{\lambda_{n+1}}, (T,\varphi)\tilde{x}^*_{\lambda_*})\big) &= \tau + F\big(\tilde{d}((T,\varphi)\tilde{x}^n_{\lambda_n}, (T,\varphi)\tilde{x}^*_{\lambda_*})\big)\\ &\tilde{\leq} F\Bigg(\max\Big\{\tilde{d}(\tilde{x}^n_{\lambda_n}, \tilde{x}^*_{\lambda_*}), \frac{\tilde{d}(\tilde{x}^n_{\lambda_n}, (T,\varphi)\tilde{x}^*_{\lambda_*}) + \tilde{d}(\tilde{x}^*_{\lambda_*}, (T,\varphi)\tilde{x}^n_{\lambda_n})}{2},\\ &\quad \frac{\tilde{d}((T,\varphi)^2\tilde{x}^n_{\lambda_n}, \tilde{x}^n_{\lambda_n}) + \tilde{d}((T,\varphi)^2\tilde{x}^n_{\lambda_n}, (T,\varphi)\tilde{x}^*_{\lambda_*})}{2}, \tilde{d}\big((T,\varphi)^2\tilde{x}^n_{\lambda_n}, T\tilde{x}^n_{\lambda_n}\big), \tilde{d}\big((T,\varphi)^2\tilde{x}^n_{\lambda_n}, \tilde{x}^*_{\lambda_*}\big),\\ &\quad \tilde{d}\big((T,\varphi)^2\tilde{x}^n_{\lambda_n}, (T,\varphi)\tilde{x}^*_{\lambda_*}\big) + \tilde{d}(\tilde{x}^n_{\lambda_n}, T\tilde{x}^n_{\lambda_n}), \tilde{d}\big((T,\varphi)\tilde{x}^n_{\lambda_n}, \tilde{x}^*_{\lambda_*}\big) + \tilde{d}\big(\tilde{x}^*_{\lambda_*}, (T,\varphi)\tilde{x}^*_{\lambda_*}\big)\Big\}\Bigg).\end{aligned} \tag{11}$$

From (F3), 10, and taking the limit as $n \to \infty$ in 11, we obtain

$$\tau + F\big(\tilde{d}(\tilde{x}^*_{\lambda_*}, (T,\varphi)\tilde{x}^*_{\lambda_*})\big) \tilde{\leq} F\big(\tilde{d}(\tilde{x}^*_{\lambda_*}, (T,\varphi)\tilde{x}^*_{\lambda_*})\big).$$

This is a contradiction. Hence, $\tilde{x}^*_{\lambda_*} = (T,\varphi)\tilde{x}^*_{\lambda_*}$. Now, let us to show that (T,φ) has at most one soft fixed point. Indeed, if $\tilde{x}^*_{\lambda_*}, \tilde{y}^*_{\lambda_*} \in \tilde{X}$ are two distinct soft fixed points of (T,φ), that is, $vT\tilde{x}^*_{\lambda_*} = \tilde{x}^*_{\lambda_*} \neq \tilde{y}^*_{\lambda_*} = (T,\varphi)\tilde{y}^*_{\lambda_*}$, then

$$\tilde{d}\big((T,\varphi)\tilde{x}^*_{\lambda_*}, (T,\varphi)\tilde{y}^*_{\lambda_*}\big) = \tilde{d}\big(\tilde{x}^*_{\lambda_*}, \tilde{y}^*_{\lambda_*}\big) > 0.$$

It follows from 3 that

$$\begin{aligned}F\big(\tilde{d}(\tilde{x}^*_{\lambda_*}, \tilde{y}^*_{\lambda_*})\big) &< \tau + F\big(\tilde{d}(\tilde{x}^*_{\lambda_*}, \tilde{y}^*_{\lambda_*})\big)\\ &= \tau + F\big(\tilde{d}((T,\varphi)\tilde{x}^*_{\lambda_*}, (T,\varphi)\tilde{y}^*_{\lambda_*})\big)\\ &\tilde{\leq} F\Bigg(\max\Big\{\tilde{d}(\tilde{x}^*_{\lambda_*}, \tilde{y}^*_{\lambda_*}), \frac{\tilde{d}(\tilde{x}^*_{\lambda_*}, (T,\varphi)\tilde{y}^*_{\lambda_*}) + \tilde{d}(\tilde{y}^*_{\lambda_*}, (T,\varphi)\tilde{x}^*_{\lambda_*})}{2},\\ &\quad \frac{\tilde{d}((T,\varphi)^2\tilde{x}^*_{\lambda_*}, \tilde{x}^*_{\lambda_*}) + \tilde{d}((T,\varphi)^2\tilde{x}^*_{\lambda_*}, (T,\varphi)\tilde{y}^*_{\lambda_*})}{2}, \tilde{d}\big((T,\varphi)^2\tilde{x}^*_{\lambda_*}, (T,\varphi)\tilde{x}^*_{\lambda_*}\big), \tilde{d}\big((T,\varphi)^2\tilde{x}^*_{\lambda_*}, \tilde{y}^*_{\lambda_*}\big),\\ &\quad \tilde{d}\big((T,\varphi)^2\tilde{x}^*_{\lambda_*}, (T,\varphi)\tilde{y}^*_{\lambda_*}\big) + \tilde{d}\big(\tilde{x}^*_{\lambda_*}, (T,\varphi)\tilde{x}^*_{\lambda_*}\big), \tilde{d}\big((T,\varphi)\tilde{x}^*_{\lambda_*}, \tilde{y}^*_{\lambda_*}\big) + \tilde{d}\big(\tilde{y}^*_{\lambda_*}, (T,\varphi)\tilde{y}^*_{\lambda_*}\big)\Big\}\Bigg)\\ &= F\Bigg(\max\Big\{\tilde{d}(\tilde{x}^*_{\lambda_*}, \tilde{y}^*_{\lambda_*}), \frac{\tilde{d}(\tilde{x}^*_{\lambda_*}, \tilde{x}^0_{\lambda_0}\tilde{y}^*_{\lambda_*}) + \tilde{d}(\tilde{y}^*_{\lambda_*}, \tilde{x}^*_{\lambda_*})}{2}, \frac{\tilde{d}(\tilde{x}^*_{\lambda_*}, \tilde{x}^*_{\lambda_*}) + \tilde{d}(\tilde{x}^*_{\lambda_*}, \tilde{y}^*_{\lambda_*})}{2},\\ &\quad \tilde{d}(\tilde{x}^*_{\lambda_*}, \tilde{x}^*_{\lambda_*}), \tilde{d}(\tilde{x}^*_{\lambda_*}, \tilde{y}^*_{\lambda_*}), \tilde{d}(\tilde{x}^*_{\lambda_*}, \tilde{y}^*_{\lambda_*}) + \tilde{d}(\tilde{x}^*_{\lambda_*}, \tilde{x}^*_{\lambda_*}), \tilde{d}(\tilde{x}^*_{\lambda_*}, \tilde{y}^*_{\lambda_*}) + \tilde{d}(\tilde{y}^*_{\lambda_*}, \tilde{x}^*_{\lambda_*})\Big\}\Bigg)\\ &= F\big(\tilde{d}(\tilde{x}^*_{\lambda_*}, \tilde{y}^*_{\lambda_*})\big),\end{aligned} \tag{12}$$

which is a contradiction. Therefore, the soft fixed point is unique. □

Theorem 2.9. *Let $(\tilde{X}, \tilde{d}, E)$ be a complete soft metric space and $(T,\varphi) : \tilde{X} \to \tilde{X}$ be a continuous modified generalized F-contraction of type (B). Then (T,φ) has a unique soft fixed point $\tilde{x}^*_{\lambda_*} \in \tilde{X}$ and for every $\tilde{x}_\lambda \in \tilde{X}$ the sequence $\{(T,\varphi)^n\tilde{x}^0_{\lambda_0}\}_{n\in\mathbb{N}}$ converges to $\tilde{x}^*_{\lambda_*}$.*

Proof. By using a similar method to that used in the proof of Theorem 2.8, we have

$$\begin{aligned}F\big(\tilde{d}(\tilde{x}^n_{\lambda_n}, \tilde{x}^{n+1}_{\lambda_{n+1}})\big) = F\big(\tilde{d}((T,\varphi)\tilde{x}^{n-1}_{\lambda_{n-1}}, (T,\varphi)\tilde{x}^n_{\lambda_n})\big) &\tilde{\leq} F\big(\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}}, \tilde{x}^n_{\lambda_n})\big) - \tau\\ &< F\big(\tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}}, \tilde{x}^n_{\lambda_n})\big)\end{aligned}$$

and

$$\lim_{n\to\infty} d(\tilde{x}^n_{\lambda_n}, (T,\varphi)\tilde{x}^n_{\lambda_n}) = \lim_{n\to\infty} d(\tilde{x}^n_{\lambda_n}, \tilde{x}^{n+1}_{\lambda_{n+1}}) = 0.$$

By simple calculates we can prove that $\{\tilde{x}^n_{\lambda_n}\}_{n=1}^{\infty}$ is a Cauchy sequence. So, by completeness of $(\tilde{X},\tilde{d},E)$, $\{\tilde{x}^n_{\lambda_n}\}_{n=1}^{\infty}$ converges to some point $\tilde{x}^*_{\lambda_*}\in\tilde{X}$. Since (T,φ) is continuous, we have

$$\tilde{d}\big(\tilde{x}^*_{\lambda_*},(T,\varphi)\tilde{x}^*_{\lambda_*}\big)=\lim_{n\to\infty}\tilde{d}(\tilde{x}^n_{\lambda_n},(T,\varphi)\tilde{x}^n_{\lambda_n})=\lim_{n\to\infty}d(\tilde{x}^n_{\lambda_n},\tilde{x}^{n+1}_{\lambda_{n+1}})=0.$$

Again by using similar method as used in the proof of Theorem 2.8, we can prove that $\tilde{x}^*_{\lambda_*}$ is the unique soft fixed point of (T,φ). □

3. Some Applications

Theorem 3.1. *Let (T,φ) be a self-mapping of a complete soft metric space $(\tilde{X},\tilde{d},E)$ into itself. Suppose there exist $F\in\mathfrak{F}$ and $\tau>0$ such that*

$$\forall\tilde{x}_\lambda,\tilde{y}_\mu\in\tilde{X},\quad\big[\tilde{d}((T,\varphi)\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)>0\Rightarrow\tau+F\big(\tilde{d}((T,\varphi)\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)\big)\tilde{\leq}F\big(\tilde{d}(x,y)\big)\big].$$

Then (T,φ) has a unique soft fixed point $\tilde{x}^*_{\lambda_*}\in\tilde{X}$ and for every $\tilde{x}^0_{\lambda_0}\in\tilde{X}$ the sequence $\{(T,\varphi)^n\tilde{x}^0_{\lambda_0}\}_{n=1}^{\inf(T,\varphi)\tilde{y}_\mu}$ converges to $\tilde{x}^*_{\lambda_*}$.

Proof. Since

$$\begin{aligned}&\max\Big\{\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu),\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda),\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu),\frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{x}_\lambda)}{2}\Big\}\\
&\tilde{\leq}\max\Big\{\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu),\frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{x}_\lambda)}{2},\frac{\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,\tilde{x}_\lambda)+\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)}{2},\\
&\quad\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda\big),\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,\tilde{y}_\mu\big),\\
&\quad\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu\big)+\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda),\tilde{d}((T,\varphi)\tilde{x}_\lambda,\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu)\Big\},\end{aligned}$$

from (F1) and Theorem 2.8 the proof is complete. □

Theorem 3.2. *Let $(\tilde{X},\tilde{d},E)$ be a complete soft metric space and let $(T,\varphi):\tilde{X}\to\tilde{X}$ be an F-contraction. Then (T,φ) has a unique soft fixed point $\tilde{x}^*_{\lambda_*}\in\tilde{X}$ and for every $\tilde{x}_\lambda\in\tilde{X}$ the sequence $\{(T,\varphi)^n\tilde{x}_\lambda\}_{n\in\mathbb{N}}$ converges to $\tilde{x}^*_{\lambda_*}$.*

Proof. Since

$$\begin{aligned}&\max\Big\{\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu),\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda),\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu),\frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{x}_\lambda)}{2}\Big\}\\
&\tilde{\leq}\max\Big\{\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu),\frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{x}_\lambda)}{2},\frac{\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,\tilde{x}_\lambda)+\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)}{2},\\
&\quad\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda\big),\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,\tilde{y}_\mu\big),\tilde{d}\big((T,\varphi)^2x,(T,\varphi)\tilde{y}_\mu\big)+\tilde{d}(x,(T,\varphi)\tilde{x}_\lambda),\\
&\quad\tilde{d}((T,\varphi)\tilde{x}_\lambda,y)+\tilde{d}(y,(T,\varphi)\tilde{y}_\mu)\Big\}.\end{aligned}$$

So from (F1) and Theorem 2.9 the proof is complete. □

Theorem 3.3 ([19]). *Let $(\tilde{X},\tilde{d},E)$ be a complete soft metric space and let $(T,\varphi):\tilde{X}\to\tilde{X}$ be an F-weak contraction. If (T,φ) or F is continuous,then (T,φ) has a unique soft fixed point $\tilde{x}^*_{\lambda_*}\in\tilde{X}$ and for every $\tilde{x}_\lambda\in\tilde{X}$ the sequence $\{(T,\varphi)^n\tilde{x}_\lambda\}_{n\in\mathbb{N}}$ converges to $\tilde{x}^*_{\lambda_*}$.*

Proof. Since

$$\max\left\{\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu),\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda),\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu),\frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{x}_\lambda)}{2}\right\}$$
$$\tilde{\leq}\max\Big\{\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu),\frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{x}_\lambda)}{2},$$
$$\frac{\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,\tilde{x}_\lambda)+\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)}{2},\tilde{d}\big((T,\varphi)^2x,(T,\varphi)\tilde{x}_\lambda\big),\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,\tilde{y}_\mu\big),$$
$$\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu\big)+\tilde{d}(x,(T,\varphi)\tilde{x}_\lambda),\tilde{d}((T,\varphi)\tilde{x}_\lambda,y)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu)\Big\},$$

if F is continuous, from (F1) and Theorem 2.8 the proof is complete. If (T,φ) is continuous, from (F1) and Theorem 2.9 the proof is complete. □

Theorem 3.4 ([8])**.** *Let $(\tilde{X},\tilde{d},E)$ be a complete soft metric space and let $(T,\varphi):\tilde{X}\to\tilde{X}$ be a generalized F-contraction. If (T,φ) or F is continuous, then (T,φ) has a unique soft fixed point $\tilde{x}^*_{\lambda_*}\in\tilde{X}$ and for every $\tilde{x}_\lambda\in\tilde{X}$ the sequence $\{(T,\varphi)^n\tilde{x}_\lambda\}_{n\in\mathbb{N}}$ converges to $\tilde{x}^*_{\lambda_*}$.*

Proof. Since

$$\max\Big\{\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu),\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda),\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu),\frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{x}_\lambda)}{2},$$
$$\frac{\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,\tilde{x}_\lambda)+\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)}{2},\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda\big),\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,\tilde{y}_\mu\big),d\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu\big)\Big\}$$
$$\tilde{\leq}\max\Big\{\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu),\frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{x}_\lambda)}{2},\frac{\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,\tilde{x}_\lambda)+\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)}{2},$$
$$\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda\big),\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,\tilde{y}_\mu\big),\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu\big)+\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda),\tilde{d}((T,\varphi)\tilde{x}_\lambda,\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu)\Big\},$$

if F is continuous, from (F1) and Theorem 2.8 the proof is complete. If (T,φ) is continuous, from (F1) and Theorem 2.9 the proof is complete. □

Theorem 3.5. *Let $(\tilde{X},\tilde{d},E)$ be a complete soft metric space and let $(T,\varphi):\tilde{X}\to\tilde{X}$ be a function with the following property:*

$$\tilde{d}((T,\varphi)\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)\tilde{\leq}\alpha\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu)+\beta\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda)+\gamma\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu),\tag{13}$$

where $\alpha,\beta,$ and γ are nonnegative and satisfy $\alpha+\beta+\gamma<1$. Then (T,φ) has a unique soft fixed point.

Proof. From 13, we have

$$\tilde{d}((T,\varphi)\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)\tilde{\leq}(\alpha+\beta+\gamma)\max\Big\{\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu),\frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{x}_\lambda)}{2},$$
$$\frac{\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,\tilde{x}_\lambda)+\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)}{2},\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda\big),d\big((T,\varphi)^2\tilde{x}_\lambda,\tilde{y}_\mu\big),$$
$$d\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu\big)+\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda),\tilde{d}((T,\varphi)\tilde{x}_\lambda,\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu)\Big\}.$$

Then if $\tilde{d}((T,\varphi)\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)>0$, we have

$$\ln\frac{1}{\alpha+\beta+\gamma}+\ln\big(\tilde{d}((T,\varphi)\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)\big)\tilde{\leq}\ln\Big(\max\Big\{\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu),\frac{\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{x}_\lambda)}{2},$$
$$\frac{\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,\tilde{x}_\lambda)+\tilde{d}((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu)}{2},\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda\big),\tilde{d}\big((T,\varphi)^2\tilde{x}_\lambda,\tilde{y}_\mu\big),$$
$$d\big((T,\varphi)^2\tilde{x}_\lambda,(T,\varphi)\tilde{y}_\mu\big)+\tilde{d}(\tilde{x}_\lambda,(T,\varphi)\tilde{x}_\lambda),\tilde{d}((T,\varphi)\tilde{x}_\lambda,\tilde{y}_\mu)+\tilde{d}(\tilde{y}_\mu,(T,\varphi)\tilde{y}_\mu)\Big\}\Big).$$

Therefore by taking $F(\alpha)=\ln(\alpha)$ and $\tau=\ln\frac{1}{\alpha+\beta+\gamma}$ in Theorem 2.8 or in Theorem 2.9the proof is complete. □

Theorem 3.6. *Let (T, φ) be a self-mapping of a complete soft metric space $(\tilde{X}, \tilde{d}, E)$ into itself and an F-contraction. Then (T, φ) has a unique soft fixed point $\tilde{x}^*_{\lambda_*} \in \tilde{X}$ and for every $\tilde{x}^0_{\lambda_0} \in \tilde{X}$ the sequence $\{(T, \varphi)^n \tilde{x}^0_{\lambda_0}\}_{n=0}^{\infty}$ converges to $\tilde{x}^*_{\lambda_*}$.*

Theorem 3.7. *Let (T, φ) be a self-mapping of a complete soft metric space $(\tilde{X}, \tilde{d}, E)$ into itself. Suppose $F \in \mathcal{F}$ and there exists $\tau > 0$ such that $\forall\ \ \tilde{x}_\lambda, \tilde{y}_\mu \in \tilde{X},\ \ \{d(Tx, Ty) > 0 \Longrightarrow \tau + F(d((T, \varphi)\tilde{x}_\lambda, (T, \varphi)\tilde{y}_\mu)) \leq F(d(\tilde{x}_\lambda, \tilde{y}_\mu))\}$. Then (T, φ) has a unique soft fixed point $\tilde{x}^*_{\lambda_*} \in \tilde{X}$ and for every $\tilde{x}^0_{\lambda_0} \in \tilde{X}$ the sequence $\{(T, \varphi)^n \tilde{x}^0_{\lambda_0}\}_{n=0}^{\infty}$ converges to $\tilde{x}^*_{\lambda_*}$.*

Theorem 3.8. *Let $(\tilde{X}, \tilde{d}, E)$ be a complete soft metric space and $T : \tilde{X} \to \tilde{X}$ be an F-weak contraction. If (T, φ) or F is continuous, then we have*

(1). (T, φ) has a unique soft fixed point $\tilde{x}^*_{\lambda_*} \in \tilde{X}$.

(2). For all $\tilde{x}_\lambda \in \tilde{X}$, the sequence $\{(T, \varphi)^n \tilde{x}_\lambda\}$ is convergent to $\tilde{x}^*_{\lambda_*}$.

Theorem 3.9. *Let $(\tilde{X}, \tilde{d}, E)$ be a complete soft metric space and $T : \tilde{X} \to \tilde{X}$ be a generalized F-contraction. If (T, φ) or F is continuous, then we have*

(1). (T, φ) has a unique soft fixed point $\tilde{x}^*_{\lambda_*} \in \tilde{X}$.

(2). For each $\tilde{x}_\lambda \in \tilde{X}$, if $(T, \varphi)^{n+1}\tilde{x}_\lambda = (T, \varphi)^n \tilde{x}_\lambda$ for all $n \in \mathbb{N} \cup \{0\}$, then $\lim_{n\to\infty}(T, \varphi)^n \tilde{x}_\lambda = \tilde{x}^*_{\lambda_*}$.

4. Conclusion

Our theorems are extensions of the above theorems in the following aspects:

(1). Theorem 2.8 gives all consequences of Theorem 3.7, without assumption (F2) used in its proof.

(2). Theorem 2.9 gives all consequences of Theorem 3.6, without assumption (F2) used in its proof.

(3). If in Theorem 3.9, F is continuous, Theorem 2.8 gives all consequences of Theorem 3.9, without assumptions (F2) and (F3) used in its proof.

(4). If in Theorem 3.9, (T, φ) is continuous, Theorem 2.9 gives all consequences of Theorem 3.9 without assumption (F2) used in its proof.

(5). Because every F-weak contraction is a generalized F-contraction, 3 and 4 are also true for Theorem 3.8.

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