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Some Rational Contractions for Coupled Coincidence and Common Coupled Fixed Point Theorems in Complex-Valued Metric Spaces

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Abstract: The aim of this paper is to obtain a coupled coincidence point theorem and a common coupled fixed point theorem of contractive type mappings involving rational expressions in the framework of a complex-valued metric spaces. We also improve the result obtain by [4]. The results of this paper generalize and extend the results of Kang [3], in complex-valued metric spaces.

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1. Introduction and Preliminaries

In 2011, Azam [2] introduced the notion of complex valued metric space which is a generalization of the classical metric space. They established some fixed point results for mappings satisfying a rational inequality. The idea of complex valued metric spaces can be exploited to define complex valued normed spaces and complex valued Hilbert spaces; additionally, it offers numerous research activities in mathematical analysis. Let $\mathbb{C}$ be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$, we define a partial order $\preceq$ on $\mathbb{C}$ as follows:

$$z_1 \preceq z_2 \quad \text{if and only if} \quad Re(z_1) \leq Re(z_2) \quad \text{and} \quad Im(z_1) \leq Im(z_2).$$

It follows that $z_1 \preceq z_2$ if one of the following conditions is satisfied:

(C1) $Re(z_1) = Re(z_2)$ and $Im(z_1) = Im(z_2)$;

(C2) $Re(z_1) < Re(z_2)$ and $Im(z_1) = Im(z_2)$;

(C3) $Re(z_1) = Re(z_2)$ and $Im(z_1) < Im(z_2)$;

(C4) $Re(z_1) < Re(z_2)$ and $Im(z_1) < Im(z_2)$.

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In particular, we will write $z_1 \prec z_2$ if $z_1 \neq z_2$ and one of (C2), (C3) and (C4) is satisfied and we will write $z_1 \prec z_2$ if only (C4) is satisfied.

Remark 1.1. *We obtained that the following statements hold:*

(1). If $a, b \in \mathbb{R}$ with $a \leq b$, then $az \prec bz$ for all $z \in \mathbb{C}$.

(2). If $0 \preceq z_1 \prec z_2$, then $|z_1| < |z_2|$.

(3). If $z_1 \preceq z_2$ and $z_2 \prec z_3$, then $z_1 \prec z_3$.

Consistent with Azam [2], we state some definitions and results about the complex-valued metric space to prove our main results.

Definition 1.2. *Let X be a nonempty set. Suppose that the mapping $d : X \times X \to \mathbb{C}$ satisfies the following conditions:*

(d1) $0 \preceq d(x, y)$ for all $x, y \in X$;

(d2) $d(x, y) = 0$ if and only if $x = y$ for all $x, y \in X$;

(d3) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(d4) $d(x, y) \preceq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called a complex-valued metric on X and (X, d) is called a complex-valued metric space.

Example 1.3. *Let $X = \mathbb{C}$. Define the mapping $d : X \times X \to \mathbb{C}$ by $d(z_1, z_2) = 2i|z_1 - z_2|$ for all $z_1, z_2 \in X$. Then (X, d) is a complex valued metric space.*

Definition 1.4. *Let (X, d) be a complex-valued metric space.*

(1). Apoint $x \in X$ is called interior point of a set $B \subseteq X$ whenever there exists $0 \prec r \in \mathbb{C}$ such that

$$N(x, r) := \{y \in X : d(x, y) \prec r\} \subseteq B.$$

(2). A point $x \in X$ is called limit point of a set $B \subseteq X$ whenever there exists $0 \prec r \in \mathbb{C}$ such that$N(x, r) \cap (B\{x\}) \neq \phi$.

(3). A subset $B \subseteq X$ is called open whenever each element of B is an interior point of B.

(4). A subset $B \subseteq X$ is called closed whenever each limit point of B belongs to B.

(5). The family $F = \{N(x, r) : x \in X, 0 \prec r\}$ is a sub-basis for a topology on X. We denote this complex topology by τ_c. Indeed, the topology τ_c is Hausdorff.

Definition 1.5. *Let (X, d) be a complex-valued metric space, and let $\{x_n\}$ be a sequence in X and $x \in X$.*

(1). If for every $c \in \mathbb{C}$ with $0 \prec c$ there is $N \in \mathbb{N}$ such that for all $n > N$, $d(x_n, x) \prec c$ then $\{x_n\}$ is said to be convergent, if $\{x_n\}$ converges to x and x is the limit point of $\{x_n\}$. We denote this by $x_n \to x$ as $n \to \infty$ or $\lim_{n\to\infty} x_n = x$.

(2). If for every $c \in \mathbb{C}$ with $0 \prec c$ there is $N \in \mathbb{N}$ such that for all $n, m > N$, $d(x_n, x_m) \prec c$ then $\{x_n\}$ is said is said to be Cauchy sequence.

(3). If every Cauchy sequence in X is convergent, then (X, d) is said to be a complete complex-valued metric space.

Lemma 1.6. *Let (X,d) be a complex-valued metric space, and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $|d(x_n,x)| \to 0$ as $n \to \infty$.*

Lemma 1.7. *Let (X,d) be a complex-valued metric space, and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $|d(x_n,x_{n+m})| \to 0$ as $n \to \infty$.*

In 2006, Bhaskar [1] introduced the notion of coupled fixed point and proved some fixed point results in this context. Similarly, Kang [3] introduce the notion of coupled fixed point for a mapping in complex valued metric spaces as follows.

Definition 1.8. *Let (X,d) be a complex-valued metric space, an element $(x,y) \in X \times X$ is said to be a coupled fixed point of the mapping $F : X \times X \to X$ if $F(x,y) = x$ and $F(y,x) = y$.*

Definition 1.9. *Let (X,d) be a complex valued metric space. An element $(x,y) \in X \times X$ is said to be*

(1). A coupled coincidence point of mappings $F : X \times X \to X$ and $g : X \to X$ if $g(x) = F(x,y)$ and $g(y) = F(y,x)$, and (gx,gy) is called a coupled point of coincidence if there exists $(u,v) \in X \times X$ such that $x = gu = F(u,v)$ and $y = gv = F(v,u)$.

(2). A common coupled fixed point of mappings $F : X \times X \to X$ and $g : X \to X$ if $x = gx = F(x,y)$ and $y = gy = F(y,x)$.

Definition 1.10. *Let (X,d) be a complex-valued metric space. The mappings $F : X \times X \to X$ and $g : X \to X$ are called w-compatible if $g(F(x,y)) = F(gx,gy)$, whenever $gx = F(x,y)$ and $gy = F(y,x)$.*

Kang [3] prove following result,

Theorem 1.11. *([3], Theorem-2.1) Let (X,d) be a complex valued metric space. Suppose that the mapping $F : X \times X \to X$ satisfies*

$$d(F(x,y),F(u,v)) \leq hd(x,u) + kd(y,v) \tag{1}$$

for all $x,y,u,v \in X$, where h and k are non-negative constants with $h+k < 1$. Then F has a unique coupled fixed point.

In [4], Jhade and Khan prove following result,

Theorem 1.12 ([4], Theorem 3.1)**.** *Let (X,d) be a complex-valued metric space. Let $F : X \times X \to X$ and $g : X \to X$ be two mappings. Suppose that there exist nonnegative constants $a_i \in [0,1), i = 1,2,\ldots,6$ such that $\Sigma_{i=1}^{6} a_i < 1$ and for all $x,y,u,v \in X$*

$$\begin{aligned} d(F(x,y),F(u,v)) \preceq\ & a_1 d(gx,gu) \\ & + a_2(gy,gv) + a_3 \frac{d(gx,F(x,y))d(gu,F(u,v))}{d(gx,gu)} + a_4 \frac{d(gx,F(u,v))d(gu,F(x,y))}{d(gx,gu)} \\ & + a_5 \frac{d(gy,F(y,x))d(gv,F(v,u))}{d(gy,gv)} + a_6 \frac{d(gy,F(v,u))d(gv,F(y,x))}{d(gy,gv)}. \end{aligned} \tag{2}$$

Suppose $F(X \times X) \subseteq g(X)$ and $g(X)$ is a complete subspace of X. Then F and g have a coupled coincidence point $(x^*,y^*) \in X \times X$.

Remark 1.13. *It should be noted that Theorem 1.12 is not true for $x = u$ and $y = v$, i.e., 2 is not valid for $x = u$ and $y = v$ and we can not obtain coupled fixed point.*

2. Main Results

First we improve the Theorem 1.12 and prove a coupled coincidence point theorem which state is as follows,

Theorem 2.1. *Let (X, d) be a complex-valued metric space. Let $F : X \times X \to X$ and $g : X \to X$ be two mappings. Suppose that there exist nonnegative constants $a_i \in [0, 1), i = 1, 2, \ldots, 6$ such that $\Sigma_{i=1}^{6} a_i < 1$ and for all $x, y, u, v \in X$*

$$\begin{aligned} d(F(x,y),F(u,v)) \preceq\ & a_1 d(gx,gu) + a_2 d(gy,gv) \\ & + a_3 \frac{[1+d(gx,F(x,y))]d(gu,F(u,v))}{d(gx,gu)+1} + a_4 \frac{[1+d(gx,F(u,v))]d(gu,F(x,y))}{d(gx,gu)+1} \\ & + a_5 \frac{[1+d(gy,F(y,x))]d(gv,F(v,u))}{d(gy,gv)+1} + a_6 \frac{[1+d(gy,F(v,u))]d(gv,F(y,x))}{d(gy,gv)+1}. \end{aligned} \tag{3}$$

Suppose $F(X \times X) \subseteq g(X)$ and $g(X)$ is a complete subspace of X. Then F and g have a coupled coincidence point $(x^*, y^*) \in X \times X$.

Proof. Let $x_0, y_0 \in X$ are arbitrary. Set $gx_1 = F(x_0, y_0)$ and $gy_1 = F(y_0, x_0)$, this can be done because $F(X \times X) \subseteq g(X)$. Continuing the process, we obtain two sequences $\{x_n\}$ and $\{y_n\}$ such that $gx_{n+1} = F(x_n, y_n)$ and $gy_{n+1} = F(y_n, x_n)$ for all $n \geq 0$. Then we have

$$\begin{aligned} d(gx_n, gx_{n+1}) =\ & d(F(x_{n-1}, y_{n-1}), F(x_n, y_n)) \\ \preceq\ & a_1 d(gx_{n-1}, gx_n) + a_2 d(gy_{n-1}, gy_n) \\ & + a_3 \frac{[1+d(gx_{n-1},F(x_{n-1},y_{n-1}))]d(gx_n,F(x_n,y_n))}{d(gx_{n-1},gx_n)+1} + a_4 \frac{[1+d(gx_{n-1},F(x_n,y_n))]d(gx_n,F(x_{n-1},y_{n-1}))}{d(gx_{n-1},gx_n)+1} \\ & + a_5 \frac{[1+d(gy_{n-1},F(y_{n-1},x_{n-1}))]d(gy_n,F(y_n,x_n))}{d(gy_{n-1},gy_n)+1} + a_6 \frac{[1+d(gy_{n-1},F(y_n,x_n))]d(gy_n,F(y_{n-1},x_{n-1}))}{d(gy_{n-1},gy_n)+1} \\ d(gx_n, gx_{n+1}) \preceq\ & a_1 d(gx_{n-1}, gx_n) + a_2 d(gy_{n-1}, gy_n) \\ & + a_3 \frac{[1+d(gx_{n-1},gx_n)]d(gx_n,gx_{n+1})}{d(gx_{n-1},gx_n)+1} + a_4 \frac{[1+d(gx_{n-1},gx_{n+1})]d(gx_n,gx_n)}{d(gx_{n-1},gx_n)+1} \\ & + a_5 \frac{[1+d(gy_{n-1},gy_n)]d(gy_n,gy_{n+1})}{d(gy_{n-1},gy_n)+1} + a_6 \frac{[1+d(gy_{n-1},d(gy_{n-1},gy_n))]d(gy_n,gy_n)}{d(gy_{n-1},gy_n)+1} \end{aligned}$$

which implies

$$|d(gx_n, gx_{n+1})| \preceq a_1|d(gx_{n-1}, gx_n)| + a_2|d(gy_{n-1}, gy_n)| + a_3|d(gx_n, gx_{n+1})| + a_5|d(gy_n, gy_{n+1})| \tag{4}$$

Similarly we have

$$|d(gy_n, gy_{n+1})| \preceq a_1|d(gy_{n-1}, gy_n)| + a_2|d(gx_{n-1}, gx_n)| + a_3|d(gy_n, gy_{n+1})| + a_5|d(gx_n, gx_{n+1})|. \tag{5}$$

Suppose that $d_n = \|d(gx_n, gx_{n+1}\| + \|d(gy_n, gy_{n+1}\|$. Adding inequalities 4 and 5, we obtain

$$d_n \leq (a_1 + a_2)d_{n-1} + (a_3 + a_5)d_n \tag{6}$$

that is $d_n \leq hd_{n-1}$, where $h = \frac{a_1+a_2}{1-(a_3+a_5)} < 1$. Thus, we have

$$d_n \leq hd_{n-1} \leq h^2 d_{n-2} \leq h^3 d_{n-3} \leq h^4 d_{n-4} \leq \cdots \leq h^n d_0. \tag{7}$$

We shall show that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences. If $m > n$, then we have

$$\begin{aligned} |d(gx_n, gx_m| + |d(gy_n, gy_m| &\leq |d(gx_n, gx_{n+1}| + |d(gy_n, gy_{n+1}| + |d(gx_{n+1}, gx_{n+2}| + |d(gy_{n+1}, gy_{n+2}| \\ &\quad +|d(gx_{n+2}, gx_{n+3}| + |d(gy_{n+2}, gy_{n+3}| + \cdots + |d(gx_{m-1}, gx_m| + |d(gy_{m-1}, gy_m| \\ &\leq h^n d_0 + h^{n+1} d_0 + h^{n+2} d_0 + h^{n+3} d_0 + \cdots + h^{m-1} d_0 \\ &\leq \frac{h^n}{1-h} d_0 \to 0 \quad \text{as} \quad n \to \infty. \end{aligned}$$

Hence $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in $g(X)$. Since $g(X)$ is complete, there exists x^* and y^* such that $gx_n \to x^*$ and $gy_n \to y^*$ as $n \to \infty$. On the other hand, we have from 3,

$$\begin{aligned} d(F(x^*, y^*), gx^*) &\preceq d(F(x^*, y^*), gx_{n+1}) + d(gx_{n+1}, gx^*) \\ &= d(F(x^*, y^*), F(x_n, y_n)) + d(gx_{n+1}, gx^*) \\ d(F(x^*, y^*), gx^*) &\preceq a_1 d(gx^*, gx_n) + a_2 d(gy^*, gy_n) \\ &\quad + a_3 \frac{[1 + d(gx^*, F(x^*, y^*))] d(gx_n, F(x_n, y_n))}{d(gx^*, gx_n) + 1} + a_4 \frac{[1 + d(gx^*, F(x_n, y_n))] d(gx_n, F(x^*, y^*))}{d(gx^*, gx_n) + 1} \\ &\quad + a_5 \frac{[1 + d(gy^*, F(y^*, x^*))] d(gy_n, F(y_n, x_n))}{d(gy^*, gy_n) + 1} + a_6 \frac{[1 + d(gy^*, F(y_n, x_n))] d(gy_n, F(y^*, x^*))}{d(gy^*, gy_n) + 1} \\ &\quad + |d(gx_{n+1}, gx^*)| \\ d(F(x^*, y^*), gx^*) &\preceq a_1 d(gx^*, gx_n) + a_2 d(gy^*, gy_n) \\ &\quad + a_3 \frac{[1 + d(gx^*, F(x^*, y^*))] d(gx_n, gx_{n+1})}{d(gx^*, gx_n) + 1} + a_4 \frac{[1 + d(gx^*, gx_{n+1})] d(gx_n, F(x^*, y^*))}{d(gx^*, gx_n) + 1} \\ &\quad + a_5 \frac{[1 + d(gy^*, F(y^*, x^*))] d(gy_n, gy_{n+1})}{d(gy^*, gy_n) + 1} + a_6 \frac{[1 + d(gy^*, gy_{n+1})] d(gy_n, F(y^*, x^*))}{d(gy^*, gy_n) + 1} \\ &\quad + |d(gx_{n+1}, gx^*)| \\ |d(F(x^*, y^*), gx^*)| &\preceq a_1 |d(gx^*, gx_n)| + a_2 |d(gy^*, gy_n)| \\ &\quad + a_3 \frac{[1 + |d(gx^*, F(x^*, y^*))|](|d(gx_n, gx^*)| + |d(gx^*, gx_{n+1})|)}{|d(gx^*, gx_n)| + 1} + a_4 \frac{[1 + |d(gx^*, gx_{n+1})|] |d(gx_n, F(x^*, y^*))|}{|d(gx^*, gx_n)| + 1} \\ &\quad + a_5 \frac{[1 + |d(gy^*, F(y^*, x^*))|](|d(gy_n, gy^*)| + |d(gy^*, gy_{n+1})|)}{|d(gy^*, gy_n)| + 1} + a_6 \frac{[1 + |d(gy^*, gy_{n+1})|] |d(gy_n, F(y^*, x^*))|}{|d(gy^*, gy_n)| + 1} \\ &\quad + |d(gx_{n+1}, gx^*)|. \end{aligned}$$

Since $gx_n \to gx^*$ and $gy_n \to gy^*$ as $n \to \infty$, we have $|d(F(x^*, y^*), gx^*)| \leq 0$. That is, $F(x^*, y^*) = gx^*$. Similarly one can show that $F(y^*, x^*) = gy^*$. Hence (x^*, y^*) is a coupled coincidence point of F and g. □

For common coupled fixed point for the mappings F and g, the condition of Theorem 2.1 are not enough. So by applying the condition of w-compatibility on F and g, we obtain the following common coupled fixed point theorem.

Theorem 2.2. *In addition to the hypotheses of Theorem 2.1 are not enough to prove the existence of a common coupled fixed point for the mappings F and g. By applying the condition of w-compatibility on F and g, we obtain the following common coupled fixed point theorem, if F and g are w-compatible, then F and g have a unique common coupled fixed point. Moreover, a common coupled fixed point of F and g is of the form (u, v) for some $u, v \in X$.*

Proof. The existence of coupled coincidence point (x^*, y^*) of F and g follows from Theorem 2.1. Then (gx^*, gy^*) is a coupled point of coincidence of F, g and so $gx^* = F(x^*, y^*)$ and $gy^* = F(y^*, x^*)$.

First we will show that this coupled point of coincidence is unique. For this, suppose that F and g have another coupled point of coincidence (gu, gv), that is, $gu = F(u, v)$ and $gv = F(v, u)$ where $(u, v) \in X \times X$. Then we have

$$d(F(x^*, y^*), F(u, v)) \preceq a_1 d(gx^*, gu) + a_2 d(gy^*, gv)$$

$$+a_3\frac{[1+d(gx^*,F(x^*,y^*))]d(gu,F(u,v))}{d(gx^*,gu)+1}+a_4\frac{[1+d(gx^*,F(u,v))]d(gu,F(x^*,y^*))}{d(gx^*,gu)+1}$$
$$+a_5\frac{[1+d(gy^*,F(y^*,x^*))]d(gv,F(v,u))}{d(gy^*,gv)+1}+a_6\frac{[1+d(gy^*,F(v,u))]d(gv,F(y^*,x^*))}{d(gy^*,gv)+1}.$$

Hence

$$|d(gx^*,gu)|\preceq a_1|d(gx^*,gu)|+a_2|d(gy^*,gv)|+a_4|d(gx^*,gu)|+a_6|d(gy^*,gv)|. \tag{8}$$

Similarly we obtain

$$|d(gy^*,gv)|\preceq a_1|d(gy^*,gv)|+a_2|d(gx^*,gu)|+a_4|d(gy^*,gv)|+a_6|d(gx^*,gu)|. \tag{9}$$

Adding 8 and 9 we obtain

$$|d(gx^*,gu)|+|d(gy^*,gv)|\leq(a_1+a_2+a_4+a_6)[|d(gx^*,gu)|+|d(gy^*,gv)|].$$

Since $(a_1+a_2+a_4+a_6)<1$. Therefore,

$$|d(gx^*,gu)|+|d(gy^*,gv)|\leq 0$$

which contradiction. Hence $d(gx^*,gu)=0$ and $d(gy^*,gv)=0$, i.e., $gx^*=gu$ and $gy^*=gv$. Thus $(gx^*,gy^*)=(u,v)$ is the unique coupled point of coincidence of F and g. Now if F and g are w-compatible, then $gu=g(F(x^*,y^*))=F(gx^*,gy^*)=F(u,v)=w(\text{say})$. Similarly, we obtain $gv=g(F(y^*,x^*))=F(gy^*,gx^*)=F(v,u)=z(\text{say})$. So, (w,z) is another coupled point of coincidence of F and g. By uniqueness, we have $(u,v)=(w,z)$, that is, $gu=F(u,v)=u$ and $gv=F(v,u)=v$. Thus (u,v) is the unique common coupled fixed point of F and g. □

Example 2.3. *Let $X=\{ix:x\in[0,1]\}$ and consider a complex valued metric $d:X\times X\to X$ defined by $d(x,y)=i|x-y|$ for all $x,y\in X$. Then (X,d) is a complex valued metric space. Define the mappings $F:X\times X\to X$ and $g:X\to X$ by $F(x,y)=i\left(\frac{x}{10}+\frac{y}{15}\right)$ and $g(x)=\frac{x}{5}i$ for all $x,y\in[0,1]$. Then we have*

$$\begin{aligned} d(F(x,y),F(u,v)) &= i|i\left(\frac{x}{10}+\frac{y}{15}\right)-i\left(\frac{u}{10}+\frac{v}{15}\right)| \\ &= i|i\left(\frac{x}{10}-\frac{u}{10}\right)-i\left(\frac{y}{15}-\frac{v}{15}\right)| \\ &\leq \frac{5}{10}i|i\left(\frac{x}{5}-\frac{u}{5}\right)|+\frac{5}{15}i|i\left(\frac{y}{5}-\frac{v}{5}\right)| \\ &\leq \frac{1}{2}d(gx,gu)+\frac{1}{3}d(gy,gv) \end{aligned}$$

where $a_1=\frac{1}{2}$, $a_2=\frac{1}{3}$, $a_i=0, i=3,4,5,6$. Note that $a_1+a_2=\frac{5}{6}+\frac{5}{6}<1$, $F(X\times X)\subseteq g(X)$ and $g(X)$ is a complete subspace of X. Hence the condition of Theorem 2.1 are satisfied, that is, F and g have a coupled coincidence point (0,0). Furthermore, since F and g are w-compatible, hence, Theorem 2.2 shows that (0,0) is the unique common coupled fixed point of F and g.

Remark 2.4. *It should be noted that Example 2.3 is valid for Theorem 2.1 as well as for Theorem 1.12 . In fact Theorem 2.1 is more general the Theorem 1.12 .*

Remark 2.5. *If we take $a_i=0$ for $i=3,4,5,6$ and $g=I_X$ (identity mapping over X) in Theorem 2.1 then we get result of Kang [3].*

Corollary 2.6 ([3]). *Let (X, d) be a complete complex valued metric space. Suppose that the mapping $F : X \times X \to X$ satisfies $d(F(x, y), F(u, v)) \leq h[(d(x, u) + d(y, v))]$ for all $x, y, u, v \in X$, where h is a non-negative constant with $h < \frac{1}{2}$. Then F has a unique coupled fixed point.*

Proof. If we take $a_1 = a_2 = h$, $a_i = 0 \quad for \quad i = 3, 4, 5, 6$ and $g = I_X$ (identity mapping over X) in Theorem 2.1, then we get required result. □

Example 2.7. *Let $X = \{ix : x \in [0, 1]\}$ and consider a complex valued metric $d : X \times X \to X$ defined by $d(x, y) = i|x - y|$ for all $x, y \in X$. Then (X, d) is a complex valued metric space. Define the mappings $F : X \times X \to X$ by $F(x, y) = i\left(\frac{x+y}{3}\right)$ for all $x, y \in [0, 1]$. Then we have $h = \frac{1}{3} < \frac{1}{2}$. So all condition of Corollary 2.6 are satisfied and we get (0,0) is a coupled fixed of F.*

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