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The Machine Interference Model; (m,n) Systems with Balking and Spares

Research Article

R. Sivaraman[1,*]

1 Department of Mathematics, D. G. Vaishnav College, Chennai, Tamil Nadu, India.

Abstract: In this paper, we have obtained an analytical solution of the queue: (m, n) system for machine interference with balking and spares considering the discipline FIFO. The probabilities have been calculated for the transient case, we have calculated the system availability and up time ratio. They had studied (m,n) system with spares only. It has been discussed the truncated Poissonion queue: $MIM/C/k/N$ for machine interference but without assumption of balking, reneging or spares, They considered the system; MIMICm m with spares only. Medhi obtained a solution for the system: $MIMIC/mn/rn$ without assumption of balking, reneging or spares, and derived an analytical solution of the queue: $M/MJC/k/N$ for machine interference with balking, reneging and spares.

Keywords: Poisson Queue, Birth-Death coefficients, Probability differential difference equations, Up Time Ratio (UTR).

1. The Model

We assume a multi-component system, which consists of n identical and independent components each with failure rate λ arranged to form an (m,n) system. Suppose r repair facilities are available and the repair time is exponentially distributed with mean $1/t$. we assume that, initially we have N spares on hand so that a failed component can be replaced immediately with n spare. Thus a component can be in any one of three possible states: (1) operating in system, (2) waiting in spares storage to be used and (3) waiting for or receiving repair service. Suppose that i indicates a system state in which exactly i components are failed. Thus the only possible system states are $0,1,\ldots,N+n-m+1$. Let $\lambda_i\Delta t$, t denote the probability that a failed component joins repair facslity during an interval of length Δt and let $\mu_i\Delta t$ denote the probability that a repaired component leaves the repair facility during an interval of length Δt. A transition from $i+j$ to i or from $i-j$ to i for $j=2$ has probability of magnitude $o(\Delta t)$, hence the only possible transition can be of the form $i\to i+1$ for $i=0,1,...,N+n-m$ or 1 for $i\to l,2,\ldots,N+n-m+1$. Now we assume that b is the probability that a unit joins the queue. Then we may say that $(I-b)$ is the probability that a unit does not join the queue (because of lack of space), we can say that the unit is in balked situation.

$0\le b<1$

0, if $r<i\le N+n-m+1$

and 1, if $i<r$.

Now we discus tow cases as follows:

* E-mail: rsivaraman1729@yahoo.co.in (Research Scholar)

Case 1: $r \leq N$. The set of birth - death coefficients are

$$\lambda_i = \begin{cases} n\lambda, & \text{if } 0 \leq i < r \\ nb\lambda, & \text{if } r \leq i < N \\ (N+n-i)b\lambda, & \text{if } N \leq i < N+n-m \\ 0, & \text{otherwise} \end{cases}$$

and

$$\mu_i = \begin{cases} i\mu, & \text{if } 1 \leq i \leq r \\ r\mu, & \text{if } r \leq i \leq N+n-m+1 \\ 0, & \text{otherwise} \end{cases}$$

We may write the probability differential - difference equations in the following manner:

$$P'_0(t) = -n\lambda P_0(t) + \mu P_1(t), \quad i = 0 \tag{1}$$

$$P'_i(t) = -(n\lambda + i\mu) P_i(t) + n\lambda P_{i-1}(t) + (i+l)\mu P_{i-1}(t), \quad 1 \leq i < r \tag{2}$$

$$P'_r(t) = -(nb\lambda + r\mu) P_r(t) + n\lambda P_{r-1}(t) + r\mu P_{r-1}(t), \quad i = r \tag{3}$$

$$P'_i(t) = -(nb\lambda + r\mu) P_i(t) + n\lambda P_{i-1}(t) + r\mu P_{i-1}(t), \quad r+1 \leq i \leq N \tag{4}$$

$$P'_i(t) = -\{(N+n-i)b\lambda + r\mu\}\ P_i(t) + (N+n-i+1)b\lambda P_{i-1}(t) + r\mu P_{i-1}(t), \quad N < i < N+n-m+1 \tag{5}$$

$$P'_{N+n-m+1}(t) = -r\mu P_{N+n-m+1} + mb\lambda P_{N+n-m} = 0, \quad i = N+n-m+1 \tag{6}$$

We assume that initial conditions are $P_0(0) = 1$ and $P_i(0) = 0$. Now, taking Laplace transform of above equations, we obtain the following system or equations:

$$(s + n\lambda) L_0(s) - \mu L_1(s) = 1, \quad i = 0 \tag{7}$$

$$(s + n\lambda + i\mu) L_i(s) - n\lambda L_{i-1}(s) - (i+1)\mu L_{i-1}(s) = 0, \quad 1 \leq i < r \tag{8}$$

$$(s + nb\lambda + r\mu) L_r(s) - n\lambda L_{r-1}(s) - r\mu L_{r-1}(s) = 0, \quad i = r \tag{9}$$

$$(s + nb\lambda + r\mu) L_i(s) - nb\lambda L_{i-1}(s) - r\mu L_{i-1}(s) = 0, \quad r+1 \leq i \leq N \tag{10}$$

$$[s + \{(N+n-i)b\lambda + r\mu\}]\ L_i(s) - (N+n-i+1)b\lambda L_{i-1}(s) - r\mu L_{i-1}(s) = 0, \quad N < i < N+n-m+1 \tag{11}$$

$$(s + r\mu) L_{N+n-m+1}(s) - mb\lambda L_{N+n-m}(s) = 0, \quad i = N+n-m+1 \tag{12}$$

To solve the above system of equations, we write these in matrix form

$$AX = B \tag{13}$$

where

$$X = \begin{bmatrix} L_0(s) \\ L_1(s) \\ \vdots \\ L_{N+n-m+1}(s) \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

And A is a $(N+n-m+2) \times (N+n-m+2)$ matrix. The entries are given by $a_{11} = (s + n\lambda)$; $a_{12} = -\mu$, $a_{13} =$ otherwise.

For $2 \leq i < r$.

$$a_{ij-1} = -n\lambda,\ a_{ij} = s + nb\lambda + r\mu,\ a_{ij-1} = -r\mu,\ a_{ij} = 0,\ otherwise;$$

For $r + 1 \leq i \leq N$.

$$a_{ij-1} = -nb\lambda,\ a_{ij} = s + nb\lambda + r\mu,\ a_{ij-1} = -r\mu,\ a_{ij} = 0,\ otherwise;$$

For $N < I \leq N + n - m + 1$.

$$a_{ij-1} = -\left(N + n - i + 2\right) b\lambda,\ a_{ij} = s + \left\{\left(N + n - i + 1\right) b\lambda + r\mu\right\},\ a_{ij-1} = -r\mu, a_{ij} = 0,\ otherwise;$$

For $i = N + n - m + 2$.

$$a_{N+n-m-2N+n-m+1} = -mb\lambda,\ a_{N+n-m-2N+n-m+2} = s + r\mu.$$

Case 2: $N < r = N + n - m$. The birth-death coefficients are

$$\lambda_i = \begin{cases} n\lambda, & \text{if } 0 \leq i < N \\ (n + N - i)\lambda, & \text{if } N \leq i < r \\ (n + N - i)b\lambda, & \text{if } r \leq i < N + n - m \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_i = \begin{cases} i\mu, & \text{if } 0 \leq i \leq r \\ r\mu, & \text{if } r < i \leq N + n - m \\ 0 & \text{otherwise} \end{cases}$$

The probability differential- difference equations are:

$$P_0'\left(t\right) = -n\lambda P_0\left(t\right) + \mu P_1\left(t\right), \quad i = 0 \tag{14}$$

$$P_i'\left(t\right) = -(n\lambda + i\mu)P_i\left(t\right) + n\lambda P_{i-1}\left(t\right) + \left(i+1\right)\mu P_{i-1}\left(t\right), \quad 1 \leq i \leq N \tag{15}$$

$$P_i'\left(t\right) = [(N + n - i)\lambda + i\mu]P_i\left(t\right) + (N + n - i + 1)\lambda P_{i-1}\left(t\right) + \left(i+1\right)\mu P_{i-1}\left(t\right), \quad N < i < r \tag{16}$$

$$P_i'\left(t\right) = [(N + n - r)b\lambda + r\mu]P_r\left(t\right) + (N + n - r + 1)\lambda P_{r-1}\left(t\right) + r\mu P_{r-1}\left(t\right), \quad i = r \tag{17}$$

$$P_i'\left(t\right) = [(N + n - i)b\lambda + r\mu]P_i\left(t\right) + (N + n - i + 1)b\lambda P_{i-1}\left(t\right) + r\mu P_{i-1}\left(t\right), \quad r < i < N + n - m + 1 \tag{18}$$

$$P_{N-n+m+1}'\left(t\right) = -r\mu P_{N-n+m+1}\left(t\right) + mb\lambda P_{N-n+m}\left(t\right), \quad i = N + n - m + 1 \tag{19}$$

$$\tag{20}$$

Taking Laplace transform, we obtain the following system of equations:

$$(s + n\lambda)L_0\left(s\right) - \mu L_1\left(s\right) = 1 \tag{21}$$

For $1 \leq i \leq N$.

$$\left(s + n\lambda + i\mu\right) L_i\left(s\right) - n\lambda L_{i-1}\left(s\right) - \left(i-1\right)\mu L_{i-1}\left(s\right) = 0 \tag{22}$$

For $N < i < r$.

$$[s + \left(N + n - i\right)\lambda + i\mu]L_i\left(s\right) - \left(N + n - i + 1\right)\lambda L_{i-1}\left(s\right) - \left(i-1\right)\mu L_{i-1}\left(s\right) = 0 \tag{23}$$

For $i = r$.

$$[s + (N + n - r)\, b\lambda + r\mu] L_r\,(s) - (N + n - r + 1)\,\lambda L_{r-1}\,(s) - r\mu L_{r-1}\,(s) = 0 \tag{24}$$

For $r < i < N + n - m + 1$.

$$[s + (N + n - i)\, b\lambda + r\mu] L_i\,(s) - (N + n - i + 1)\, b\lambda L_{i-1}\,(s) - r\mu L_{i-1}\,(s) = 0 \tag{25}$$

For $i = N + n - m + 1$.

$$(s + rr\mu) L_{N+n-m+1}\,(s) - mb\lambda L_{N+n-m}\,(s) = 0 \tag{26}$$

In this case the entries of matrix A can be written as:

$$a_{11} = s + n\lambda,\ a_{12} = -\mu,\ a_{ij} = 0,\ otherwise;$$

For $2 \leq i \leq N + 1$.

$$a_{ii-1} = -n\lambda,\ a_{ii} = s + n\lambda + (i-1)\,\mu,\ a_{ij+1} = -i\mu,\ a_{ij} = 0,\ otherwise;$$

For $N + 1 < i < r + 1$.

$$a_{ij-1} = -\,(N + n - i + 2)\,\lambda,\ a_{ij} = s + (N + n - i + 1)\lambda + (i-1)\,\mu,\ a_{ij-1} = -i\mu,\ a_{ij} = 0,\ otherwise;$$

For $i = r + 1$.

$$a_{ij-1} = -\,(N + n - i + 2)\,\lambda,\ a_{ij} = s + (N + n - i + 1)\lambda + (i-1)\,\mu,\ a_{ij-1} = -i\mu,\ a_{ij} = 0,\ otherwise;$$

For $r < i \leq N + n - m + 1$.

$$a_{ij-1} = -\,(N + n - i + 2)\, b\lambda,\ a_{ij} = s + (N + n - i + 1) b\lambda + r\mu,\ a_{ij-1} = -r\mu,\ a_{ij} = 0,\ otherwise;$$

For $i = N + n - m + 2$.

$$a_{N+n-m+2N-n+m-1} = -mb\lambda a_{N+n-m+2N-n-m-2} = s + r\mu$$

The system availability is given by

$$A(t) = 1 - P_{N+n-m+1} \tag{27}$$

Using the method of determinants, we obtain

$$L_{N+n-m+1} = \frac{\det A'}{\det A} \tag{28}$$

where the matrix A can be obtained from A replacing the $(N + n - m + 2)^{th}$ column vector B.

$$\det A' = n^N . b^{i-r} . \lambda^{N+n-m+1} . \frac{n!}{(m-1)!}$$

So

$$L_{N+n-m+1}\,(s) = n^N . b^{i-r} . \lambda^{N+n-m+1} . \frac{n!}{(m-1)! \det A} \tag{29}$$

From equation (28), we can obtain the value of P (t) by expanding det A and taking inverse Laplace transform. With the help of Equations (1)-(6) and (14)-(19), we can also obtain steady - state solution in both cases.

Case: $r \leq N$

$$P_i = \begin{cases} \dfrac{(n\lambda)^i}{i!\ \mu^i} P_0, & \text{if } 0 \leq i < r \quad (30) \\ \dfrac{(n\lambda)^i b^{i-r}}{i!\ \mu^i . r^{i-r}} P_0, & \text{if } r \leq i \leq N \quad (31) \\ \dfrac{n^N.\ \lambda^i.\ b^{i-r} n\,(n-1)\,(n-2)\ldots(n+N-i+1)}{r!\mu^i.r^{i-r}}, & \text{if } N < i \leq N+n-m \quad (32) \end{cases}$$

Case: $N < r \leq N+n-m$

$$P_i = \begin{cases} \dfrac{(n\lambda)^i}{i!\ \mu^i} P_0, & \text{if } 1 \leq i < N \quad (33) \\ \dfrac{n^N.\ \lambda^i.\ n\,(n-1)\ldots(n+N-i+1)}{i!\ \mu^i.} P_0, & \text{if } N < i < r \quad (34) \\ \dfrac{n^N.\ \lambda^i.\ b^{i-r} n\,(n-1)\,(n-2)\ldots(n+N-i+1)}{r!\mu^i.r^{i-r}}, & \text{if } r \leq i \leq N+n-m+1 \quad (35) \end{cases}$$

Where

$$P_0 = 1 - \sum_{i=1}^{N+n-m+1} P_i \tag{36}$$

The computation of the up time ratio thus follows from the fact that

$$UTR = 1 - P_{N+n-m+1} \tag{37}$$

Special Case:

Let $b \Rightarrow (m, n)$ systems with spares only. From equations (29)-(34), we have

Case 1: $r = N$.

$$P_i = \begin{cases} \frac{(n\lambda)^i}{i!\mu^i} P_0, & \text{if } 0 \leq i < r \\ \frac{(n\lambda)^i}{i!\mu^i.r^{i-r}} P_0, & \text{if } r \leq i \leq N \\ \frac{n^N,\lambda^i.b^{i-r} n(n-1)(n-2)\ldots(n+N-i+1)}{r!\mu^i.r^{i-r}}, & \text{if } N < i \leq N+n-m+1 \end{cases}$$

Case 2: $N < r \leq N+n-m$.

$$P_i = \begin{cases} \frac{(n\lambda)^i}{i!\mu^i} P_0, & \text{if } 1 \leq i \leq N \\ \frac{n^N.\lambda^i.n(n-1)\ldots(n+N-i+1)}{i!\mu^i} P_0, & \text{if } N \leq i \leq r \\ \frac{n^N \lambda^i \cdot n(n-1)(n-2)\ldots(n+N-i+1)}{r!\mu^i r^{i-r}}, & \text{if } r \leq i \leq N+n-m+1 \end{cases}$$

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