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A New Generalization of Power Lomax Distribution

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Abstract: A new generalization of the power Lomax (PL) distribution, called exponentiated power Lomax (EPL), is introduced for modeling lifetime data. Some statistical properties of the EPL distribution are provided. Explicit expressions for the quntile, moments, moment generating function, probability weighted moments and order statistics are studied. Maximum likelihood estimation technique is employed to estimate the model parameters are presented. In addition, the superiority of the EPL distribution is illustrated with applications to two real data sets.

Keywords: Exponentiated-G family; Power Lomax distribution, Order statistics; moments.

1. Introduction

Lomax [10] introduced The Lomax (L) distribution. The L distribution has found wide applications such as the analysis of the business failure life time data, income and wealth inequality, medical and biological sciences, engineering, lifetime and reliability modeling. The L distribution is used for reliability modelling and life testing by Hassan and Al-Ghamdi [7]. Corbelini [2] proposed it to model firm size and queuing problems. Many researchers introduced several generalizations of the L distribution. Ghitany [6] investigated the Marshal–Olkin extended L distribution, Abdul-Moniem and Abdel-Hameed [1] introduced the exponentiated L distribution, Lemonte and Cordeiro [9] proposed the McDonald L, Cordeiro [3] investigated the gamma L distribution. The exponential L distribution is studied by ElBassiouny [4]. Al-Weighted L introduced by Kilany [8], and Tahir [12] introduced Weibull L distribution. Rady [11] introduced Power Lomax (PL) distribution it has The cumulative distribution function (cdf) of is

$$G\left(x\right) = 1 - \left(1 + \frac{x^{\beta}}{\lambda}\right)^{-\alpha}; \quad x > 0; \alpha, \lambda, \beta > 0.$$

Where α and β are two shape parameters and λ is a scale parameter. The corresponding probability density function (pdf) of PL distribution is

$$g\left(x\right) = \frac{\alpha\beta}{\lambda} x^{\beta-1} \left(1 + \frac{x^{\beta}}{\lambda}\right)^{-\alpha-1}; \quad x > 0; \alpha, \lambda, \beta > 0.$$

The construction of the exponentiated distribution is rather simple and is based on the observation that by raising an arbitrary cdf $G(x)$ to an arbitrary power $\gamma > 0$, a new cdf $F(x) = G(x)^{\gamma}$ emerges with one additional parameter. The corresponding density functions is $f(x) = \gamma g(x) G\left(x\right)^{\gamma-1}$. In this article we wish to introduce a new flexible distribution called the EPL distribution. The rest of the paper is arranged as follows: In Section 2, we define the EPL distribution.

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Reliability analysis is studied in Section 3. In Section 4, we derive a very useful expansion for the EPL density and distribution functions. Further, we derive some mathematical properties of the new distribution. The maximum likelihood (ML) method is used to estimate the model parameters in Section 5. Simulation study is carried out to estimate the model parameters of EPL distribution in Section 6. In Section 7, we using two real data sets to show the importance of the EPL distribution. Finally, concluding remarks in Section 8.

2. The EPL Distribution

In this section,we introduce the four-parameter EPL distribution, the cdf and pdf of the EPL distribution is given by

$$F(x) = \left[1 - \left(1 + \frac{x^{\beta}}{\lambda}\right)^{-\alpha}\right]^{\gamma}, \quad x, \alpha, \beta, \lambda, \gamma > 0. \tag{1}$$

and

$$f(x) = \frac{\alpha\beta\gamma}{\lambda} x^{\beta-1} \left(1 + \frac{x^{\beta}}{\lambda}\right)^{-\alpha-1} \left[1 - \left(1 + \frac{x^{\beta}}{\lambda}\right)^{-\alpha}\right]^{\gamma-1}, \quad x, \alpha, \beta, \lambda, \gamma > 0. \tag{2}$$

Where λ is scale parameter and α, β, γ are three shape parameters. The *EPL* distribution is a very flexible model that includes some distributions. Note that:

(1). When $\gamma = 1$ the EPL distribution reduce to PL distribution which studied by Rady [11].

(2). When $\beta = 1$ the EPL distribution reduce to EL distribution which studied by Abdul-Moniem and Abdel-Hameed [1].

(3). When $\gamma = \beta = 1$ the EPL distribution reduce to L distribution which studied by Lomax [10].

Figure 1 displays some plots of the pdf for the *EPL* pdf for some different values of parameters.

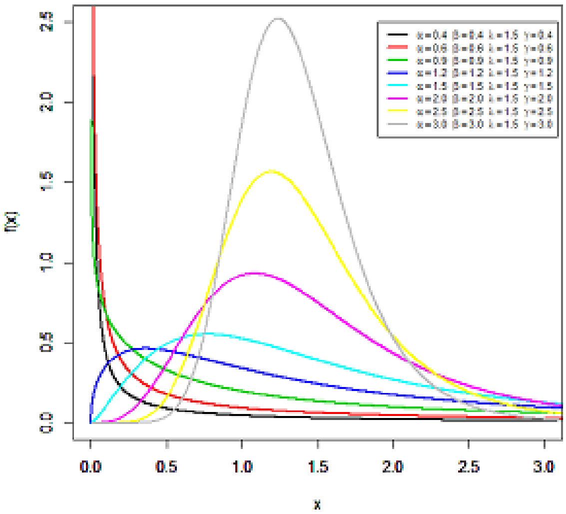

Figure 1. **Plots of the pdf for *EPL* distribution for different values of parameters**

From Figure 1, we conclude that pdf of *EPL* distribution can be uni-model, symmetric, decreasing and right skewed.

3. The Reliability Analysis

The survival function (sf), hazard rate function (hrf), reversed hrf and cumulative hrf of X are given, respectively, as follows:

$$R(x) = 1 - \left[1 - \left(1 + \frac{x^{\beta}}{\lambda}\right)^{-\alpha}\right]^{\gamma},$$

$$h(x)=\frac{\frac{\alpha\beta\gamma}{\lambda}x^{\beta-1}\left(1+\frac{x^{\beta}}{\lambda}\right)^{-\alpha-1}\left[1-\left(1+\frac{x^{\beta}}{\lambda}\right)^{-\alpha}\right]^{\gamma-1}}{1-\left[1-\left(1+\frac{x^{\beta}}{\lambda}\right)^{-\alpha}\right]^{\gamma}},$$

$$\tau(x)=\frac{\frac{\alpha\beta\gamma}{\lambda}x^{\beta-1}\left(1+\frac{x^{\beta}}{\lambda}\right)^{-\alpha-1}}{1-\left(1+\frac{x^{\beta}}{\lambda}\right)^{-\alpha}},$$

$$\text{and}\quad H(x)=-ln\left(1-\left[1-\left(1+\frac{x^{\beta}}{\lambda}\right)^{-\alpha}\right]^{\gamma}\right).$$

Figure 2 displays some plots of the hrf for the *EPL* for some different values of parameters.

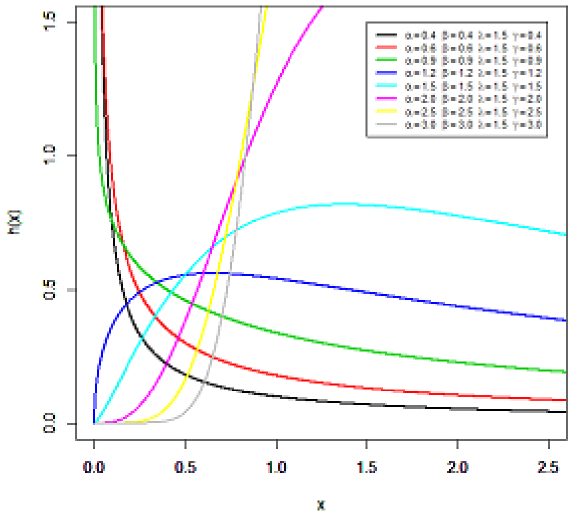

Figure 2. **Plots of the hrf for EPL distribution for different values of parameters**

From Figure 2, we conclude that the hrf of EPL distribution can be J-shaped, decreasing and increasing.

4. Statistical Properties

In this section, we study some statistical properties for EPL distribution.

4.1. Useful expansions

In this subsection we present the two useful expansions of pdf and cdf for EPL distribution. Now, consider the following well-known binomial expansions (for $0 < a < 1$),

$$(1-a)^{n}=\sum_{k=0}^{\infty}(-1)^{k}\binom{a}{k}a^{k} \tag{3}$$

Thus, using (3), the following term in (2) can be expressed as

$$\left[1-\left(1+\frac{x^{\beta}}{\lambda}\right)^{-\alpha}\right]^{\gamma-1}=\sum_{k=0}^{\infty}(-1)^{k}\binom{\gamma-1}{k}\left(1+\frac{x^{\beta}}{\lambda}\right)^{-\alpha k} \tag{4}$$

Therefore, from (4) and (2) the pdf of EPL can be write as

$$f(x)=\frac{\beta}{\lambda}\sum_{k=0}^{\infty}w_{k}x^{\beta-1}\left(1+\frac{x^{\beta}}{\lambda}\right)^{-\alpha(k+1)-1} \tag{5}$$

Where $w_k = \alpha\gamma(-1)^k \binom{\gamma-1}{k}$. By using binomial theory for $[F(x)]^h$, where h is an integer, leads to:

Since,

$$[F(x)]^h = \left[1 - \left(1 + \frac{x^\beta}{\lambda}\right)^{-\alpha}\right]^{\gamma h}.$$

Then,

$$[F(x)]^h = \sum_{i=0}^{\infty} w_i \left(1 + \frac{x^\beta}{\lambda}\right)^{-\alpha i}. \tag{6}$$

Where, $w_i = (-1)^i \binom{\gamma h}{i}$.

4.2. Quantile and Median

The quantile function, say $Q(u) = F^{-1}(u)$ of X is given by

$$Q(u) = \sqrt[\beta]{\lambda\left(1 - u^{\frac{1}{\gamma}}\right)^{\frac{-1}{\alpha}} - \lambda}. \tag{7}$$

Where, u is considered as a uniform random variable on the unit interval $(0, 1)$. The median can be calculated by setting $u = 0.5$ in (7). Then, the median (M) is given by

$$M = \sqrt[\beta]{\lambda\left(1 - (0.5)^{\frac{1}{\gamma}}\right)^{\frac{-1}{\alpha}} - \lambda}.$$

4.3. Probability Weighted Moments (PMWs)

The PMWs can be calculated from the following

$$\tau_{r,s} = E[X^r F(x)^s] = \int_{-\infty}^{\infty} x^r f(x)(F(x))^s dx. \tag{8}$$

By inserting (5) and (6) into (8), replacing h with s, leads to:

$$\tau_{r,s} = \frac{\beta}{\lambda} \sum_{i,k=0}^{\infty} w_i w_k \int_0^{\infty} x^{r+\beta-1} \left(1 + \frac{x^\beta}{\lambda}\right)^{-\alpha(i+k+1)-1} dx$$

Let $y = \frac{x^\beta}{\lambda}$, then,

$$\tau_{r,s} = \sum_{i,k=0}^{\infty} w_i w_k \lambda^{\frac{r}{\beta}} \int_0^{\infty} y^{\frac{r}{\beta}} (1+y)^{-\alpha(i+k+1)-1} dy$$

Again make the following transformation $y = \frac{w}{1-w}$

$$\tau_{r,s} = \sum_{i,k=0}^{\infty} w_i w_k \lambda^{\frac{r}{\beta}} \int_0^{1} w^{\frac{r}{\beta}} (1-w)^{\alpha(k+1)-\frac{r}{\beta}-1} dw$$

Hence, the PWM of EPL distribution takes the following form

$$\tau_{r,s} = \sum_{i,k=0}^{\infty} w_i w_k \lambda^{\frac{r}{\beta}} B\left[\frac{r}{\beta} + 1,\ \alpha(i+k+1) - \frac{r}{\beta}\right]$$

4.4. Moments

In this subsection, we intend to derive the moments and the moment generating function of the EPL model. If X has the pdf (2), then its r$^{\text{th}}$ moment is given by

$$\mu_r' = \int_0^\infty x^r f(x)\, dx$$

Using binomial expansions, we have

$$\mu_r' = \sum_{k=0}^\infty w_i w_k \lambda^{\frac{r}{\beta}} B\left[\frac{r}{\beta}+1, \alpha(k+1)-\frac{r}{\beta}\right].$$

The moment generating function (mgf) of the EPL distribution is

$$M_x(t) = E\left(e^{tX}\right) = \int_0^\infty e^{tx} f(x)\, dx = \sum_{r=0}^\infty \frac{t^r}{r!} E\left(X^r\right),$$

then,

$$M_x(t) = \sum_{r=0}^\infty \frac{t^r}{r!} \sum_{k=0}^\infty w_i w_k \lambda^{\frac{r}{\beta}} B\left[\frac{r}{\beta}+1, \alpha(k+1)-\frac{r}{\beta}\right].$$

4.5. Order Statistics

Let $X_{1:n} < X_{2:n} < \cdots < X_{n:n}$ be the order statistics of a random sample of size n following the EPL distribution, with parameters α, β, λ and γ then,the pdf of the k$^{\text{th}}$ order statistic can be written as follows

$$f_{k:n}(x) = \frac{1}{B(r, n-k+1)} f(x) F(x)^{k-1} [1-F(x)]^{n-k}.$$

The binomial expansion yields

$$f_{k:n}(x) = \frac{f(x)}{B(k, n-k+1)} \sum_{v=0}^{n-k} (-1)^v \binom{n-k}{v} F(x)^{v+k-1}, \tag{9}$$

where, $B(.,.)$ is the beta function. By substituting (5) and (6) in (9), replacing h with $v+k-1$, leads to

$$f_{k:n}(x) = \frac{\beta}{\lambda B(k, n-k+1)} \sum_{i,k=0}^\infty \eta x^{r+\beta-1} \left(1+\frac{x^\beta}{\lambda}\right)^{-\alpha(i+k+1)-1}. \tag{10}$$

Where $\eta = (-1)^v \binom{n-k}{v} w_i w_k$. Further, the r$^{\text{th}}$ moment of k$^{\text{th}}$ order statistics for EPL distribution is defined by:

$$E\left(X_{k:n}^r\right) = \frac{1}{B(k, n-k+1)} \sum_{k=0}^\infty \eta \lambda^{\frac{r}{\beta}} B\left[\frac{r}{\beta}+1, \alpha(i+k+1)-\frac{r}{\beta}\right].$$

5. ML Estimation

The ML estimates of the unknown parameters for the *EPL* distribution are determined based on complete samples. Let $X_1, \ldots, X_n$ be observed values from the *EPL* model with set of parameters $\varphi = (\alpha, \beta, \lambda, \gamma)^T$. The total log-likelihood function for the vector of parameters φ can be expressed as

$$\ln L(\varphi) = n\ln\gamma + n\ln\beta - n\ln\lambda + n\ln\alpha + (\beta-1)\sum_{i=1}^n \ln x_i - (\alpha+1)\sum_{i=1}^n \ln\left(1+\frac{x_i^\beta}{\lambda}\right) + (\gamma-1)\sum_{i=1}^n \ln\left(1-\left(1+\frac{x_i^\beta}{\lambda}\right)^{-\alpha}\right).$$

The elements of the score function $U(\varphi) = (U_\alpha, U_\beta, U_\lambda, U_\gamma)$ are given by

$$U_\alpha = \frac{n}{\alpha} - \sum_{i=1}^{n} \ln\left(1 + \frac{x_i^\beta}{\lambda}\right) + (\gamma - 1)\sum_{i=1}^{n} \frac{\left(1+\frac{x_i^\beta}{\lambda}\right)^{-\alpha} \ln\left(1+\frac{x_i^\beta}{\lambda}\right)}{1-\left(1+\frac{x_i^\beta}{\lambda}\right)^{-\alpha}},$$

$$U_\beta = \frac{n}{\beta} + \sum_{i=1}^{n} \ln x_i - \frac{1}{\lambda}(\alpha+1)\sum_{i=1}^{n}\left(\frac{x_i^\beta \ln x_i}{1+\frac{x_i^\beta}{\lambda}}\right) + \frac{(\gamma-1)\alpha}{\lambda}\sum_{i=1}^{n} \frac{x_i^\beta \ln x_i \left(1+\frac{x_i^\beta}{\lambda}\right)^{-\alpha-1}}{1-\left(1+\frac{x_i^\beta}{\lambda}\right)^{-\alpha}},$$

$$U_\lambda = \frac{-n}{\lambda} + (\alpha+1)\sum_{i=1}^{n}\left(\frac{\lambda^{-2} x_i^\beta}{1+\frac{x_i^\beta}{\lambda}}\right) - (\gamma-1)\alpha\sum_{i=1}^{n} \frac{\lambda^{-2} x_i^\beta \left(1+\frac{x_i^\beta}{\lambda}\right)^{-\alpha-1}}{1-\left(1+\frac{x_i^\beta}{\lambda}\right)^{-\alpha}},$$

and $$U_\gamma = \frac{n}{\gamma} + \sum_{i=1}^{n} \ln\left(1 - \left(1+\frac{x_i^\beta}{\lambda}\right)^{-\alpha}\right).$$

Then the ML estimators of the parameters α, β, λ and γ are obtained by setting U_α, U_β, U_λ and U_γ to be zero and solving them. Clearly, it is difficult to solve them, therefore applying the Newton-Raphson's iteration method and using the computer package such as Maple or R or other softare.

6. Simulation Study

It is very difficult to compare the theoretical performances of the different estimators (MLE) for the EPL distribution. A numerical study is performed using Mathematica 9 software. Different sample sizes are considered through the experiments at size $n = 10$, 30 and 100. In addition, the different values of parameters α, β, λ and γ. The experiment will be repeated 3000 times. In each experiment, the estimates of the parameters will be obtained by maximum likelihood methods of estimation. The means, MSEs and biases for the different estimators will be reported from these experiments.

Table 1. **The parameter estimation for EPL distribution using MLE**

n	Par	Set 1: (0.5,1.5,0.5,1.5)			Set 2: (0.5,2.0,0.5,1.5)		
		MLE	Bais	MSE	MLE	Bais	MSE
10	α	1.1480	0.6480	3.4857	0.5548	0.0548	0.0242
	β	2.0117	0.5117	1.9109	2.9479	0.9479	2.8223
	λ	0.5513	0.0512	0.0379	0.4726	-0.0274	0.0031
	γ	1.7538	0.2538	0.8843	1.8590	0.3590	0.3448
30	α	0.4607	-0.0393	0.0095	0.5324	0.0324	0.0233
	β	1.3979	-0.1022	0.0555	2.1288	0.1288	0.3342
	λ	0.4843	-0.0157	0.0019	0.5047	0.0046	0.0029
	γ	1.6741	0.1741	0.1686	1.5908	0.0908	0.1463
100	α	0.5019	0.0019	0.0034	0.5055	0.0055	0.0021
	β	1.5307	0.0307	0.0326	2.0123	0.0123	0.0411
	λ	0.5000	0.0000	0.0005	0.5028	0.0028	0.0003
	γ	1.5286	0.0286	0.0147	1.5019	0.0019	0.0130

Continued of table 1

n	Par	Set 3: (1.5,1.5,0.5,1.5)			Set 4: (1.5,2.0,0.5,1.5)		
		MLE	Bais	MSE	MLE	Bais	MSE
10	α	1.9310	0.4310	0.7613	2.4406	0.9406	4.7223
	β	2.7147	1.2147	3.8499	2.0608	0.0607	0.4008
	λ	0.5262	0.0262	0.0163	0.5921	0.0921	0.0645
	γ	1.8778	0.3778	0.5559	1.6085	0.1085	0.5392
30	α	1.6751	0.1751	0.2929	1.8785	0.3785	0.2488
	β	1.4573	-0.0427	0.0851	1.7888	-0.2112	0.3203
	λ	0.5264	0.0264	0.0088	0.5888	0.0888	0.0141
	γ	1.4628	-0.0372	0.0677	1.2896	-0.2104	0.1152
100	α	1.4807	-0.0193	0.0453	1.5675	0.0675	0.0191
	β	1.5429	0.0429	0.0695	1.9112	-0.0888	0.0690
	λ	0.4935	-0.0065	0.0024	0.5172	0.0172	0.0011
	γ	1.5598	0.0598	0.0464	1.4345	-0.0655	0.0153

Continued of table 1

n	Par	Set 5: (2.0,1.5,0.5,1.5)			Set 6: (2.0,2.0,0.5,1.5)		
		MLE	Bais	MSE	MLE	Bais	MSE
10	α	2.1969	0.1969	0.7931	2.1355	0.1355	0.5294
	β	1.7826	0.2826	0.7807	2.1401	0.1401	0.6707
	λ	0.5211	0.0211	0.0208	0.5240	0.0240	0.0175
	γ	1.8125	0.3125	0.6191	1.6241	0.1241	0.2434
30	α	2.3795	0.3795	0.7796	2.0739	0.0739	0.1189
	β	1.7392	0.2392	0.2160	2.1193	0.1193	0.3245
	λ	0.5379	0.0379	0.0149	0.5145	0.0145	0.0041
	γ	1.5693	0.0693	0.0725	1.5337	0.0336	0.0585
100	α	1.8989	-0.1011	0.0291	2.0609	0.0609	0.0544
	β	1.5777	0.0777	0.0505	2.0612	0.0612	0.0537
	λ	0.4810	-0.0190	0.0010	0.5036	0.0036	0.0014
	γ	1.5675	0.0675	0.0214	1.5061	0.0061	0.0177

7. Data Analysis

In this section, we use two real data sets to explain the importance and flexibility of the EPL model. We compare the fits of the EPL model with some models namely: the gamma L (GL) (Cordeiro [3]), beta L (BL) (Eugene [5]), exponentiated L (EL) (Abdul-Moniem and Abdel-Hameed, [1]) and L distributions. The maximized log-likelihood ($-\ell$), Akaike information criterion (AIC), the corrected Akaike information criterion (CAIC), Anderson-Darling (A^*) and Cramér-Von Mises (W^*) statistics are used for model selection.

Data set 1:

The first data set is obtained from Tahir [12] represents failure times of 63 aircraft windshield. The data are:

0.046, 1.436, 2.592, 0.140, 1.492, 2.600, 0.150, 1.580, 2.670, 0.248, 1.719, 2.717, 0.280, 1.794, 2.819, 0.313, 1.915, 2.820, 0.389, 1.920, 2.878, 0.487, 1.963, 2.950, 0.622, 1.978, 3.003, 0.900, 2.053, 3.102, 0.952, 2.065, 3.304, 0.996, 2.117, 3.483, 1.003, 2.137, 3.500, 1.010, 2.141, 3.622, 1.085, 2.163, 3.665, 1.092, 2.183, 3.695, 1.152, 2.240, 4.015, 1.183, 2.341, 4.628, 1.244, 2.435, 4.806, 1.249, 2.464, 4.881, 1.262, 2.543, 5.140.

For the first data set, Table 2 gives the MLEs of the fitted models and their standard errors (SEs) in parenthesis. The values of goodness-of-fit statistics are listed in Table 3. It is noted, from Table 3, that the EPL distribution provides a better fit than other competitive fitted models. It has the smallest values for goodness-of-fit statistics among all fitted models. Plots of the histogram, fitted densities are shown in Figure 3. This figure supported the conclusion drawn from the numerical values in Table 3.

Data set 2:

The second data set is obtained from Tahir [12] and represents failure times of 84 Aircraft Windshield. The data are: 0.040, 1.866, 2.385, 3.443, 0.301, 1.876, 2.481, 3.467, 0.309, 1.899, 2.610, 3.478, 0.557, 1.911, 2.625, 3.578, 0.943, 1.912, 2.632, 3.595, 1.070, 1.914, 2.646, 3.699, 1.124, 1.981, 2.661, 3.779, 1.248, 2.010, 2.688, 3.924, 1.281, 2.038, 2.82,3, 4.035, 1.281, 2.085, 2.890, 4.121, 1.303, 2.089, 2.902, 4.167, 1.432, 2.097, 2.934, 4.240, 1.480, 2.135, 2.962, 4.255, 1.505, 2.154, 2.964, 4.278, 1.506, 2.190, 3.000, 4.305, 1.568, 2.194, 3.103, 4.376, 1.615, 2.223, 3.114, 4.449, 1.619, 2.224, 3.117, 4.485, 1.652, 2.229, 3.166, 4.570, 1.652, 2.300, 3.344, 4.602, 1.757, 2.324, 3.376, 4.663.

Table 4 lists the MLEs of the fitted models and their SEs in parenthesis. The values of goodness-of-fit statistics are presented in Table 5.

Table 2. **The MLEs and SEs of the model parameters for data set 1**

Model	Estimates (SEs)			
EPL$(\alpha, \beta, \lambda, \gamma)$	3.138 (2.649)	2.447 (0.718)	34.458 (21.239)	0.583 (0.218)
GL(a, α, β)	1.9073 (0.3213)	35842.4330 (6945.0743)	39197.5715 (151.6530)	
BL(a, b, α, β)	1.9218 (0.3184)	31.2594 (316.8413)	4.9684 (50.5279)	169.5719 (339.2067)
EL(a, α, β)	1.9145 (0.3482)	22971.1536 (3209.5329)	32881.9966 (162.2299)	

Table 3. **Goodness-of-fit statistics for data set 1**

Model	$-\ell$	AIC	CAIC	A^*	W^*
EPL	100.188	208.376	209.065	0.58888	0.09779
GL	102.8332	211.6663	212.0731	1.112	0.1836
BL	-102.9611	213.9223	214.6119	1.1336	0.1872
EL	103.5498	213.0995	213.5063	1.2331	0.2037

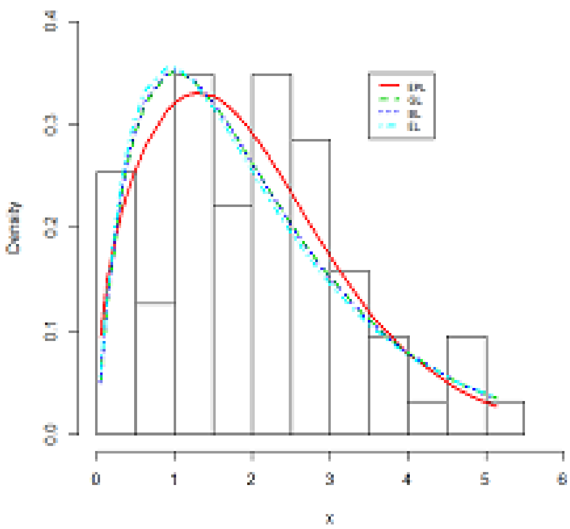

Figure 3. **Estimated densities of the fitted models for data set 1**

Table 4. **The MLEs and SEs for data set 2**

Model	Estimates (SEs)			
EPL$(\alpha, \beta, \lambda, \gamma)$	4.176 (2.874)	2.665 (0.56)	64.337 (34.988)	0.874 (0.257)
GL(a, α, β)	3.5876 (0.5133)	52001.4994 (7955.0003)	37029.6583 (81.1644)	
BL(a, b, α, β)	3.6036 (0.6187)	33.6387 (63.7145)	4.8307 (9.2382)	118.8374 (428.9269)
EL(a, α, β)	3.6261 (0.6236)	20074.5097 (2041.8263)	26257.6808 (99.7417)	

Table 5. **Goodness-of-fit statistics for data set 2**

Model	$-\ell$	AIC	CAIC	A^*	W^*
EPL	131.693	271.386	271.892	0.7286	0.0732
GL	138.4042	282.8083	283.1046	1.3666	0.1618
BL	138.7177	285.4354	285.9354	1.4084	0.1680
EL	141.3997	288.7994	289.0957	1.7435	0.2194

It is observed, from Table 5, that the EPL distribution gives a better fit than other fitted models. Plots of the histogram, fitted densities are displayed in Figure 4.

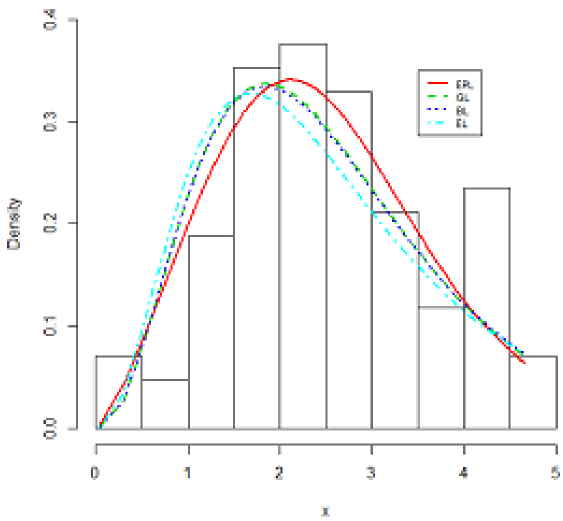

Figure 4. **Estimated pdfs for data set 2**

8. Conclusions

In this paper, we study a four-parameter model, named the exponentiated power Lomax (EPL) distribution. The EPL pdf can be expressed as a mixture of PL densities. We derive explicit expressions for the quantile function, moments, moment generating function, probability weighted moments, and order statistics. The maximum likelihood estimation method is used to estimate the model parameters. We provide some simulation results to assess the performance of the proposed model. The practical importance of the EPL distribution is demonstrated by means of two real data sets.

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