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Application of the Grey-Taguchi Method for the Optimization of Lost Wax Mold

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Abstract: Investment casting technology is negligible in Ethiopia on account of the cost of investment casting is considerably higher than the sand casting due to the cost of the importation of the wax and refractory materials. In this work an attempt has been made to develop an alternative local mold material which is capable of providing hard, high strength, thermal stable shell for lost wax casting technology to manufacture intricate part. It optimizes the lost wax mold compositions based on Taguchi method and Grey relational analysis in order to obtain the particular refractory composition with optimum multiple characteristics of the mold. From Grey relational analysis response it is found optimum result that the local investment slurry with the refractory type of fired clay, its quantity of 35%, marl clay of 4%, water content of 23%, wheat husk of 35%, cow dung of 2%, mold drying time of 24 hours and the firing temperature of 8000C. The confirmation experiments showed that the results of mold compositions were optimized for achieving the combined objectives of higher compression strength of material, lower linear and volumetric expansion value on the dried mold.

Keywords: Lost wax, Mold, Optimization, Taguchi, Grey relation.

1. Introduction

In nations like Ethiopia, investment casting process isn't generally pertinent when contrasted with sand casting process. This circumstance constrained our nation to import tools, equipment and other extra parts which are unreasonable to be delivered with sand casting and other assembling forms. Hence, tolerance capability, design freedom, alloy selection freedom, product size range, machining cost factors and different factors may drive engineers towards the advancement of an investment casting process. Most of the yield of the foundry industry comprises of castings made in refractory molds. Since the properties of the molding material are vital to the creation of sound, dimensionally exact castings, its choice and testing constitutes one of the crucial strides in establishment of the material [1]. Investment casting mold utilize diverse Percentage of slurry arrangements for building of investment shell. The most existing pattern is the utilization of layer of local/gypsum, potters clay, water with other binding additives as a coat with various slurry thickness and viscosity. The coat building component is done either by brushing or sinking process on the surface of the wax pattern till to get the demanded thickness. A machine component with great surface finish is a print of shell working from appropriate arrangement of slurry done at all around examined and controlled conditions. The surface nature of the cast component which depend up on the admixture of local slurry materials have high refractoriness and cohesive nature and can be crushed after the casting procedure.

The properties of molding materials can be estimated by an extensive variety of strategies, which give the premise both to routine foundry control and for the advancement of new materials. The quality of investment casting shell must be

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satisfactory to avert fracture or chipping of the mold in the course of heating and casting of the alloy. It is estimated that the compression strength of investment mold can be an essential factor to be considered, notwithstanding the development while assessing the dimensional accuracy of metal castings the quality of an investment is normally estimated under compression strength [2]. The Past work examination demonstrate that there is not as such clear study, especially, concentrating on elective indigenous mold materials for copper based alloys. Mohd Hasbullah Idris, Ali Ourdjini and Esah Hamsah were cooperated to research the mold cracking during investment casting of magnesium alloy. Their work focuses the control of slurry viscosity, shell drying time and dewaxing temperature. As indicated by the paper, several mixes of the viscosity of the slurry, stucco framework and the dewaxing temperature were created and are extremely helpful to the present investment casting of industrial area [3]. As indicated by Allene Endayilalu thesis work on improvement of local blend wax and slurry to produce complex formed parts held at Bahir Dar University, 2012 in his theory he created nearby bee wax as a pattern and slurry material synthesis with 60% blue Nile sand 10% nearby gypsum, 10% local pottery clay and 22% water to bind compositions [4]. This shell is developed from silica sand, gypsum, clay and water for aluminum alloys. Kim K D, Yang D Y, Jeong J H. 2006 [5] expressed that plaster casting is an advanced strategy for lost wax casting and thin-walled complex quality structure molding which is high in quality and accuracy, wide range in size and weight, short cycle of innovative work.

In this paper parts having thin wall portion, complex shape geometry can be accomplished through lost wax casting process with range of shape and measurements using plaster as molding item. Barnali Mandal and Prasanta Kumar Datta [6] state old casting forms for manufacturing of utensils and symbols made of metals were made of hot molds utilizing clay shaped investment casting or piece mold process. Those symbols had got intricate shape, complex geometry of various cross segments with smooth surface finish.

This paper chiefly underlined on development of optimum investment casting mold composition utilizing local materials focusing on the multi performance assessment of the mold regarding its refractory composition. Examinations are conducted utilizing L18 Grey taguchi's technique for various alternative designs utilizing the local slurry materials. In this way the significant elements and their optimum level of composition are produced.

2. Materials and Methods

Firstly, the investment casting mould material composition was carried out in small foundry as reported by Hurst [7]. In this investigation, the investment shell was utilized as the test specimens. Once the coveted blend is acquired, put the green blend in a holder having a measurement of 5 cm X 5 cm and let it remain to harden. In the compression strength testing, the investment slurry were blended, dried and let go before setting on compression strength testing machine. What's more, the whole roundabout mold is utilized as specimens for volumetric and direct expansion.

Investment mold is set up from refractory materials as slurry that is being blended by utilizing the silica sand/fired clay, marl, dung and husk. Figure 1 and 2 demonstrates the blend of first coat and reinforcement investment being mixed by the blender separately. Before the trial, the control factors and their levels should first be resolved. This is appeared in Table 1. To produce the patterns, PVC channels with 3cm height and 5cm in diameter have been utilized as the dies. The wax patterns utilized for preparation of investment casting molds comprises a blend of 70% paraffin wax and 30% bees wax.

At the point when the slurry is prepared, the pattern is dunked into the slurry. The primary coat was left to dry gradually (at least 24 hours) as appeared in Figure 3. The coat is then left to dry in the room temperature. The reinforcement investment demonstrates a difference in method. A firm paste like porridge is plied altogether and pushed onto the wax with the fingers. This thick slurry of admixture comprises designed amounts of clay, dung and husk.

Control factors	Level			Base level [7]
	1	**2**	**3**	
A. Refractory type	Silica sand	Fired clay	-	Silica sand
B.Refractory percentage	30	35	40	40
C. Pottery clay (marl) percentage	3	4	6	6
D. Water percentage	15	23	30	15
E. Dust (wheat husk) percentage	30	35	40	40
F. Cow dung percentage	2	3	4	12
G. Mould drying time	20	21	24	24
H. Mould firing temperatures	600	700	800	800

Table 1: The Control Factors And Their Levels In Lost Wax Mould Experiments

The last stage before the metal pouring procedure is dewaxing. The dewaxing temperature is around 200°C. Also, the dewaxing time is around 60 minutes. This is trailed by the mold firing to expel all the wax deposits inside the molds. All things considered, the last stage is to empty the liquefying metal into the form as observed Fig. 4. The non-ferrous metal utilized for the casting is bronze. The metal is melted in furnace, and the pouring temperature is at 977°C. Figure 5 demonstrates the last casting test subsequent to completing procedure.

Figure 1: The mixture of the investment

Figure 2: Mixture of reinforcement investment

Figure 3: Coat drying process

Figure 4: Pouring the melted metal on fired mold

Figure 5: Sample product after finishing operation

Figure 6: Compression testing

A Taguchi design or an orthogonal array the technique is designing the experimental strategy utilizing distinctive kinds of design like, two, three, four, five, and mixed level [8]. In the examination, a three factor mixed level setup is picked with a sum of eighteen quantities of experiments to be conducted and thus the OA L18 was selected. By giving the parameters and levels for MINITAB [9], the software demonstrates the experimental design format as found in table 2. An aggregate of 18 groups of experiments were directed as per the orthogonal array. Each group of experiment ought to take after the assessing strategies required by universal compression machine, Volume measuring flask and Vernier caliper to test compression strength, volumetric expansion and also linear expansion separately.

(a) Linear expansion: The dimension of expansion of mold is taken from the cast samples utilizing Vernier caliper. We can straightforwardly compute the linear shrinkage by taking the distinction between the standard measurement of the wax pattern (considering the pattern and flexible die shrinkage) and the measurement that we get from the cast test (thinking about the shrinkage of bronze).

(b) Volumetric expansion: Firstly fill the water in measuring flask and take the initial reading then place the wax pattern in the measuring flask, now the volume of the water raises that is the initial volume of mold. Similarly, bronze casting volume is measured. Finally, volumetric expansion can be find out as follows:

$$\left(\frac{Volume\ of\ the\ wax\ pattern - volume\ of\ casting}{Volume\ of\ the\ wax\ pattern}\right) * 100. \tag{1}$$

(c) Compression strength: The compression strength testing is carried out on universal compression machines such as that shown in Figure 6.

Refractory Type	**Refractory %**	**Marl %**	**Water %**	**Husk %**	**Dung %**	**Drying time (hr)**	**Firing temperature (°C)**
Silica sand	30	3	15	30	2	20	600
Silica sand	30	4	23	35	3	21	700
Silica sand	30	6	30	40	4	24	800
Silica sand	35	3	15	35	3	24	800
Silica sand	35	4	23	40	4	20	600
Silica sand	35	6	30	30	2	21	700
Silica sand	40	3	23	30	4	21	800
Silica sand	40	4	30	35	2	24	600
Silica sand	40	6	15	40	3	20	700

Refractory Type	Refractory %	Marl %	Water %	Husk %	Dung %	Drying time (hr)	Firing temperature (°C)
Fired clay	30	3	30	40	3	21	600
Fired clay	30	4	15	30	4	24	700
Fired clay	30	6	23	35	2	20	800
Fired clay	35	3	23	40	2	24	700
Fired clay	35	4	30	30	3	20	800
Fired clay	35	6	15	35	4	21	600
Fired clay	40	3	30	35	4	20	700
Fired clay	40	4	15	40	2	21	800
Fired clay	40	6	23	30	3	24	600

Table 2: The L18 orthogonal array layout for MINITAB

3. Results and Discussion

The tests were directed in view of changing the mold parameters, which influence the shelling procedure to get the required quality attributes. Quality attributes are the response values or yield values expected out of the tests. There are 64 such quality attributes. The most regularly utilized are:1) Larger the better, 2) Smaller the better, 3) Nominal the best, 4) Classified property, 5) Signed target.

A. Experimental Result: The response table for compression strength, linear and volumetric expansion of mold is appeared in Table 3. An aggregate of 18 groups of analyses were led as per the orthogonal array. Taguchi method for compression strength is utilized to break down the consequence of response of mold parameter for larger is better criteria. The S/N ratios for linear and volumetric expansion are additionally calculated. Taguchi method used to investigate the consequence of response of mold parameter is smaller the better criteria.

Optimization using Grey Relational Analysis: Taguchi's method 10 is centered on the successful utilization of designing systems as opposed to cutting edge measurable strategies. The essential objective of Taguchi method is decrease in the variety of an item configuration to enhance quality and lower the misfortune granted to society. And it is also an appropriate item or process execution technique, which can additionally decrease the level of variety.

The steps involved in Taguchi's Grey Relational Analysis are:

Step 1: The collected experimental data were used to calculate the SN ratio for each group of experiment in accordance with Equations 2, 3 and 4. This is shown in table 3.

$$lower - the - better - 10 \log \frac{1}{n} \sum y^2, \tag{2}$$

$$Higher - the - better - 10 \log \frac{1}{n} \sum \frac{1}{y^2} \tag{3}$$

Where n is the number of observations and y is the observed data.

Step 2: In the grey relational analysis [11, 12], data preprocessing is first performed in order to normalize the raw data for analysis. A linear data preprocessing method for raw data can be expressed as

$$x^*{}_i\,(k) = \frac{\max x_i(k) - x_i(k)}{\max x_i(k) - \min x_i(k)}, \quad i = 1, 2 \ldots, m; \quad k = 1, 2 \ldots, n \tag{4}$$

Where m is the number of experiments, n is the number of response variables. Where $x_i(k)$ is the original sequence of the response, $x_i^*(k)$ is the comparable sequence after data normalization, $\max x_i(k)$ and $\min x_i(k)$ are the largest value and smallest value of $x_i(k)$ in this paper, $m = 18$, $n = 3$ is taken.

No	Compression strength		Linear expansion		Volumetric expansion	
	Response (Kpsi)	*SN ratio (dB)*	*Response (%)*	*SN ratio(dB)*	*Response (%)*	*SN ratio (dB)*
1	25.00	27.9588	4.62	-13.2928	11.010	-20.8357
2	30.42	29.6632	3.20	-10.103	8.410	-18.4959
3	32.50	30.2377	3.00	-9.5424	9.360	-19.4255
4	33.00	30.3703	2.90	-9.248	6.540	-16.3116
5	31.50	29.9662	4.00	-12.0412	10.400	-20.3407
6	27.00	28.6273	3.40	-10.6296	9.440	-19.4994
7	29.40	29.3669	3.80	-11.5957	8.840	-18.929
8	32.50	30.2377	2.90	-9.248	6.540	-16.3116
9	27.75	28.8653	5.29	-14.4691	14.110	-22.9905
10	29.92	29.5192	3.60	-11.1261	8.640	-18.7303
11	29.50	29.3964	3.40	-10.6296	9.420	-19.481
12	31.50	29.9662	2.85	-9.0969	6.720	-16.5474
13	34.00	30.6296	3.05	-9.686	5.640	-15.0256
14	32.50	30.2377	2.35	-7.4214	7.320	-17.2902
15	29.50	29.3964	4.32	-12.7097	11.808	-21.4435
16	30.80	29.7710	4.10	-12.2557	9.840	-19.8599
17	33.80	30.5783	2.80	-8.9432	6.720	-16.5474
18	30.60	29.7144	3.41	-10.6551	8.184	-18.2593

Table 3: The responses and SN ratio of L18 orthogonal array

Step 3: The grey relational coefficient [13] is calculated to express the relationship between the ideal (best) and actual normalized experimental results. After normalization of the original sequence, the grey relational coefficient is calculated using Equation 5. It can be expressed as

$$\gamma\left(x^{*}{}_{0}\left(k\right), x^{*}{}_{i}(k)\right) = \frac{\Delta_{\min} + \tau\Delta_{\max}}{\Delta_{oi}\left(k\right) + \tau\Delta_{\max}} \tag{5}$$

ζ is the distinguishing coefficient and $\zeta \in [0,1]$. ζ is set at 0.5.

$$\Delta_{\min} = \min_{\forall\ i} 1 \quad \min_{\forall\ k} \Delta_{oi}(k)$$

$$\Delta_{\max} = \max_{\forall\ i} 1 \quad \max_{\forall\ k} \Delta_{oi}(k)$$

Based on Equation 5 the Grey relational coefficient is given in Table 4.

Step 4: Principal component analysis is specially introduced here to determine the corresponding weighting values for each performance characteristic.The Eigenvalue and Eigenvector are calculated using MINITAB software. There is one of three Eigenvalue larger than one. The eigenvector that corresponded to the largest Eigenvalue 2.4062 was [-0.597, -0.543, and -0.591]. Hence, for this study, the squares of its corresponding eigenvectors are selected as the weighting values of the related performance characteristic. The contributions of compression strength, linear expansion, and volumetric expansion of the mould are indicated as 0.3564, 0.2949, and 0.3493.

Step 5: The grey relational grade represents the level of correlation between the reference sequence and Comparability sequence [13]. It is determined as

$$\Psi\left(x^{*}{}_{o}, x^{*}{}_{i}\right) = \sum_{k=1}^{n} \omega_{k}\gamma\left(x^{*}{}_{o}\left(k\right), x^{*}{}_{i}\left(k\right)\right) \tag{6}$$

Where Ψ is the weight of the kth performance characteristics. Based on Equation (6) and the grey relational coefficient, the grey relational grade can be calculated as follows:

$$\Psi\left(x^{*}{}_{0}, x^{*}{}_{1}\right) = 0.3564 \times 0.333333 + 0.2949 \times 0.375067 + 0.3493 \times 0.406682 = 0.372662$$

Comparability sequence, No	Grey relational coefficient			Overall Grey Relational Grade
	Compression strength	Linear expansion	Volumetric expansion	
1	0.333333	0.375067	0.406682	0.372662
2	0.580155	0.567864	0.53436	0.562704
3	0.773114	0.624259	0.475099	0.627589
4	0.837399	0.658608	0.755905	0.758824
5	0.668101	0.432711	0.428333	0.516722
6	0.400096	0.523444	0.470948	0.463139
7	0.513991	0.457753	0.505012	0.496044
8	0.773114	0.658608	0.755905	0.735911
9	0.430816	0.333333	0.333333	0.369345
10	0.545997	0.487491	0.518066	0.52088
11	0.519894	0.523444	0.471974	0.506189
12	0.668101	0.677748	0.723523	0.692877
13	1	0.608773	1	0.887174
14	0.773114	1	0.637493	0.796323
15	0.519894	0.399885	0.382915	0.438254
16	0.60866	0.421606	0.451691	0.500387
17	0.963006	0.698394	0.723523	0.804138
18	0.593353	0.521469	0.55188	0.559695

Table 4: Grey relational coefficients for 18 comparability sequence

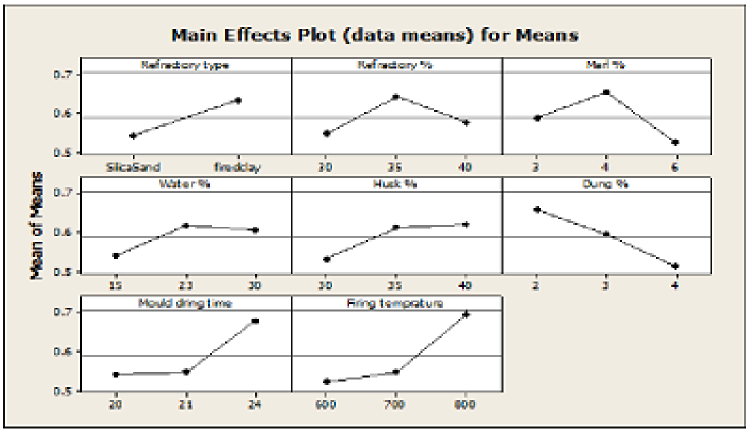

Figure 7: The plots of the control factor effects for grey relation grade

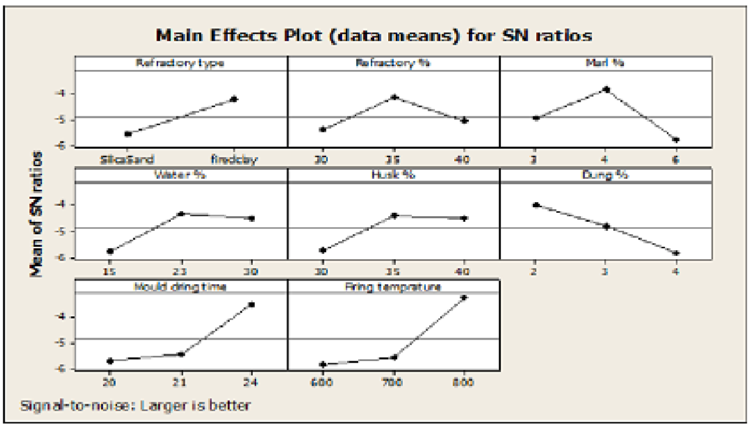

Figure 8: The plots of the control factor effects for SN ratio

Step 6: Determination of the Optimal Factor and its Level gives best mix to multiple performance characteristics using Minitab. It is obvious from the experiments that experiment no. 13 has large value of grade. In this way, it gives best mix to multiple performance characteristics. Keeping in mind the end goal to isolate out of each process variable on grey relational grade at different levels using Taguchi methodology, Grey relational graph is plotted as shown in Figure 7 and 8.

Mean value of Grey relational grade is 0.59. Fundamentally, the larger the Grey relational grade, the better is the multiple performance characteristics. Combination of A2B2C2D2E3F1G3 and H3 showed larger value of Grey relational grade for factors A, B, C, D, E, F, G and H respectively. . Accordingly, A2B2C2D2E3F1G3 is optimal parameter combination for three performance characteristics. However, significant contributions of process parameters still need to be known to predict optimal values of performance characteristics.

Step 7: Analysis of variance (ANOVA) of the overall grade is done to demonstrate the significant parameters. This is accomplished by setting the experimental response in Minitab. It demonstrates that the seven parameters A, C, D, E, F, G and H are observed to be the major factors with the selected multiple performance characteristics, because their corresponding P ratio is less than 0.05.

Step 8: The optimal grey relational grade (μGRG) is anticipated at the chosen optimal setting of process parameters. The significant parameters with optimal levels are already selected as: A2, C2, D2, E3, F1, G3 and H3. The estimated mean of the response characteristic is registered as Equation (7) [10].

$$\begin{aligned}\mu_{GRG} = \overline{T}_{GRG} &+ \left(A_2 - \overline{T}_{GRG}\right) + \left(C_2 - \overline{T}_{GRG}\right) + \left(D_2 - \overline{T}_{GRG}\right) + \left(E_3 - \overline{T}_{GRG}\right) + \left(F_1 - \overline{T}_{GRG}\right)\\ &+ \left(G_3 - \overline{T}_{GRG}\right) + \left(H_3 - \overline{T}_{GRG}\right)\end{aligned} \tag{7}$$

A confidence interval for the predicted mean on a confirmation run is ±3.32. And the 95% confidence interval of the predicted optimal grey relational grade is:

$$[\mu_{GRG} - CI] < \mu_{GRG} < [\mu_{GRG} + CI] \quad i.e. \quad -1.69 < \mu_{GRG} < 4.949$$

Step 9: After the optimal level of mould parameters has been recognized, a confirmation test should be done so as to check the accuracy of analysis. Predicting value for multiple performance characteristics at optimal setting of process parameters are affirmed through experimental results.

At that point three experiments on optimal combination of multiple quality characteristics are directed. The result of verification experiment in terms of SN ratio, compression strength, linear expansion and volumetric expansions are 30.71dB, -8.71 dB, and -14.45 dB, respectively. It tends to be seen from the above that SN ratio of each quality characteristic falls into 95% confidence interval, which therefore bears out the reliability of the optimal combination of processing parameters for multiple quality characteristics.

4. Conclusion

Taguchi's Signal-to-Noise ratio and Grey Relational Analysis were connected in this work to enhance the multi-response characteristics such as compression strength, linear and volumetric expansion during development process of local investment casting mould. In this work, the optimal parameters combination was determined as A2B2C2D2E3F1G3H3. Specifically, the refractory type of fired clay, its quantity of 35%, marl clay of 4%, water content of 23%, wheat husk of 35%, cow dung of 2%, mould drying time of 24 hours and the firing temperature of $800°C$. And The anticipated results were checked with experimental results and a good agreement was found. Finally this work demonstrates the method of using Taguchi methods for optimizing local investment casting mould for multiple response characteristics.

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