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Hyers-Ulam Stability of the Trigintic Functional Equation in Menger Probabilistic Normed Spaces

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Abstract: In this paper we examine the Hyers-Ulam stability of the trigintic functional equation in Menger probabilistic normed spaces, using the fixed point method.

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1. Introduction and Preliminaries

The fundamental problem about the solutions of the inequality that lie near the solutions of the strict equation in the case of replacing a given functional equation by a functional inequality, was formulated by Ulam in 1940 (cf. [15]) and solved the next year for the Cauchy functional equation by Hyers [4]. In 1950, Aoki [2] and Rassias [12], proved a generalization of Hyers' theorem for additive and linear mappings, respectively. Several stability results have been recently obtained for various equations and for mappings with more general domains and ranges (see [3, 5, 9, 11] and the references there in). The theory of probabilistic metric spaces introduced in 1942 by Menger [7], who proposed transferring the probabilistic notions of quantum mechanic from physics to the underlying geometry. Many years later, Alsina, Schweizer and Sklar in [1] gave a general definition of probabilistic normed spaces based on the definition of Menger for probabilistic metric spaces. In this section we recall some basic facts concerning Menger probabilistic normed spaces and some preliminary results as in [1, 3, 5, 14].

Definition 1.1. *A function $F:\mathbb{R}\to[0,1]$ is called a distribution function if it is non-decreasing and left continuous, with $\sup F(t)=1$ and $\inf F(t)=0$. The class of all distribution functions F with $F(0)=0$ is denoted by D_+. ε_0 is the element of D_+ defined by* $\varepsilon_0=\begin{cases}1, & t>0\\ 0, & t\leq 0\end{cases}$.

Definition 1.2. *A binary operation $\tau:[0,1]\times[0,1]\to[0,1]$ is said to be t-norm if it satisfies the following conditions:*

(a). τ is commutative and associative,

(b). τ is continuous,

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(c). $p\tau 1 = p$ *for all* $p \in [0,1]$,

(d). $p\tau q \leq r\tau s$ *whenever* $p \leq r$ *and* $q \leq s$ *for all* $p, q, r, s \in [0,1]$.

Definition 1.3. *Let* X *be a real vector space,* F *a mapping from* X *to* D_+ *(for any* $x \in X$, $F(x)$ *is denoted by* F_X*) and* τ *a t-norm. The triple* (X, F, τ) *is called a Menger probabilistic normed space (briefly, MPN space), if the following are satisfied:*

(a). $F_X(0) = 0$, *for all* $x \in X$,

(b). $F_X(0) = \varepsilon_0$ *if-f* $x = \theta$ *(*θ *is the null vector in* X*),*

(c). $F_{ax}(t) = F_X\left(\frac{t}{|a|}\right)$ *for all* $a \in \mathbb{R}$, $a \neq 0$ *and* $x \in X$,

(d). $F_{x+y}(t_1 + t_2) \geq F_x(t_1)\tau F_y(t_2)$ *for all* $x, y \in X$ *and* $t_1, t_2 > 0$.

Theorem 1.4 (Margolis-Diaz [6])**.** *Let* (X, d) *be a complete generalized metric space and let* $J : X \to X$ *be a strictly contractive mapping with Lipschitz constant* $L < 1$. *Then for each given element* $x \in X$, *either*

(B_1*)* $d(T^n x, T^{n+1}x) = \infty, \quad \forall \; n \geq 0$ *or*

(B_2*) there exists a positive integer* n_0 *such that:*

(1). $d(T^n x, T^{n+1}x) < \infty, \quad \forall \; n \geq n_0$,

(2). The sequence $\{T^n x\}$ *is convergent to a fixed point* y^* *of* J,

(3). y^* *is the unique fixed point of* T *in the set* $Y = \{y \in X : d(T^{n_0}x, y) < \infty\}$,

(4). $d(y^*, y) \leq \frac{1}{1-L} d(y, Ty)$ *for all* $y \in Y$.

Definition 1.5. *Let* (X, F, τ) *be a MPN space and let* $\{x_n\}$ *be a sequence in* X. *Then* $\{x_n\}$ *is said to be convergent if there exists* $x \in X$ *such that* $\lim\limits_{n\to\infty} F_{x_n - x}(t) = 1$ *for all* $t > 0$. *In this case* x *is called the limit of* $\{x_n\}$.

Definition 1.6. *The sequence* $\{x_n\}$ *in MPN space* (X, F, τ) *is called Cauchy if for each* $\varepsilon > 0$ *and* $\delta > 0$, *there exists some* n_0 *such that* $F_{x_n - x_m}(\delta) > 1 - \varepsilon$ *for all* $m, n \geq n_0$.
Clearly, every convergent sequence in a MPN space is Cauchy. If each Cauchy sequence is convergent in a MPN space (X, F, τ), *then* (X, F, τ) *is called Menger probabilistic Banach space (briefly, MPB space).*

Let X, Y be vector spaces and $f : X \to Y$ be a mapping. Define a mapping $Df : X^2 \to Y$ by

$$\begin{aligned}
Df(x,y) := {} & f(x+15y) - 30f(x+14y) + 435f(x+13y) - 4060f(x+12y) \\
& + 27405f(x+11y) - 142506f(x+10y) + 593775f(x+9y) - 2035800f(x+8y) \\
& + 5852925f(x+7y) - 14307150f(x+6y) + 30045015f(x+5y) - 54627300f(x+4y) \\
& + 86493225f(x+3y) - 119759850f(x+2y) + 145422675f(x+y) - 155117520f(x) \\
& + 145422675f(x-y) - 119759850f(x-2y) + 86493225f(x-3y) - 54627300f(x-4y) \\
& + 30045015f(x-5y) - 14307150f(x-6y) + 5852925f(x-7y) - 2035800f(x-8y) \\
& + 593775f(x-9y) - 142506f(x-10y) + 27405f(x-11y) - 4060f(x-12y) + 435f(x-13y) \\
& - 30f(x-14y) + f(x-15y) - 30!f(y)
\end{aligned}$$

where $30! = 265252859812191058636308480000000$. The functional equation

$$Df(x,y) = 0 \tag{1}$$

$x, y \in X$, is called trigintic functional equation.

In this paper we give a solution of the trigintic functional equation as introduced in Ramdoss et. al. [10], but in the spirit of Nazarianpoor et. al. [9], avoiding calculations with nine decimal points and powers of ten, and we study the stability of the functional equation (1) in Menger probabilistic normed spaces by using the standard fixed point method.

2. General Solution of Trigintic Functional Equation

In this section, we solve the trigintic functional equation (1) in vector spaces.

Theorem 2.1. *If X and Y are real vector spaces and $f : X \to Y$ is a mapping satisfying trigintic functional equation (1) for all $x, y \in X$, then f is a trigintic mapping, i.e., $f(x) = x^{30}$ for all $x \in X$.*

Proof. Let $f_i : X \to Y$ $(i = 1, 2, \ldots, 17)$ be mappings defined by the following relations:

$$\begin{aligned}
f_1(x) := {} & f(16x) - 30f(15x) + 435f(14x) - 4060f(13x) + 27405f(12x) - 142506f(11x) \\
& + 593775f(10x) - 2035800f(9x) + 5852925f(8x) - 14307150f(7x) + 30045015f(6x) \\
& - 54627300f(5x) + 86493225f(4x) - 119759850f(3x) + 145422675f(2x) \\
& - 155117520f(x) + 145422675f(0) - 119759850f(-x) + 86493225f(-2x) \\
& - 54627300f(-3x) + 30045015f(-4x) - 14307150f(-5x) + 5852925f(-6x) \\
& - 2035800f(-7x) + 593775f(-8x) - 142506f(-9x) + 27405f(-10x) \\
& - 4060f(-11x) + 435f(-12x) - 30f(-13x) + f(-14x)
\end{aligned} \tag{2}$$

$$\begin{aligned}
f_2(x) := {} & f(30x) - 30f(28x) + 435f(26x) - 4060f(24x) + 27405f(22x) \\
& - 142506f(20x) + 593775f(18x) - 2035800f(16x) + 5852925f(14x) \\
& - 14307150f(12x) + 30045015f(10x) - 54627300f(8x) + 86493225f(6x) \\
& - 119759850f(4x) - 132626429906095529318154094577733f(2x)
\end{aligned} \tag{3}$$

$$\begin{aligned}
f_3(x) := {} & 30f(29x) - 465f(28x) + 4060f(27x) - 26970f(26x) \\
& + 142506f(25x) - 597835f(24x) \ + 2035800f(23x) - 5825520f(22x) \\
& + 14307150f(21x) - 30187521f(20x) + 54627300f(19x) \ - 85899450f(18x) \\
& + 119759850f(17x) - 147458475f(16x) + 155117520f(15x) \ - 139569750f(14x) \\
& + 119759850f(13x) - 100800375f(12x) + 54627300f(11x) \ + 14307150f(9x) \\
& - 60480225f(8x) + 2035800f(7x) + 85899450f(6x) + 142506f(5x) \\
& - 119787255f(4x) + 4060f(3x) + (145422240 - 30!/2)\, f(2x) + (30 + 30!)\, f(x)
\end{aligned} \tag{4}$$

$$\begin{aligned}
f_4(x) := {} & -435f(28x) + 8990f(27x) - 94830f(26x) + 679644f(25x) \\
& - 3677345f(24x) + 15777450(23x) - 55248480f(22x) + 161280600f(21x) \\
& - 399026979f(20x) + 846723150f(19x) - 1552919550f(18x) + 2475036900f(17x) \\
& - 3445337025f(16x) + 4207562730f(15x) - 4513955850f(14x) + 4242920400f(13x)
\end{aligned}$$

$$- 3491995125f(12x) + 2540169450f(11x) - 1638819000f(10x) + 887043300f(9x)$$
$$- 368734275f(8x) + 173551950f(7x) - 146973450f(6x) + 17670744f(5x) + 115512075f(4x)$$
$$+ 818090f(3x) + (-145544040 + 30!/2)f(2x) + (13050 - (31)30!)f(x) \quad (5)$$
$$f_5(x) := -4060f(27x) + 94395f(26x) - 1086456f(25x) + 8243830f(24x)$$
$$- 46212660f(23x) + 203043645f(22x) - 724292400f(21x) + 2146995396f(20x)$$
$$- 5376887100f(19x) + 11516661975f(18x) - 21287838600f(17x)$$
$$+ 34179215850f(16x) - 47887972020f(15x) + 58744907775f(14x)$$
$$- 63233200800f(13x) + 59766868500f(12x) - 49555365300f(11x)$$
$$+ 35985733875f(10x) - 22875832200f(9x) + 12700847250f(8x)$$
$$- 6050058300f(7x) + 2399048925f(6x) - 867902256f(5x)$$
$$+ 373804200f(4x) - 61172020f(3x) + (-133622430 + 30!/2)f(2x)$$
$$- (1766100 + (466)30!)f(x) \quad (6)$$
$$f_6(x) := -27405f(26x) + 679644(25x) - 8239770f(24x) + 65051640f(23x)$$
$$- 375530715f(22x) + 1686434100f(21x) - 6118352604f(20x)$$
$$+ 18385988400f(19x) - 46570367025f(18x) + 100694922300f(17x)$$
$$- 187607622150f(16x) + 303274521480f(15x) - 427480083225f(14x)$$
$$+ 527182859700f(13x) - 570010262700f(12x) + 540860695200f(11x)$$
$$- 450239257125f(10x) + 328286661300f(9x) - 209085990750f(8x)$$
$$+ 115932702600f(7x) - 55687980075f(6x) + 22894973244f(5x)$$
$$- 7891543800f(4x) + 2349558540f(3x) + (-712318590 + 30!/2)f(2x)$$
$$+ (111264300 - (4526)30!)f(x) \quad (7)$$
$$f_7(x) := -142506f(25x) + 3681405f(24x) - 46212660f(23x) + 375503310f(22x)$$
$$- 2218942830f(21x) + 10154051271f(20x) - 37405110600f(19x)$$
$$+ 113829042600f(18x) - 291392523450f(17x) + 635776013925f(16x)$$
$$- 1193786635020f(15x) + 1942866747900f(14x) - 2754835829550(13x)$$
$$+ 3415298145675f(12x) - 3710134940400f(11x) + 3535069151250f(10x)$$
$$- 2953732027950f(9x) + 2161260840375f(8x) - 1381128453900f(7x)$$
$$+ 767695656000f(6x) - 369192472506f(5x) + 152507893230f(4x)$$
$$- 53442362610f(3x) + (15572006460 + 30!/2)f(2x)$$
$$- (3905376930 + (31931)30!)f(x) \quad (8)$$
$$f_8(x) := -593775f(24x) + 15777450f(23x) - 203071050f(22x)$$
$$+ 1686434100f(21x) - 10153908765f(20x) + 47211389550f(19x)$$
$$- 176284672200f(18x) + 542684406600f(17x) - 1403078703975f(16x)$$
$$+ 3087808272570f(15x) - 5841851265900f(14x) + 9570967692300f(13x)$$
$$- 13651199038425f(12x) + 17013468783150f(11x) - 18570108153870f(10x)$$

$$
\begin{aligned}
&+ 17769871695600f(9x) - 14905236343725f(8x) + 10944675067950f(7x) \\
&- 7017022357800f(6x) + 3912402577590f(5x) - 1886351099850f(4x) \\
&+ 780696557550f(3x) + (-275120282700 + 30!/2)\, f(2x) \\
&+ (84616500150 + (-174437)\, 30!)\, f(x) \qquad (9) \\
f_9(x) := &-2035800f(23x) + 55221075f(22x) - 724292400f(21x) \\
&+ 6118495110f(20x) - 37405110600f(19x) + 176284078425f(18x) \\
&- 666122738400f(17x) + 2072241837900f(16x) - 5407419718680f(15x) \\
&+ 11998127515725f(14x) - 22865357365200f(13x) + 37706315635950f(12x) \\
&- 54096936150600f(11x) + 67778240694255f(10x) - 74335033742400f(9x) \\
&+ 71443112504400f(8x) - 60165729865800f(7x) + 44340492910350f(6x) \\
&- 28523940293160f(5x) + 15953885973900f(4x) - 7716942160200f(3x) \\
&+ (3216472663050 + 30!/2)\, f(2x) - (1208807145000 + (768212)\, 30!)\, f(x) \qquad (10) \\
f_{10}(x) := &-5852925f(22x) + 161280600f(21x) - 2146852890f(20x) \\
&+ 18385988400f(19x) - 113829636375f(18x) + 542684406600f(17x) \\
&- 2072239802100f(16x) + 6507964996320f(15x) - 17128368454275f(14x) \\
&+ 38300284171800f(13x) - 73503941704050f(12x) + 121985971304400f(11x) \\
&- 176028861935745f(10x) + 221716448022600f(9x) - 244345134711600f(8x) \\
&+ 235885753935000f(7x) - 199466670793650f(6x) + 147559852734840\, f(5x) \\
&- 95264636714100f(4x) + 53504490475800f(3x) + (-26200137021750 + 30!/2)\, f(2x) \\
&+ (11915384715000 + (-2804012)\, 30!)\, f(x) \qquad (11) \\
f_{11}(x) := &-14307150f(21x) + 399169485f(20x) - 5376887100f(19x) \\
&+ 46569773250f(18x) - 291392523450f(17x) + 1403080739775f(16x) \\
&- 5407419718680f(15x) + 17128362601350f(14x) - 45438391741950f(13x) \\
&+ 102347277714825f(12x) - 197743518548100f(11x) + 330209496997380f(10x) \\
&- 479228972038650f(9x) + 606802881215700f(8x) - 672005632398750f(7x) \\
&+ 651683885303100f(6x) - 553409330201910f(5x) + 411134121628650f(4x) \\
&- 267059076306750f(3x) + (153126402939000 + 30!/2)\, f(2x) \\
&- (83738675913750 + (8656937)\, 30!)\, f(x) \qquad (12) \\
f_{12}(x) := &-30045015f(20x) + 846723150f(19x) - 11517255750f(18x) \\
&+ 100694922300f(17x) - 635773978125f(16x) + 3087808272570f(15x) \\
&- 11998133368650f(14x) + 38300284171800f(13x) - 102347263407675f(12x) \\
&+ 232115017809150f(11x) - 451351478197620f(10x) + 758242586327250f(9x) \\
&- 1106619685926300f(8x) + 1408584615837750f(7x) - 1567663827993900f(6x) \\
&+ 1527566781870090f(5x) - 1304326871016750f(4x) + 978907695743250f(3x) \\
&+ (-657561068226000 + 30!/2)\, f(2x) + (429858536357250 + (-22964087)\, 30!)\, f(x) \qquad (13)
\end{aligned}
$$

$$\begin{aligned}
f_{13}(x) :=& -54627300f(19x) + 1552325775f(18x) - 21287838600f(17x)\\
&+ 187609657950f(16x) - 1193786635020f(15x) + 5841845412975f(14x)\\
&- 22865357365200f(13x) + 73503956011200f(12x) - 197743518548100f(11x)\\
&+ 451351478197620f(10x) - 883036362932700f(9x) + 1492083626178600f(8x)\\
&- 2189723856570900f(7x) + 2802386007357300f(6x) - 3137223028200300f(5x)\\
&+ 3082739559480000f(4x) - 2680444435441500f(3x) + (2116980393716250 + 30!/2)f(2x)\\
&- (1641278047909500 + (53009102)30!)f(x) \qquad (14)\\
f_{14}(x) :=& -86493225f(18x) + 2475036900f(17x) - 34177180050f(16x)\\
&+ 303274521480f(15x) - 1942872600825f(14x) + 9570967692300f(13x)\\
&- 37706301328800f(12x) + 121986025931700f(11x) - 330211135816380f(10x)\\
&+ 758265447852300f(9x) - 1492280065949400f(8x) + 2536664554628100f(7x)\\
&- 3747555964561500f(6x) + 4839261390884700f(5x) - 5502121998156000f(4x)\\
&+ 5583333148438500f(3x) + (-5206737835383750 + 30!/2)f(2x)\\
&+ (4724891350042500 + (-107636402)30!)f(x) \qquad (15)\\
f_{15}(x) :=& -119759850f(17x) + 3447372825f(16x) - 47887972020f(15x)\\
&+ 427474230300f(14x) - 2754835829550f(13x) + 13651299838800f(12x)\\
&- 54099476320050f(11x) + 176064847669620f(10x) - 479557258699950f(9x)\\
&+ 1108780523405100f(8x) - 2200552598936250f(7x) + 3784879521013500f(6x)\\
&- 5695237168586550f(5x) + 7582192509654000f(4x) - 9070752954422250f(3x)\\
&+ (9970028556016500 + 30!/2)f(2x) - (10358415652016250 + (194129627)30!)f(x) \qquad (16)\\
f_{16}(x) :=& -145422675f(16x) + 4207562730f(15x) - 58750760700f(14x)\\
&+ 527302619550f(13x) - 3418790140800f(12x) + 17063024148450f(11x)\\
&- 68228479951380f(10x) + 224670180050550f(9x) - 621708111706500f(8x)\\
&+ 1468744295645250f(7x) - 3001084835521500f(6x) + 5364123903490950f(5x)\\
&- 8473651300296000f(4x) + 11943231279824250f(3x) + (-15148979625460500 + 30!/2)f(2x)\\
&+ (17415797744598750 + (-313889477)30!)f(x) \qquad (17)\\
f_{17}(x) :=& -155117520f(15x) + 4653525600f(14x) - 67476121200f(13x)\\
&+ 629777131200f(12x) - 4250995635600f(11x) + 22105177305120f(10x)\\
&- 92104905438000f(9x) + 315788247216000f(8x) - 907891210746000f(7x)\\
&+ 2219289626268000f(6x) - 4660508215162800f(5x) + 8473651300296000f(4x)\\
&- 13416614558802000f(3x) + (18576850927572000 + 30!/2)f(2x)\\
&- (22557604697766000 + (459312152)30!)f(x) \qquad (18)
\end{aligned}$$

Replacing (x, y) by (x, x) in (1), we get

$$f_1(x) = 30!f(x) \qquad (19)$$

for all $x \in X$. Replacing (x, y) by $(0, 2x)$ in (1), we obtain

$$f_2(x) = 0 \tag{20}$$

for all $x \in X$. Replacing (x, y) by (ix, x) $(i = 1, 2, \ldots, 15)$ in (1), we get

$$f_i(x) = 0 \quad (i = 3, 4, \ldots, 17) \tag{21}$$

for all $x \in X$. Replacing (x, y) by $(0, x)$ in (1), we have

$$f(2x) = 1073741824 f(x) = 2^{30} f(x) \tag{22}$$

for all $x \in X$. Hence f is a trigintic mapping. $\square$

3. Hyers-Ulam Stability of the Trigintic Functional Equation (1) in Menger Probabilistic Normed Spaces

In this section, we prove the Hyers-Ulam stability of trigintic functional equation (1) in Menger probabilistic normed spaces.

Theorem 3.1. *Let (X, F, τ) be a Menger PN space and (Y, G, τ) be a Menger PB space. If $f : X \to Y$ satisfies*

$$G_{Df(x,y)}(t + s) \geq F_x(t)\tau F_y(s) \tag{23}$$

for all $t, s > 0$, and there exists positive number k, $0 < k < \frac{1}{2^{30}}$ such that

$$F_x(2t) \geq F_{kx}(t) \tag{24}$$

Then there exists a unique trigintic mapping $J : X \to Y$ such that

$$G_{f(x)-J(x)}(t) \geq F_x\left(\frac{1 - 2^{30}k}{2^{29}k}\right) \tag{25}$$

for all $x \in X$ and $t > 0$.

Proof. From Theorem 2.1 and using (22), inequality (23) is leading to

$$G_{f(2x)-2^{30}f(x)}(2t) \geq F_x(t) \tag{26}$$

for all $x \in X$ and $t > 0$. Replacing x by $\frac{x}{2}$ in (26) we have

$$G_{f(x)-2^{30}f\left(\frac{x}{2}\right)}(2t) \geq F_{\frac{x}{2}}(t) \tag{27}$$

for all $x \in X$ and $t > 0$. By Definition 1.3 and replacing t with $\frac{t}{2}$ in (27), we obtain

$$G_{f(x)-2^{30}f\left(\frac{x}{2}\right)}(t) \geq F_x(t) \tag{28}$$

for all $x \in X$ and $t > 0$. Let $P := \{m : X \to Y : m(0) = 0\}$ and the generalized metric d defined on P by

$$d(n, m) = \inf\{\lambda \in [0, \infty]/G_{m(x)-n(x)}(\lambda t) \geq F_x(t)\}. \tag{29}$$

Then it is easy to show that (P, d) is a generalized complete metric space [8]. We define an operator $T : P \to P$ by

$$Tn(x) = 2^{30} n\left(\frac{x}{2}\right)$$

for all $x \in X$. We assume that T is a strictly contractive operator. Indeed given $m, n \in P$, let $\lambda \in (0, \infty)$ be an arbitrary constant with $d(m, n) < \lambda$. From the definition of d, it follows that

$$G_{m(x)-n(x)}(\lambda t) \geq F_x(t) \tag{30}$$

for all $x \in X$ and $t > 0$. Therefore,

$$\begin{aligned} G_{Tm(x)-Tn(x)}(2^{30}k\lambda t) &= G_{2^{30}m\left(\frac{x}{2}\right)-2^{30}n\left(\frac{x}{2}\right)}(2^{30}k\lambda t) \\ &= G_{m\left(\frac{x}{3}\right)-n\left(\frac{x}{3}\right)}(k\lambda t) \\ &\geq F_{\frac{x}{2}}(kt) \\ &\geq F_x(t) \end{aligned}$$

for all $x \in X$ and $t > 0$. That means,

$$d(Tm, Tn) < 2^{30}k\lambda$$

or

$$d(Tm, Tn) \leq 2^{30}kd(m, n)$$

for all $m, n \in P$. Hence, T is a strictly contractive operator on P with the Lipschitz constant $L = 2^{30}k$.It follows from (28) that $d(f, Tf) \leq 2^{29}k$. According to Theorem 1.4, there exists a unique fixed point J of T in the set $K := \{n \in P : d(m, n) < \infty\}$ that is the mapping $J : X \to Y$ with

$$J\left(\frac{x}{2}\right) = \frac{1}{2^{30}}J(x)$$

for all $x \in X$. This implies that there exists $\lambda \in [0, \infty]$ satisfying

$$G_{f(x)-T(x)}(\lambda t) \geq F_x(t)$$

for all $x \in X$ and $t > 0$. Moreover, $d(T^n f, J) \to 0$, which implies

$$J(x) = \lim_{n\to\infty} 2^{30n} f\left(\frac{x}{2^n}\right)$$

for all $x \in X$. Also,

$$d(f, J) \leq \frac{d(f, Jf)}{1 - L} \leq \frac{2^{29}k}{1 - 2^{30}k}$$

with $f \in K$ hence,

$$G_{f(x)-J(x)}\left(\frac{2^{29}kt}{1 - 2^{30}k}\right) \geq F_x(t)$$

that means

$$G_{f(x)-J(x)}(t) \geq F_x\left(\frac{1-2^{30}k}{2^{29}k}\right)$$

for all $x \in X$ and $t > 0$. Since, $F_{\frac{x}{2^n}}(t) \geq F_{k^n x}(t)$ and $F_{\frac{y}{2^n}}(t) \geq F_{k^n y}(t)$ then

$$F_{\frac{x}{2^n}}(t) \cdot F_{\frac{y}{2^n}}(t) \geq F_{k^n x}(t) \tau F_{k^n y}(t)$$

for all $x \in X$ and $t, s > 0$. This implies

$$\lim_{n\to\infty} F_{k^n x}(t) \cdot F_{k^n y}(t) = 1$$

and

$$G_{DJ(x,y)}(t+s) = 1$$

$x, y \in X$. Hence J is a trigintic mapping and the proof is complete. □

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