Int. J. Math. And Appl., 9(4)(2021), 89–90
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

Approximation of the Series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4n-3}$

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Abstract: In this paper we give a rational approximation to an alternating series, by applying a correction function to the series. The introduction of correction function certainly improves the value of sum of the series and gives a better approximation to it.

Keywords: Correction function, error function, remainder term, rational approximation.

1. Introduction

Commenting on the Lilavati rule for finding the value of circumference of a circle from its diameter, the commentator Sankara refers to various infinite series for computing the circumference from the diameter. One such series attributed to illustrious mathematician Madhava of 14^{th} century is $C = \frac{4d}{1} - \frac{4d}{3} + \frac{4d}{5} - \cdots \pm \frac{4d}{2n-1} \mp \frac{4d\left(\frac{2n}{2}\right)}{(2n)^2+1}$, where $+$ or $-$ indicates that n is odd or even and C is the circumference of a circle of diameter d.

2. The Alternating Series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4n-3}$

The alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4n-3}$ is a convergent series and converges to $\frac{[\pi + 2\ln(\sqrt{2}+1)]}{4\sqrt{2}}$

Definition 2.1. *The remainder term after n terms of the series is $R_n = (-1)^n G_n$ where G_n is the correction function after n terms.*

Definition 2.2. *If G_n denotes the correction function, then the error function is defined as $E_n = G_n + G_{n+1} - \frac{1}{4n+1}$.*

Theorem 2.3. *The correction function for the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4n-3}$ is $G_n = \frac{1}{8n-2}$.*

Proof. For a fixed n and for any $r \in R$ and for $G_n(r) = \frac{1}{8n+2-r}$ the error function $E_n(r)$ is minimum for $r = 4$. Thus the correction function for the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4n-3}$ is $G_n = \frac{1}{8n-2}$. The corresponding error function is $|E_n| = \frac{16}{256n^3 + 192^2 - 16n - 12}$. Hence the proof. □

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Remark 2.4. *Clearly* $G_n < \dfrac{1}{4n+1}$, *absolute value of* $(n+1)^{th}$ *term. Also* $\dfrac{1}{8n+2} < \dfrac{1}{8n-2} < \dfrac{1}{8n-6}$ *i.e.* $\dfrac{1}{2}\{|t_{n+1}|\} < G_n < \dfrac{1}{2}\{|t_n|\}$ *where tn denotes* n^{th} *term of the series.*

Theorem 2.5. *The error function* $E_n(r)$ *is continuous at* $r = 4$.

Proof. The proof is left to the reader. □

Theorem 2.6. *The error function* $E_n(r)$ *has a local minimum at the point* $r = 4$.

Proof. The proof is obtained from the proof of Theorem 2.3.
Now for $G_n(r) = \dfrac{1}{6n+2-r}$ where $r \in R$ and n is fixed, we have the error function $E_n(r)$ is minimum for $r = 3$. Thus the correction function for the series $\sum_{n=1}^{\infty}(-1)^{n-1}\dfrac{1}{3n-2}$ is $G_n = \dfrac{1}{6n-1}$. The corresponding error function is $|E_n| = \dfrac{9}{108n^3+108n^2+9n-5}$. Hence the proof. □

Remark 2.7. *Clearly* $G_n < \dfrac{1}{3n+1}$, *absolute value of* $(n+1)^{th}$ *term. Also* $\dfrac{1}{6n+2} < \dfrac{1}{6n-1} < \dfrac{1}{6n-4}$ *i.e.* $\dfrac{1}{2}\{|t_{n+1}|\} < G_n < \dfrac{1}{2}\{|t_n|\}$.

Proposition 2.8. *The error function* $|E_n(r)|$ *is continuous at* $r = 3$.

Theorem 2.9. *The error function* $|E_n(r)|$ *has a local minimum at the point* $r = 3$.

Proof. The proof is obtained from Theorem 2.3. □

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