Int. J. Math. And Appl., 6(4)(2018), 107–116
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

The Magnetic, Joule's and Ohmic Effect on Incompressible Viscous Flow Over a Hyperbolic Stretching Circular Cylinder

K. Divya Joseph[1,*] and P. A. Dinesh[1]

1 Department of Mathematics, M.S. Ramaiah Institute of Technology, Bangalore, Karnataka, India.

Abstract: In we study the heat transfer of boundary layer flow of an incompressible viscous fluid over hyperbolic stretching cylinder. The governing nonlinear partial differential equations are converted into ordinary differential equations by using suitable transformations, which are then tackled using the homotopy method. The homotopy method gives us solutions in the form of series. The influence of the Magnetic, Joule's and Ohmic effect on velocity as well as temperature profiles are investigated and results can be seen visually in graphs. The computational results without these effects agree excellently with the previous results by [1].

Keywords: Joule's effect, Ohmic effect, Magnetohydrodynamic, incompressible, boundary layer.

Accepted on: 21.04.2018

1. Introduction

The heat transfer of the boundary layer flow over stretching boundaries has attained exceptional recognition in modern industrial and engineering fields. Here we study the boundary layer flow of an incompressible viscous fluid over hyperbolic stretching cylinder stretching sheet when subjected to the Magnetic, Joule's and Ohmic effect. Its significance to the real-world, has drawn attention among scientists and engineers in order to understand this phenomenon. Magnetohydrodynamic fluid flows have various applications in the area of polymer and metallurgical industry. The study of mutual interaction of fluid flow and magnetic field related phenomena in MHD flows is used in the cooling of filaments or continuous strips for metallurgical use. The characteristics of final products highly depend on the cooling rate. It also has wide applications in nuclear reactor technology and also in aerodynamics, in the study of aircraft design in order to analyse the prospects of enhancing speed and proficiency of the aircraft. Reddy in [6] have studied the effect of thermophoresis and Brownian moment on hydro-magnetic motion of a nanofluid over a slendering stretching sheet by reaching a similarity solution. M.M. Rashidia in a similar manner in [4] analyzes a magnetic field to which the convective flow of non-Newtonian fluid due to a linearly stretching sheet, is subject to. This is achieved by transforming the governing equations to a system of ordinary differential equations by a similarity method. The optimal homotopy analysis method is used to solve the resulting system of ordinary differential equations. On parallel lines, C. Sulochana in [5] have studied the effects of thermal radiation and slip effects on magneto hydrodynamic forced convective flow of a nano-fluid over a slendering stretching sheet in porous medium. Self-similarity transformation reduce the governing partial differential equations are transformed into nonlinear ordinary

* *E-mail: divyakj@msrit.edu*

differential equations which are solved numerically using Matlab. Swati Mukhopadhyay in [3] investigates an axi-symmetric laminar boundary layer flow of a viscous incompressible fluid and heat transfer towards a stretching cylinder embedded in a porous medium by converting the partial differential equations corresponding to the momentum and heat equations into highly nonlinear ordinary differential equations with the help of similarity transformations. Numerical solutions of these equations are obtained by shooting method. Swati Mukhopadhyay in [2] also considers the boundary layer flow of a viscous incompressible fluid along a porous nonlinearly stretching sheet by converting the partial differential equation corresponding to the momentum equation into nonlinear ordinary differential equation by carrying out similarity transformations. A Numerical solution of this is attained using the shooting method.

Nomenclature

u, v	: velocity components in x, r directions
f	: dimensionless velocity of the fluid
N	: coefficient related to stretching sheet
n	: velocity power index parameter
c	: physical parameter related to stretching sheet
$B(x)$	: magnetic field parameter
T	: temperature of the fluid (K)
T_w	: surface fluid temperature (K)
T_∞	: free stream temperature
k	: thermal conductivity $(Wm^{-1}K)$
k_0	: chemical reaction parameter
C_p	: specific heat at constant pressure $(JkgK^{-1})$
B_0	: magnetic field strength
a_1, b_1	: constants
Greek Symbols	
ϕ	: dimensionless concentration
η	: similarity variable
σ	: electrical conductivity of the fluid (mXm^{-1})
α	: the thermal diffusivity
θ	: dimensionless temperature
ρ	: density of the fluid (kgm^{-3})
μ	: dimensional variable viscosity parameter
ν	: kinematic viscosity (m^2s^{-1})

2. Problem Formulation

Consider the two-dimensional steady incompressible flow of a viscous fluid over a hyperbolic stretching circular cylinder of a fixed radius R. The governing equations are,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial r} = \nu\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - \mu\sigma B_0^2 u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\alpha\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)-\frac{\sigma B_0^2u^2}{\rho_0 c_p}, \tag{3}$$

With boundary conditions,

$$\begin{aligned} &u\left(r,x\right)=U\left(x\right),\quad v\left(r,x\right)=0,\quad T=T_w+T_\infty+AU\left(x\right)\quad at\quad r=R,\\ &u\left(r,x\right)\to 0,\quad T=T_\infty\quad as\quad r\to\infty, \end{aligned} \tag{4}$$

where A is constant, u,v are velocity components along x and r directions, T represents temperature, $\alpha=\frac{k}{\rho c_p}$ is the thermal diffusivity of the fluid. We introduce the stream function $u=\frac{1}{r}\frac{\partial\psi}{\partial r}$, $v=-\frac{1}{r}\frac{\partial\psi}{\partial x}$ by introducing similarity transformations (5),

$$\eta=\frac{r^2-R^2}{2R}\left(\frac{U}{\nu x}\right)^{\frac{1}{2}},\quad \psi=\left(U\nu x\right)^{\frac{1}{2}}Rf\left(\eta\right),\quad \theta\left(\eta\right)=\frac{T-T_\infty}{T_w-T_\infty}, \tag{5}$$

that convert (1)-(4) to the system of ODE,

$$\begin{aligned} K_6\frac{d^3f}{d\eta^3}+K_5\frac{d^2f}{d\eta^2}+K_4\frac{df}{d\eta}+\left(K_3^*\frac{d^2f}{d\eta^2}+K_3\frac{df}{d\eta}\right)\left(f-\eta\frac{df}{d\eta}\right)=0,\\ K_7\frac{d^2\theta}{d\eta^2}+K_8\frac{d\theta}{d\eta}+K_9\left(\frac{df}{d\eta}\right)^2+K_{10}\eta\frac{d\theta}{d\eta}+K_{11}\frac{d\theta}{d\eta}\left[f-\eta\frac{df}{d\eta}\right]=0, \end{aligned} \tag{6}$$

Here,

$$\begin{aligned} &L_2=-\frac{\sigma B_0^2}{\rho_0 c_p},\quad K_3\left(x\right)=\frac{U}{4}\left(\frac{U\gamma}{x}\right)^{\frac{1}{2}}\frac{R}{r^2}\left(2-r\right),\quad K_3^*\left(x\right)=\frac{U^2}{2x},\quad K_4=-\mu\sigma B_0^3U,\\ &K_5\left(x\right)=-\frac{3\gamma U}{R}\left(\frac{U}{\gamma x}\right)^{\frac{1}{2}},\quad K_6\left(x\right)=\left(\frac{U^2}{x}\right)\frac{r^2}{R^2},\quad K_7\left(r,x\right)=\frac{\alpha\left(T_w-T_\infty\right)}{R^2}\left(\frac{U}{\gamma}\right)\left(\frac{r^2}{x}\right),\\ &K_8\left(x\right)=\frac{2\alpha\left(T_w-T_\infty\right)}{R}\left(\frac{U}{\gamma}\right)^{\frac{1}{2}}\frac{1}{x^{\frac{1}{2}}},\quad K_9=L_2U^2,\quad K_{10}\left(x\right)=\frac{U}{2x}\left(T_w-T_\infty\right),\quad K_{11}\left(x\right)=-\frac{T_w-T_\infty}{2}\frac{U}{x}, \end{aligned} \tag{7}$$

with corresponding boundary conditions,

$$f\left(0\right)=0,\quad \frac{df}{d\eta}|_{\eta=0}=1,\quad \theta\left(0\right)=\frac{T_w-AU\left(x\right)}{T_w-T_\infty},\quad \frac{df}{d\eta}\ \ is\ \ bounded\ \ as\ \eta\to\infty,\quad \theta\left(\eta\right)=0\ \ as\ \ \eta\to\infty. \tag{8}$$

3. Solution Methodology

First we establish the following homotopy equations for (5), (6) with (7)

$$H(f,p)=(1-p)\left(\frac{d^3f}{d\eta^3}+\frac{K_5}{K_6}\frac{d^2f}{d\eta^2}+\frac{K_4}{K_6}\frac{df}{d\eta}\right)+p\left(\frac{d^3f}{d\eta^3}+\frac{K_5}{K_6}\frac{d^2f}{d\eta^2}+\frac{K_4}{K_6}\frac{df}{d\eta}+\left(\frac{K_3^*}{K_6}\frac{d^2f}{d\eta^2}+\frac{K_3}{K_6}\frac{df}{d\eta}\right)\left(f-\eta\frac{df}{d\eta}\right)\right)=0, \tag{9}$$

$$H(\theta,p)=(1-p)\left(\frac{d^2\theta}{d\eta^2}+\frac{K_8}{K_7}\frac{d\theta}{d\eta}\right)+p\left(\frac{d^2\theta}{d\eta^2}+\frac{K_8}{K_7}\frac{d\theta}{d\eta}+\frac{K_9}{K_7}\left(\frac{df}{d\eta}\right)^2+\frac{K_{10}}{K_7}\eta\frac{d\theta}{d\eta}+\frac{K_{11}}{K_7}\frac{d\theta}{d\eta}\left[f-\eta\frac{df}{d\eta}\right]\right)=0, \tag{10}$$

According to the generalized homotopy method, assume the solution for (9) and (10) in the form

$$f_p=p^0f_0+f_1p+f_2p^2+f_3p^3+\ldots, \tag{11}$$

$$\theta_p=p^0\theta_0+\theta_1p+f_2p^2+\theta_3p^3+\ldots, \tag{12}$$

Substituting (11), (12) into (9), (10) and rearranging the terms of order p, we have, concerning f,

$$K_6\frac{d^3f_0}{d\eta^3}+K_5\frac{d^2f_0}{d\eta^2}+K_4\frac{df_0}{d\eta}=0, \tag{13}$$

$$K_6\frac{d^3f_1}{d\eta^3}+K_5\frac{d^2f_1}{d\eta^2}+K_4\frac{df_1}{d\eta}+\left(K_3^*\frac{d^2f_0}{d\eta^2}+K_3\frac{df_0}{d\eta}\right)\left(f_0-\eta\frac{df_0}{d\eta}\right)=0, \quad (14)$$

$$K_6\frac{d^3f_2}{d\eta^3}+K_5\frac{d^2f_2}{d\eta^2}+K_4\frac{df_2}{d\eta}+\left(K_3^*\frac{d^2f_2}{d\eta^2}+K_3\frac{df_2}{d\eta}\right)\left(f_0-\eta\frac{df_0}{d\eta}\right)+\left(K_3^*\frac{d^2f_1}{d\eta^2}+K_3\frac{df_1}{d\eta}\right)\left(f_1-\eta\frac{df_1}{d\eta}\right)=0. \quad (15)$$

Concerning θ,

$$K_7\frac{d^2\theta_0}{d\eta^2}+K_8\frac{d\theta_0}{d\eta}=0, \quad (16)$$

$$K_7\frac{d^2\theta_1}{d\eta^2}+K_8\frac{d\theta_1}{d\eta}+K_9\left(\frac{df_0}{d\eta}\right)^2-K_{10}\eta\frac{d\theta_0}{d\eta}+K_{11}\frac{d\theta_0}{d\eta}\left[f_0-\eta\frac{df_0}{d\eta}\right]=0, \quad (17)$$

$$K_7\frac{d^2\theta_2}{d\eta^2}+K_8\frac{d\theta_2}{d\eta}+2K_9\frac{df_0}{d\eta}\frac{df_1}{d\eta}-K_{10}\eta\frac{d\theta_1}{d\eta}+K_{11}\frac{d\theta_1}{d\eta}\left[f_0-\eta\frac{df_0}{d\eta}\right]+K_{11}\frac{d\theta_0}{d\eta}\left[f_1-\eta\frac{df_1}{d\eta}\right]=0 \quad (18)$$

The boundary conditions (8) reducing to,

$$\begin{aligned}
&f_0(0)=1,\quad f_1(0)=f_1(0)=f_2(0)=f_3(0)=\cdots=0,\\
&\frac{df_0}{d\eta}\,|_{\eta=0}=1,\quad \frac{df_1}{d\eta}\,|_{\eta=0}=\frac{df_2}{d\eta}\,|_{\eta=0}=\frac{df_3}{d\eta}\,|_{\eta=0}=\cdots=0,\\
&\theta_0(0)=\frac{T_w-AU(x)}{T_w-T_\infty},\quad \theta_1(0)=\theta_2(0)=\theta_3(0)=\cdots=0,\\
&\frac{df}{d\eta}\ \text{ is bounded as } \eta\to\infty,\ \theta_1(\infty)=\theta_2(\infty)=\theta_3(\infty)=\cdots=0.
\end{aligned} \quad (19)$$

The solutions to (13)-(18) with boundary conditions (19) are, denote

$$K_{12}=\frac{-K_5+\sqrt{K_5^2-K_6K_4}}{2K_6},\quad K_{13}=\frac{-K_5-\sqrt{K_5^2-K_6K_4}}{2K_6}.$$

Now $M=\frac{K_4}{K_6}$ giving Joule's effect is positive, which means that K_4 and K_6 are of the same sign and $\sqrt{K_5^2-K_6K_4}<K_5$ gives $K_{12}<0$, so that both K_{12} and K_{13} are negative.

$$f_0=c_1+c_2\exp(K_{12}\eta)+c_3\exp(K_{13}\eta), \quad (20)$$

$$\begin{aligned}
f_1&=c_4+(c_6+L_{12})\exp(K_{13}\eta)+L_{11}\exp(K_{12}\eta)+L_{13}\exp(2K_{12}\eta)+L_{14}\exp(2K_{13}\eta)+L_{15}\exp((K_{12}+K_{13})\eta)\\
&+L_{16}\eta\exp(2K_{12}\eta)+L_{17}\eta\exp(2K_{13}\eta)+L_{18}\eta\exp((K_{12}+K_{13})\eta)+L_{19}\eta\exp(K_{12}\eta)+L_{20}\eta\exp(K_{13}\eta),
\end{aligned} \quad (21)$$

$$\begin{aligned}
f_2&=c_7+(c_8+R_{27})\exp(K_{12}\eta)+(c_9+R_{10}^*)\exp(K_{13}\eta)+R_{11}^*\eta\exp(K_{13}\eta)+R_{11}^*\eta^2\exp(K_{13}\eta)\\
&+R_{12}^*\exp(2K_{13}\eta)+R_{13}^*\eta\exp(2K_{13}\eta)+R_{14}^*\eta^2\exp(2K_{13}\eta)+R_{15}^*\exp(3K_{13}\eta)+R_{16}^*\eta\exp(3K_{13}\eta)\\
&+R_{16}^*\eta^2\exp(3K_{13}\eta)+R_{17}^*\exp((K_{12}+K_{13})\eta)+R_{18}^*\eta\exp((K_{12}+K_{13})\eta)+R_{19}^*\exp((K_{12}+2K_{13})\eta)\\
&+R_{20}^*\eta\exp((K_{12}+2K_{13})\eta)+R_{21}\eta^2\exp((K_{12}+2K_{13})\eta)+R_{22}\exp(3K_{12}\eta)+R_{23}\eta\exp(3K_{12}\eta)\\
&+R_{24}\exp((2K_{12}+K_{13})\eta)+R_{25}\eta\exp((2K_{12}+K_{13})\eta)+R_{26}\eta^2\exp((2K_{12}+K_{13})\eta)+R_{27}\exp(K_{12}\eta)\\
&+R_{28}\exp(2K_{12}\eta)+R_{29}\eta\exp(2K_{12}\eta)+R_{30}\eta\exp(K_{12}\eta)+R_{31}\eta^2\exp(3K_{13}\eta)+R_{32}\eta^2\exp(3K_{12}\eta)
\end{aligned} \quad (22)$$

and

$$\theta_0=d_2\exp\left(-\frac{K_8}{K_7}\eta\right), \quad (23)$$

$$\begin{aligned}
\theta_1&=d_4\exp\left(-\frac{K_8}{K_7}\eta\right)+L_{30}\exp(2K_{13}\eta)+L_{31}\eta^2\exp\left(-\frac{K_8}{K_7}\eta\right)+L_{32}\eta\exp\left(-\frac{K_8}{K_7}\eta\right)+L_{33}\exp\left(\left(K_{13}-\frac{K_8}{K_7}\right)\eta\right)\\
&+L_{34}\eta\exp\left(\left(K_{13}-\frac{K_8}{K_7}\right)\eta\right)+L_{35}\exp(2K_{12}\eta)+L_{36}\exp((K_{12}+K_{13})\eta)+L_{37}\eta\exp\left(\left(K_{12}-\frac{K_8}{K_7}\right)\eta\right),
\end{aligned} \quad (24)$$

$$
\begin{aligned}
\theta_2 &= d_5 + (d_6 + P_{31} + P_{30}\eta + P_{35}\eta^2 + P_{25}\eta^4)\exp\left(-\frac{K_8}{K_7}\eta\right) + (P_{20} + P_{29}\eta)p(\exp(2K_{13}\eta) \\
&+ (P_{21} + P_{28}\eta)p(\exp(3K_{13}\eta) + P_{22}\exp((2K_{13} + K_{12})\eta) + (P_{23} + P_{32}\eta + P_{33}\eta^2 + P_{39}\eta^3)\exp\left(\left(K_{13} - \frac{K_8}{K_7}\right)\eta\right) \\
&+ P_{24}\eta\exp((K_{12} + 2K_{13})\eta)\, P_{26} + P_{27} + (P_{36}\eta^2 + P_{37} + P_{113}\eta^3)\exp\left(\left(2K_{13} - \frac{K_8}{K_7}\right)\eta\right) + P_{61}\exp(2K_{12}\eta) \\
&+ P_{62}\exp((2K_{12} + K_{13})\eta) + P_{63}\exp\left(\left(2K_{12} - \frac{K_8}{K_7}\right)\eta\right) + \left(P_{64} + P_{101}\eta^2\right)\exp\left(\left(K_{13} - \frac{2K_8}{K_7}\right)\eta\right) \\
&+ \left(P_{65} + P_{100}\eta^2\right)\exp\left(\left(K_{12} - \frac{2K_8}{K_7}\right)\eta\right) + (P_{68} + P_{93}\eta + P_{102}\eta^2)\exp\left(\left(3K_{13} - \frac{K_8}{K_7}\right)\eta\right) \\
&+ (P_{69} + P_{103} + P_{92}\eta)\exp\left(\left(3K_{12} - \frac{K_8}{K_7}\right)\eta\right) + \left(P_{70} + P_{94}\eta + P_{104}\eta^2 + P_{116}\eta^3\right)\exp\left(\left(K_{12} + 2K_{13} - \frac{K_8}{K_7}\right)\eta\right) \\
&+ (P_{71} + P_{95}\eta + P_{105}\eta^2 + P_{115}\eta^3)\exp\left(\left(2K_{12} + K_{13} - \frac{K_8}{K_7}\right)\eta\right) + (P_{72} + P_{97}\eta)\exp\left(\left(3K_{12} - 2\frac{K_8}{K_7}\right)\eta\right) \\
&+ (P_{73} + P_{96}\eta + P_{106}\eta^2)\exp\left(\left(3K_{13} - 2\frac{K_8}{K_7}\right)\eta\right) \\
&+ (P_{66} + P_{74}\eta + P_{75}\eta^2 + P_{76}\eta^3 + P_{77}\eta^4 + P_{111}\eta^3)\exp\left(\left(2K_{13} - 2\frac{K_8}{K_7}\right)\eta\right) \\
&+ (P_{67} + P_{78}\eta + P_{79}\eta^2 + +P_{107}\eta^2 + P_{80}\eta^3 + P_{81}\eta^4 + P_{112}\eta^3)\exp\left(\left(2K_{12} - 2\frac{K_8}{K_7}\right)\eta\right) + (P_{82} + P_{83}\eta)\exp(3K_{12}\eta) \\
&+ P_{84}\exp((2K_{12} + K_{13})\eta) + (P_{85} + P_{98}\eta^2 + P_{114}\eta^3)\exp\left(\left(2K_{12} - \frac{K_8}{K_7}\right)\eta\right) + (P_{86} + P_{87}\eta)\exp((K_{12} + K_{13})\eta) \\
&+ (P_{88} + P_{89}\eta + P_{108}\eta^3)\exp\left(\left(K_{12} - \left(\frac{K_8}{K_7}\right)\right)\eta\right) + (P_{90}\eta + P_{109}\eta^3)\exp\left(\left(K_{12} - 2\frac{K_8}{K_7}\right)\eta\right) \\
&+ (P_{91}\eta + P_{110}\eta^3)\exp\left(\left(K_{13} - 2\frac{K_8}{K_7}\right)\eta\right). \qquad (25)
\end{aligned}
$$

Now the solutions to (6) with (7) satisfying boundary conditions (8) is given by

$$
f = \lim_{p\to 1} f_p = f_0 + f_1 + f_2 + f_3 + f_4 + f_5 + \ldots, \qquad (26)
$$

$$
\theta = \lim_{p\to 1} \theta_p = \theta_0 + \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \ldots, \qquad (27)
$$

So we can write the first and second approximations to f, θ respectively as,

$$
\begin{aligned}
f &= f_0 + f_1 = c_1 + c_4 + (c_6 + L_{12} + c_3)\exp(K_{13}\eta) + (L_{11} + c_2)\exp(K_{12}\eta) + L_{13}\exp(2K_{12}\eta) \\
&+ L_{14}\exp(2K_{13}\eta) + L_{15}\exp((K_{12} + K_{13})\eta) + L_{16}\eta\exp(2K_{12}\eta) + L_{17}\eta\exp(2K_{13}\eta) + L_{18}\eta\exp((K_{12} + K_{13})\eta) \\
&+ L_{19}\eta\exp(K_{12}\eta) + L_{20}\eta\exp(K_{13}\eta), \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
f &= f_0 + f_1 + f_2 = c_1 + c_4 + c_7 + (c_8 + R_{27} + L_{11} + c_2 + L_{19}\eta)\exp(K_{12}\eta) + (c_9 + R_{10} + c_6 + L_{12} + c_3)\exp(K_{13}\eta) \\
&+ R_{11}\eta\exp(K_{13}\eta) + \left(R^*_{11}\eta^2 + L_{20}\eta\right)\exp(K_{13}\eta) + (R_{12} + L_{14})\exp(2K_{13}\eta) + (R_{13} + L_{17})\eta\exp(2K_{13}\eta) \\
&+ R_{14}\eta^2\exp(2K_{13}\eta) + R_{15}\exp(3K_{13}\eta) + R_{16}\eta\exp(3K_{13}\eta) + R^*_{16}\eta^2\exp(3K_{13}\eta) + (R_{17} + L_{15})\exp((K_{12} + K_{13})\eta) \\
&+ (R_{18}\eta + L_{18}\eta)\exp((K_{12} + K_{13})\eta) + R_{19}\exp((K_{12} + 2K_{13})\eta) + R_{20}\eta\exp((K_{12} + 2K_{13})\eta) \\
&+ R_{21}\eta^2\exp((K_{12} + 2K_{13})\eta) + R_{22}\exp(3K_{12}\eta) + R_{23}\eta\exp(3K_{12}\eta) + R_{24}\exp((2K_{12} + K_{13})\eta) \\
&+ R_{25}\eta\exp((2K_{12} + K_{13})\eta) + R_{26}\eta^2\exp((2K_{12} + K_{13})\eta) + R_{27}\exp(K_{12}\eta) + (R_{28} + L_{13})\exp(2K_{12}\eta) \\
&+ (R_{29}\eta + L_{16}\eta)\exp(2K_{12}\eta) + R_{30}\eta\exp(K_{12}\eta) + R_{31}\eta^2\exp(3K_{13}\eta) + R_{32}\eta^2\exp(3K_{12}\eta), \qquad (29)
\end{aligned}
$$

$$
\begin{aligned}
\theta &= \theta_0 + \theta_1 = (d_4 + d_2)\exp\left(-\frac{K_8}{K_7}\eta\right) + L_{30}\exp(2K_{13}\eta) + L_{31}\eta^2\exp\left(-\frac{K_8}{K_7}\eta\right) \\
&+ L_{32}\eta\exp\left(-\frac{K_8}{K_7}\eta\right) + L_{33}\exp\left(\left(K_{13} - \frac{K_8}{K_7}\right)\eta\right) + L_{34}\eta\exp\left(\left(K_{13} - \frac{K_8}{K_7}\right)\eta\right) + L_{35}\exp(2K_{12}\eta) \\
&+ L_{36}\exp((K_{12} + K_{13})\eta) + L_{37}\eta\exp\left(\left(K_{12} - \frac{K_8}{K_7}\right)\eta\right), \qquad (30)
\end{aligned}
$$

$$\theta = \theta_0 + \theta_1 + \theta_2 = d_5 + (d_4 + d_2 + d_6 + P_{31} + P_{30}\eta + L_{32}\eta + P_{35}\eta^2 + L_{31}\eta^2 + P_{25}\eta^4) \exp\left(-\frac{K_8}{K_7}\eta\right)$$
$$+ (P_{20} + L_{30} + P_{29}\eta)p(\exp(2K_{13}\eta) + (P_{21} + P_{28}\eta)p(\exp(3K_{13}\eta) + P_{22}\exp((2K_{13} + K_{12})\eta)$$
$$+ (P_{23} + L_{33} + (P_{32} + L_{34})\eta + P_{33}\eta^2 + P_{39}\eta^3)\exp\left(\left(K_{13} - \frac{K_8}{K_7}\right)\eta\right) + P_{24}\eta\exp((K_{12} + 2K_{13})\eta)$$
$$+ P_{26} + P_{27} + (P_{36}\eta^2 + P_{37} + P_{113}\eta^3)\exp\left(\left(2K_{13} - \frac{K_8}{K_7}\right)\eta\right) + (P_{61} + L_{35})\exp(2K_{12}\eta) + P_{62}\exp((2K_{12} + K_{13})\eta)$$
$$+ P_{63}\exp\left(\left(2K_{12} - \frac{K_8}{K_7}\right)\eta\right) + \left(P_{64} + P_{101}\eta^2\right)\exp\left(\left(K_{13} - \frac{2K_8}{K_7}\right)\eta\right) + \left(P_{65} + P_{100}\eta^2\right)\exp\left(\left(K_{12} - \frac{2K_8}{K_7}\right)\eta\right)$$
$$+ (P_{68} + P_{93}\eta + P_{102}\eta^2)\exp\left(\left(3K_{13} - \frac{K_8}{K_7}\right)\eta\right) + (P_{69} + P_{103} + P_{92}\eta)\exp\left(\left(3K_{12} - \frac{K_8}{K_7}\right)\eta\right)$$
$$+ \left(P_{70} + P_{94}\eta + P_{104}\eta^2 + P_{116}\eta^3\right)\exp\left(\left(K_{12} + 2K_{13} - \frac{K_8}{K_7}\right)\eta\right) + (P_{71} + P_{95}\eta + P_{105}\eta^2 + P_{115}\eta^3)$$
$$\exp\left(\left(2K_{12} + K_{13} - \frac{K_8}{K_7}\right)\eta\right) + (P_{72} + P_{97}\eta)\exp\left(\left(3K_{12} - 2\frac{K_8}{K_7}\right)\eta\right) + (P_{73} + P_{96}\eta + P_{106}\eta^2)\exp\left(\left(3K_{13} - 2\frac{K_8}{K_7}\right)\eta\right)$$
$$+ (P_{66} + P_{74}\eta + P_{75}\eta^2 + P_{76}\eta^3 + P_{77}\eta^4 + P_{111}\eta^3)\exp\left(\left(2K_{13} - 2\frac{K_8}{K_7}\right)\eta\right)$$
$$+ (P_{67} + P_{78}\eta + P_{79}\eta^2 + P_{107}\eta^2 + P_{80}\eta^3 + P_{81}\eta^4 + P_{112}\eta^3)\exp\left(\left(2K_{12} - 2\frac{K_8}{K_7}\right)\eta\right)$$
$$+ (P_{82} + P_{83}\eta)\exp(3K_{12}\eta) + P_{84}\exp((2K_{12} + K_{13})\eta) + (P_{85} + P_{98}\eta^2 + P_{114}\eta^3)\exp\left(\left(2K_{12} - \frac{K_8}{K_7}\right)\eta\right)$$
$$+ (P_{86} + P_{87}\eta + L_{36})\exp((K_{12} + K_{13})\eta) + (P_{88} + (P_{89} + L_{37})\eta + P_{108}\eta^3)\exp\left(\left(K_{12} - \left(\frac{K_8}{K_7}\right)\right)\eta\right)$$
$$+ (P_{90}\eta + P_{109}\eta^3)\exp\left(\left(K_{12} - 2\frac{K_8}{K_7}\right)\eta\right) + (P_{91}\eta + P_{110}\eta^3)\exp\left(\left(K_{13} - 2\frac{K_8}{K_7}\right)\eta\right), \quad (31)$$

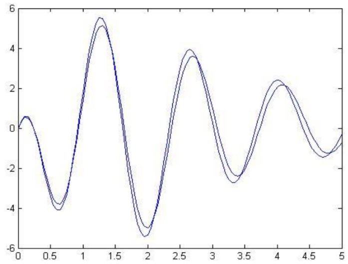
where the evaluated constants P_i, R_i, d_i, L_i as they occupy immense space are not mentioned in this paper. As the series is convergent, we ignore terms f_3, θ_3, Φ_3 onwards as their effects are negligible.

4. Results and Discussion

The system of partial differential equations with the boundary conditions, are converted to the system of ordinary differential equations (6) using similarity transformations. These ODE are solved using the homotopy technique. First we calculate f^p, θ^p and then taking $p \to 1$ we get the solution to f, θ. The 1^{st} approximations to f and θ are (27), (29) respectively and the 2^{nd} approximations to f and θ are (28), (30) respectively. We analyse the profiles of velocity, temperature through graphs, for impacts of the Joule's effect and the effect of the magnetic field on them. We use $Pr = 0.71$, $Sc = 0.01$, $Ec = 0.01$, $Kr = 1$, $A = 1$, $a_1 = 0.5$, $n = 1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$. We analyse the profile showing change in velocity with η, with M taking values ranging 5 to 6 and for the effect of J between values 1 to 8. When the effect of magnetic field is least, as seen in figures 1 and 4; velocity of the fluid is 0 at $\eta = 0$ and it initially increases steeply for a short span of η and then begins to reduce steeply until some particular point where it again begins to increase steeply and this phenomenon continues. As we increase the effect of magnetic field, as we can see in figures 2, 3, 5 and 6; there is no change in this phenomenon and velocity continues to follow this consistent pattern. Whereas for temperature, at the minimum effect of magnetic field and when the joule's effects are absent, seen in figure 7 and 10, the temperature tends to reduce steeply as η increases until it reaches a particular stage for some value of η after which it slowly increases until some point and then finally begins to reduce to 0, as the value of η increases further. At increasing values of the effect of magnetic field and Joule's effect, seen in figures 8, 9, 11 and 12; the temperature initially increases steeply as η increases until some point after which it reduces as η increases. Then it slowly tends to remain constant with temperature 0 after some particular value of η. As the values of the effect of magnetic field increase further to 7 and 8, these fluctuations in temperature continue but tend to be more steep. Eventually after some particular η the temperature reduces till it reaches 0 and continues to remain constant at 0, as the value of η increases further. Here $M = \frac{K_4}{K_6}$ gives the Joule's effect and $J = \frac{K_8}{K_6}$ gives the effect of the magnetic field.

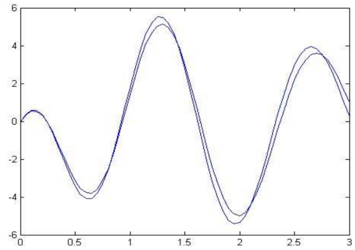
Iterations for f:

1^{st} Iteration: M, take values ranging 5-6



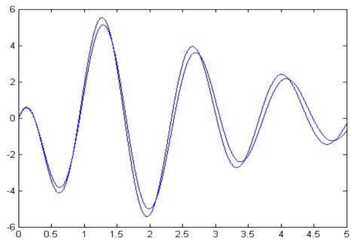
η

Figure 1: J takes values ranging 1 - 2



η

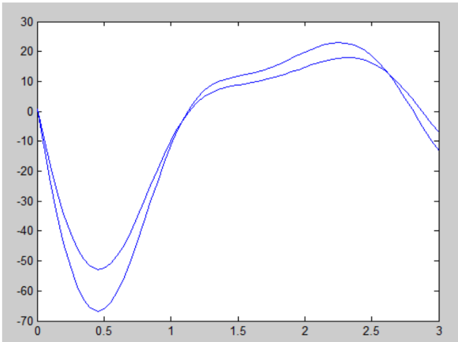
Figure 2: J takes values ranging 5 - 6



η

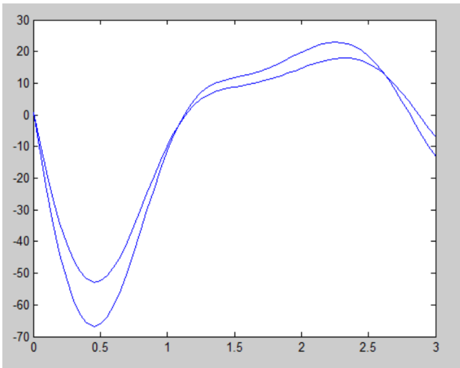
Figure 3: J takes values ranging 7 - 8

2^{nd} iteration for f: M, take values ranging 5 - 6



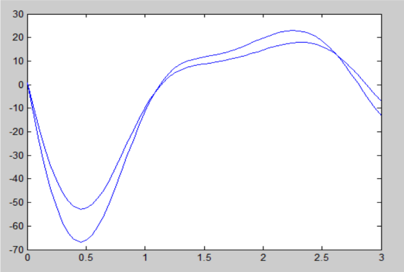
η

Figure 4: J takes values ranging 1 - 2



η

Figure 5: J takes values ranging 5 - 6

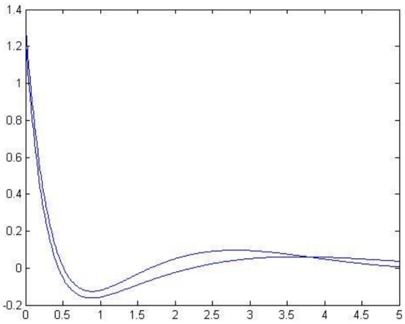


η

Figure 6: J takes values ranging 7 - 8

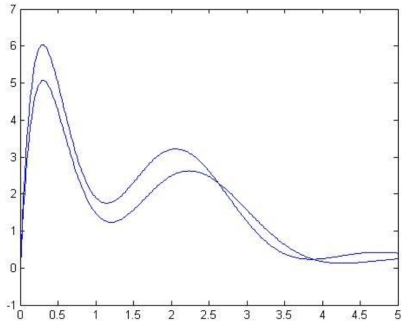
Iterations for θ:

1^{st} iteration for θ:



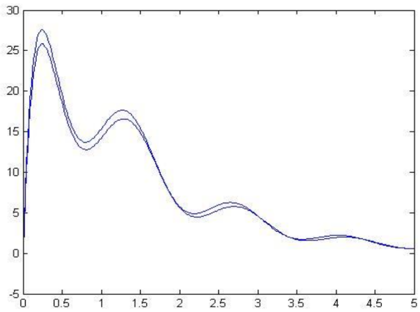
η

Figure 7: M take values ranging 0 - 0.5; J takes values ranging 1 - 2



η

Figure 8: M take values ranging 1 - 2; J takes values ranging 5 - 6



η

Figure 9: M take values ranging 5 - 6; J takes values ranging 7 - 8

2^{nd} iteration for θ:

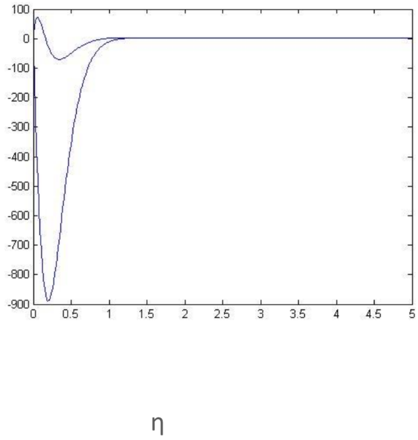


Figure 10: M take values ranging 0 - 1; J takes values ranging 1 - 2

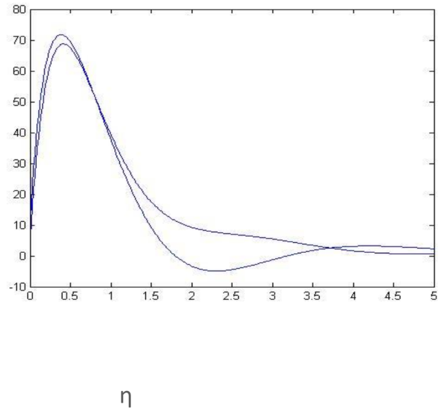


Figure 11: M take values ranging 1 - 2; J takes values ranging 5 - 6

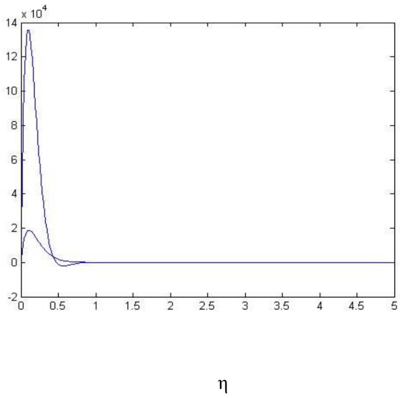


Figure 12: M take values ranging 5 - 6; J takes values ranging 7 - 8

Swati Mukhopadhyay in [2] studied the velocity of a steady axially-symmetric flow of an incompressible viscous fluid along a stretching cylinder in presence of uniform magnetic field and in the absence of Joule's effect and obtained the following Table 1 using D denoting the magnetic parameter. Whereas our Table 2 describing the Joule's effect and the effects of magnetic field on the velocity of a steady incompressible flow of a viscous fluid over a hyperbolic stretching circular cylinder helps validate our analytical result.

D	Analytical solution	Numerical solution
0	-1.0000000	-0.99005806
0.5	-1.1180340	-1.1056039
1	-1.4142135	-1.3943545
1.5	-1.802775638	-1.7705669

Table 1: Attained by [2] values of $f^{''}(0)$ obtained from analytical and numerical solutions

M - Joule's effect	J – Magnetic field	Solution
0	1	-0.010974
1	5	0.015217
5	5	0.94973
5	7	0.83461

Table 2: $f^{''}(0)$ obtained by us using the generalized homotopy method

5. Conclusion

This paper presents the boundary layer flow and heat transfer of an incompressible viscous fluid over a hyperbolic stretching cylinder in the presence of the effects of magnetic field and Joule's effect. The velocity fluctuations are consistent with increasing effects of magnetic field whereas temperature fluctuations are faster than velocity with increasing effects of magnetic field and Joule's effects. The major discoveries are:

(1). $U(x)$, T_w, T_∞, a have no effect on temperature.

(2). The behaviour of temperature when Joule's effect is absent is opposite to that when these effects are prominent

Acknowledgement

The authors are thankful to the Research centre, M.S. Ramaiah Institute of Technology for encouraging our research.

References

[1] Abid Majeed, Tariq Javed and Irfan Mustafa, *Heat transfer analysis of boundary layer flow over hyperbolic stretching cylinder*, Alexandria engineering Journal, 55(2016), 1333-1339.

[2] Swati Mukhopadhyay, *MHD boundary layer slip flow along a stretching cylinder*, Ain Shams Eng. Journal, 4(2013), 317-324.

[3] Swati Mukhopadhyay, *Analysis of Boundary Layer Flow and Heat Transfer along a Stretching Cylinder in a Porous Medium*, ISRN Thermodynamics, 2012(2012).

[4] M. M. Rashidia, S. Bagheri, E. Momoniat and N. Freidoonimehr, *Entropy analysis of convective MHD flow of third grade non-Newtonian fluid over a stretching sheet*, Ain Shams Engineering Journal, 8(1)(2017), 77-85.

[5] C. Sulochana and N. Sandeep, *Dual solutions for radiative MHD forced convective flow of a nanofluid over a slendering stretching sheet in porous medium*, Journal of Naval Architecture and Marine Engineering, 12(2)(2015).

[6] J. V. Ramana Reddy, V. Sugunamma and N. Sandeep, *Thermophoresis and Brownian motion effects on unsteady MHD nanofluid flow over a slendering stretching surface with slip effects*, Alexandria Engineering Journal, (2017).