Int. J. Math. And Appl., 9(4)(2021), 101–117
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

# Agricultural Inputs Allocation Model of Ethiopian Agricultural System

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**Abstract:** Farmers and investors like to use some important models for agricultural planning and decision making process. Researchers have developed linear programming model for agricultural farm land allocation. In this research, we construct cropping and tillage arrangement models. Cropping arrangement model is constructed from existing cropping arrangements in Ethiopia. We construct Tillage arrangement model to determine tillage day and aggregate farm land using the number of oxen and workers. We also construct agricultural inputs allocation models to solve agricultural inputs allocation problem. Finally, we derive a technique to determine revenue for farm land, inputs and labor contribution using share cropping and labor sharing arrangements.

**MSC:** 97M30, 97M40, 97M10.

**Keywords:** Constraints, Mekenajo, Minda, Model, Tillage.



## 1. Introduction

Throughout most of the Ethiopian highlands the land is tilled using a pair of oxen of one of the indigenous Zebu breeds which pull the locally-made traditional cultivation tool, the maresha, [1]. Tillage work requires three agents, namely, a pair of oxen and one worker. In Ethiopia, oxen are not distributed evenly among households, [1]. Thus, farmers use different tillage arrangements and strategies to deal with oxen constraint. Available data for the Debre Berhan area indicate that half of the farmers owned two or more oxen, around 30% had one and 20% owned no oxen, [1]. In the Debre Zeit area relatively more farmers had two or more oxen, but around 25% of the smallholder had none or only one ox, [1]. In Ethiopia farmers use different farming strategies to produce crops. Since modern agricultural tools are expensive for Ethiopian farmers, they prefer to use traditional farming practice. They use draught oxen to plough their crops production farm land. However, some farmers are not able to buy enough number of oxen required for their farm work because of their financial constraint. They use different farming strategies to solve this limitation. The most common strategy is grouping one farmer's ox to another farmer's ox to form a pair of oxen. This is because farmers need a pair of oxen to plough farm. Most of farmers are not familiar with one ox ploughing practice in Ethiopia. A farmer who uses grouping strategy plough his farm one day by pairing his ox to his friend's ox. This farmer will recieve oxen labor from his *mekanajo* friend next day. For farmers with

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one ox, the usual arrangement is a *mekanajo* agreement with another farmer also having one ox, whereby the two oxen are used on the partners' fields on alternate days, [2]. Another common farmers farming practice is getting a pair of oxen from oxen holder for one day to plough farm land in exchange with two day human labor for oxen holder. If a farmer has no oxen, the common exchange system is to give 2 days of human labour for every day a pair of oxen is borrowed, [1, 3, 4]. Researchers have investigated average maximum cropped area using corresponding number of oxen using these two farming strategies in Debre Zeit, Ethiopia. The result shows that a farmer with 0, 1, 2 and 3 can plough maximum average farm area of 1.2 ha, 1.9 ha, 2.7 ha and 3.6 ha, respectively. Economically active pair of oxen can plough $\frac{1}{4}$ ha on average per day in Ethiopia, [4–7].

There are many crop sharing arrangements for share cropping practice in Ethiopia. Common crop sharing options are

1. $\frac{1}{4}$ to $\frac{3}{4}$ crop sharing arrangement. If a lessor landholder provides land only and the tenant provides the other variable inputs (i.e., labor, oxen and seeds), the lessor gets only one-fourth of the produce, [8].

2. $\frac{1}{2}$ to $\frac{1}{2}$ crop sharing arrangement. When the lessor landholder provides his land and labor, and the tenant brings oxen and labor, the landholder gets a 50 percent share, [8].

3. $\frac{1}{3}$ to $\frac{2}{3}$ crop sharing arrangement. If the landholder provides land, labor, and one ox, he gets two-thirds of the crop, [8].

Research has been done on output shares in sharecropping contracts in northern Ethiopia. The result shows that 52.8% of the contracts had 0.5 share to the landlord,17.7% had 0.33 share, 10.3% had 0.25 share and 19% were fixed rent contracts, [8].

## 1.1. Statement of the problem

Farming practice for crops production is traditional in Ethiopia. Modern technology and modern agricultural tools are not affordable by farmers in Ethiopia. As a result, farmers follow their traditional farming experience to produce crops. Farmers' challenge is agricultural inputs allocation for producing crops. They use their expectation and guess to allocate agricultural inputs for producing crops. Their expectation or guess of agricultural inputs allocation may not be efficient. Thus, it is expected that efficient allocation of agricultural inputs allocation improves productivity. Once farmers allocate crop production inputs, they want to know revenue for labor, inputs and farm land contribution.

The statement of this research problem is determining agricultural revenue for labor, inputs and farm land contribution based on share cropping and labor sharing arrangements. This research work is designed to answer the following fundamental questions for farmers in Ethiopia.

- What is expected number of oxen $N_O(L)$ required for producing crops in average area $L(N_O)$ hectares?
- What is expected area $L(N_O)$ required for number of oxen $N_O(L)$?
- How do land owner and tenant share revenue based on share cropping and labor sharing arrangements?

## 1.2. Objectives of the study

The general objective of this study is to determine agricultural revenue for labor, inputs and farm land contribution based on share cropping and labor sharing arrangements. Specific objectives are to

1. Construct agricultural inputs allocation models.

2. Define tillage day function and reserve labor function.

3. Build tillage arrangement model and cropping arrangement model.

### 1.3. Research gap

Distribution of revenue for land owner and tenant using Ethiopian farmers share cropping and labor sharing arrangements is not studied. In this research, we build a technique for determining agricultural revenue share for land owner and tenant using Ethiopian farmers share cropping and labor sharing arrangements.
It is clear that seasonal constraints affect agricultural inputs allocation. Thus, we consider seasonal constraints to construct agricultural inputs allocation model. We know that risk and reward are positively correlated. For instance, maximizing reward leads to maximizing risk. Thus, investors prefer to maximize Sharpe ratio for examining the relative return per unit risk. Moreover, we would like to construct Sharpe ratio maximization model subject to aggregate farm land constraint, seasonal constraint and unit weight sum constraint.

## 2. Literature review

Tillage frequencies and the interval between the ploughings varied depending on the soil type, roughness of the field and the crop type which the farmers propose to plant during the main cropping season, [9]. Thus, farmers decide tillage frequencies and the interval between the ploughings based on soil type, roughness of the field and the crop type.

### 2.1. Agricultural investment linear programming

Mellaku [10] applied Linear programming to study the impact of cropland allocation decisions on the performance of rural smallholder crop production systems. Some models in agriculture and industry rely on linear programming, [10].

### 2.2. Existing agricultural investment model

Given $n$ crop choices with productivity per unit of land $q_i,\ i=1,2,3,\dots,n$ and total land size $L$, the linear programming LP problem was to estimate the area of land $L_i,\ i=1,2,3,\dots,n$ that maximizes profit subject to a given set of food crop production, ecological,and financial constraints, [10]. Given the market price $P_i,\ i=1,2,3,\dots,n$ per kilogram of each crop and the cost of production per hectare of land for each crop $C_i,\ i=1,2,3,\dots,n$, the profit maximization objective function $Z$ is modeled as, [10]:

$$Z=\sum_{i=1}^{n}(P_iL_iq_i-C_iL_i) \tag{1}$$

Let $Y$, $L$ and $Q_i$ be aggregated crop budget, aggregated land and aggregated food crop production requirement of crop $i$, respectively. Three constraints are associated with profit maximization objective function in (1). These constraints are aggregated crop budget (financial) constraint $\sum_{i=1}^{n} C_iq_i\le Y$, aggregated land (ecological) constraint $\sum_{i=1}^{n} L_i\le L$ and aggregated food crop production requirement of crop $i$ food (production) constraint $L_iq_i\ge Q_i$, [10].
Studies indicate that financial access is one of the major constraining factors limiting the performance of smallholder agriculture in low-income countries, [10]. Mellaku [10] compared maximizing profit through a LP method of cropland allocation based on two assumptions. These assumptions are assuming farmers know the theoretical potential yield per hectare for each crop and current average yield per hectare is the same as estimated yield using survey for each crops.

$$E:\quad \max_{L_1,L_2,\dots,L_n}\sum_{i=1}^{n}(P_iL_iq_i-C_iL_i)$$

$$
\begin{aligned}
\text{Subject to}: \quad & \sum_{i=1}^{n} C_i q_i \leq Y, \\
& \sum_{i=1}^{n} L_i = L, \\
& L_i q_i \geq Q_i \ \forall i = 1, 2, \ldots, n, \\
& L_i \geq 0 \ \forall i = 1, 2, \ldots, n.
\end{aligned} \tag{2}
$$

This model was developed by Melaku in 2018, [10]. The model is so important because it maximizes profit based on given assumptions and constraints.

## 2.3. Cropping seasons

In the main agricultural regions in Ethiopia there are two rainy seasons, the *Meher* and the *Belg*, and consequently there are two crop seasons.*Meher* is the main crop season. It encompasses crops harvested between Meskerem (September) and Yeaktit (February). Crops harvested between Megabit (March) and Nehase (August) are considered part of the *Belg* season crop, [9]. In most of the cases the land preparation with repeated ploughing for meher season took place between the months of March and June, which include the dry and short rainy, [9].

## 2.4. Length of tillage period

Length of tillage period is the length of period used for ploughing starting from the begging of ploughing day to the end of ploughing day of a given season. In Ethiopia, length of tillage period is four months(March, April, May and June) for meher season, [9].

## 2.5. Tillage gap

Tillage gap is an interval between ploughings. Farmers would plough the entire farm land once within tillage gap and start second time ploughing after the first tillage.

## 2.6. Tillage frequency

Tillage frequency is the number of ploughing times of a farm within a given length of tillage period. How often a farmer plough farm within a given length of tillage period?

## 2.7. Crop rotation constraint

Crop rotation is an act of changing growing crop type in the plot for specific season, [11]. It improves the soil structure and fertility and controls weeds, pests and diseases, [11]. Farmers know importance crop rotation in advance. They would like to apply crop rotation. Crop rotation is planting different crops in two successive years. If a farmer grows crop $i$ on farmland area $L_i$ this year, then he/she will not grow crop $i$ on same farmland area $L_i$ next year provided crop rotation is for one year.

## 2.8. Markowitz' mean variance model

H.M. Markowitz' was the first who introduced Markowitz' mean variance model for portfolio selection, [12, 13]. The model is developed based on some assumptions.

# 3. Methodology

## 3.1. Data

Secondary data is used to derive the relationship between maximum number of oxen and average farm land area cropped by farmers in Debre Zait, Ethiopia. Research has been carried on determination of maximum number of oxen given that average farm land area cropped by farmers in Debre Zait, Ethiopia. The result of the research is given in the following table.

| Number of oxen | Average area cropped |
|---|---|
| 0 | 1.2 |
| 1 | 1.9 |
| 2 | 2.7 |
| 3 | 3.6 |

**Table 1.** **Maximum average area harvested (ha) by number of oxen owned by farmers, [1]**

## 3.2. Method

Quadratic sequence expansion is applied to estimate parameters of maximum average farm land area. As shown in Table 1, average farm land area cropped has quadratic sequence numbers pattern. Thus, parameters of maximum average farm land area can be obtained using data of Table 1.

Cropping arrangement model ($CAM$) consists of linear equations. We use linear algebra substitution method to solve system of linear equation of ($CAM$).

We determine the number oxen and workers required for a given level of farm land by solving tillage arrangement model ($TAM$). Tillage arrangement model ($TAM$)is built based on labor sharing arrangement in Ethiopia.

## 3.3. Cropping arrangement model

Consider the following notations.

1. $L_{ic}$: One hectare farm land cost of crop $i$.
2. $HL_{ic}$: Cost of human labor per hectare of crop $i$.
3. $OL_{ic}$: Cost of oxen labor per hectare of crop $i$.
4. $I_{ic}$: Cost of inputs per hectare of crop $i$.
5. $LH_{ic}$: Land lord Cost of human labor per hectare of crop $i$.
6. $TH_{ic}$:Tenant Cost of human labor per hectare of crop $i$.
7. $Rev_{ic}$:Revenue of crop $i$ per hectare.

The following model, cropping arrangement model ($CAM$), is constructed from the above cropping arrangements.

$$CAM: \quad L_{ic} = \frac{Rev_{ic}}{4} \tag{3}$$

$$OL_{ic} + I_{ic} + HL_{ic} = \frac{3Rev_{ic}}{4} \tag{4}$$

$$L_{ic} + LH_{ic} = \frac{Rev_{ic}}{2} \tag{5}$$

$$OL_{ic} + I_{ic} + TH_{ic} = \frac{Rev_{ic}}{2} \tag{6}$$

$$TH_{ic} + LH_{ic} = HL_{ic} \tag{7}$$

$$L_{ic} + LH_{ic} + \frac{OL_{ic}}{2} = \frac{2Rev_{ic}}{3} \tag{8}$$

$$TH_{ic} + I_{ic} + \frac{OL_{ic}}{2} = \frac{Rev_{ic}}{3} \tag{9}$$

Yield and crop price online secondary data is available in Ethiopia. Thus, agricultural inputs costs for crops production will be estimated by using revenue and discounting rate. Therefore, crops' unit revenue and cost is known.

## 3.4. Tillage day function

Tillage day of $x$ is maximum number of days that farmers used a pair of oxen to plow their farm starting from day one to day $x$. Unused farmer labor up to day $x$ that a farmer will use in future is defined as reserved labor. $Td(n, m, x)$ stands for tillage day of $x$ of order $(n, m)$, where $n$ and $m$ are non negative integers representing the number of oxen and number of personnel, respectively . That is, $Td(n, m, x)$ represents maximum number of a pair of oxen days that a farmer who owns $n$ oxen can plow by using his $m$ workers starting from day one to day $x$. Clearly, $Td(0, 0, x) = 0$. $rs(n, m, x)$ is reserved labor of a farmer who owns $n$ oxen and $m$ workers starting from day one to day $x$.

**Definition 3.1.** *Tillage day function $Td : \mathbb{N}_0^3 \to \mathbb{N}_0$ is defined by $Td(n, m, x) = k$, where $x$ is day variable, $(n, m)$ is oxen to worker pair and $\mathbb{N}_0$ is the set of non negative integers.*

## 3.5. Reserve labor function

In Ethiopia, farmers share labor using minda and mekenajo labor sharing arrangement. A farmer shares his labor or his oxen labor today to receive from labor match farmer tomorrow. This labor which is expected to be received tomorrow from match farmer is called reserve labor. A farmer can't use his reserve labor today rather he can use it tomorrow or days after tomorrow. Suppose there are $m$ number of workers who are seeking for oxen labor share. These $m$ worker should work for oxen owners today in order to receive $m$ oxen labor tomorrow. Thus,$m$ workers who don't own oxen have $m$ oxen reserve labor today. Mathematically, reserve labor function of $m$ workers who don't own oxen up to day $x$ is defined by $rs(0, m, x)$. On day one all $m$ workers work for oxen owners. Thus, $rs(0, m, 1) = m$. These worker receive $m$ oxen labor on day two. Thus, $\lfloor \frac{m}{2} \rfloor$ plow their farm using $m$ oxen labor on day two and the rest workers $m - \lfloor \frac{m}{2} \rfloor$ work for oxen owners on day two. Thus, $rs(0, m, 2) = m - 2\lfloor \frac{m}{2} \rfloor + m - \lfloor \frac{m}{2} \rfloor = 2m - 3\lfloor \frac{m}{2} \rfloor$. One can derive reserve labor function of $m$ workers who don't own oxen up to day $x$ as given below. Reserve labor function $rs(0, m, x)$ has the following recursive formula.

$$\begin{aligned} rs(0, m, x) &= m + rs(0, m, x-1) \\ -3\left\lfloor \frac{rs(0, m, x-1)}{2} \right\rfloor &= mx - 3Td(0, m, x), \end{aligned}$$

where $rs(0, m, 1) = m$.

**Lemma 3.2.**

$$Td(1, 1, x) + Td(1, 0, x) = x. \tag{10}$$

*Proof.* Since $(1, 1)$ and $(1, 0)$ are disjoint pairs, $(1, 1) + (1, 0) = (2, 1)$. Clearly

$$Td(1, 1, x) + Td(1, 0, x) = Td(2, 1, x)$$

Since $(2, 1)$ is labor neutral pair, $Td(2, 1, x) = x$. Hence the proof followed. □

**Theorem 3.3** (Tillage day function of order $(1,1,x)$).

$$Td(1,1,x) = \begin{cases} x - 1 - \lfloor \frac{x}{3} \rfloor & \textit{if } \lfloor \frac{x}{3} \rfloor \textit{ is odd or } \lfloor \frac{x}{3} \rfloor = 0 \\ x - \lceil \frac{x}{3} \rceil & \textit{else} \end{cases} \tag{11}$$

*and*

$$rs(1,1,x) = \begin{cases} 0 & \textit{if } x\%6 = 0 \ \textit{ or } x\%6 = 5 \\ 1 & \textit{if } x\%6 = 1 \ \textit{ or } x\%6 = 2 \\ 2 & \textit{if } x\%6 = 3 \ \textit{ or } x\%6 = 4 \end{cases} \tag{12}$$

General tillage day function is defined by:

$$Td(n,m,x) = \begin{cases} \frac{n}{2}x + Td(0, m - \frac{n}{2}, x) & \text{if } n \text{ is even and } m > \frac{n}{2} \\ \lfloor \frac{n}{2} \rfloor x + Td(0, m - \lfloor \frac{n}{2} \rfloor - 1, x) + Td(1,1,x) & \text{if } n \text{ is odd and } m > \lfloor \frac{n}{2} \rfloor \\ \lfloor \frac{n}{2} \rfloor x + Td(1,0,x) & \text{if } n \text{ is odd and } m = \lfloor \frac{n}{2} \rfloor \\ \frac{n}{2}x & \text{if } n \text{ is even and } m = \frac{n}{2} \\ mx + Td(n-2m, 0, x) & \text{if } m < \frac{n}{2} \end{cases} \tag{13}$$

Tillage day function with zero number of workers is given by:

1. If $n$ is even, then

$$Td(n,0,x) = \begin{cases} \frac{n}{2}x & \text{if } x\%3 \neq 0 \\ \frac{n}{2}(x-1) & \text{else} \end{cases}$$

2. If $n$ is odd, then

$$Td(n,0,x) = \begin{cases} \frac{n-1}{2}x + Td(1,0,x) & \text{if } x\%3 \neq 0 \\ \frac{n-1}{2}(x-1) + Td(1,0,x) & \text{else} \end{cases} \tag{14}$$

**Lemma 3.4.**

$$Td(1,0,x) = \begin{cases} 0 & \textit{if } x = 1 \\ \frac{x+2}{3} & \textit{if } (x+2)\%3 = 0 \\ \frac{x+1}{3} & \textit{if } (x+1)\%3 = 0 \\ \frac{x}{3} & \textit{else} \end{cases} \tag{15}$$

**Lemma 3.5.**

$$rs(n,0,x) = n - 3Td(n,0,x). \tag{16}$$

**Theorem 3.6.**

$$rs(n,m,x) = \begin{cases} rs(0, m - \frac{n}{2}, x) & \textit{if } n \textit{ is even and } m > \frac{n}{2} \\ rs(0, m - \lfloor \frac{n}{2} \rfloor - 1, x) + rs(1,1,x) & \textit{if } n \textit{ is odd and } m > \lfloor \frac{n}{2} \rfloor \\ rs(1,0,x) & \textit{if } n \textit{ is odd and } m = \lfloor \frac{n}{2} \rfloor \\ 0 & \textit{if } n \textit{ is even and } m = \frac{n}{2} \\ rs(n-2m, 0, x) & \textit{if } m < \frac{n}{2} \end{cases} \tag{17}$$

## 3.6. Estimation of total crop production cost

We study crops production arrangements and strategies to estimate total production for a given level of farm land $L_t$ hectare. There are four direct crops' production cost in Ethiopia. These are labor cost, harvesting and threshing cost, farm land cost and inputs cost. Labor cost consists of oxen and workers labor cost. Inputs costs are seed cost, weeding(pesticide) cost and fertilizer cost. In this section, total crops' production cost estimation will be explained using share cropping and labor sharing arrangements in Ethiopia.

## 3.7. Harvesting and threshing cost

Threshing and harvesting ability of economically active workers and oxen depends on crop type. Some crops are threshing and harvesting labor intensive. Let $har_{im}$ and $thr_{inm}$ be harvesting and threshing ability in $ha$ of one worker per day for crop $i$, respectively. Then $\frac{1}{har_{im}}(\frac{1}{thr_{inm}})$ is the number of days required for harvesting(threshing) one hectare which is assigned for crop $i$ using one worker. Suppose that $w$ is wage of one worker. Define $har_{av} = \frac{\sum_{i=1}^{k} har_{im}}{k}$ and $thr_{av} = \frac{\sum_{i=1}^{k} thr_{inm}}{k}$. Let $har_p$ and $thr_p$ be length of harvesting and threshing period respectively. Note that the number of harvesting workers $m$ lies between $\frac{L}{har_{av}}$ and $har_p\frac{L}{har_{av}}$. That is, $\frac{L}{har_{av}} \geq m \geq har_p\frac{L}{har_{av}}$. Harvesting cost doesn't depend on the number of harvesting workers. That is, harvesting cost $HaC$ of farm land $L$ is given by $HaC = w\frac{L}{har_{av}}$, where $w$ daily wage per worker. Note that if $m$ workers are assigned to harvest $L$ hectare, then it takes $\frac{L}{mhar_{av}}$ days to finish harvesting task. Similarly, threshing cost doesn't depend on the number of harvesting workers and oxen. That is, threshing cost $ThC$ of farm land $L$ is given by $ThC = w\frac{L}{thr_{av}}$, where $w$ daily wage per worker. Note that if $m$ workers and $n$ oxen are assigned to thresh $L$ hectare, then it takes $\frac{L}{(m+n)thr_{av}}$ days to finish threshing task. Moreover, $\frac{L}{thr_{av}} \geq m+n \geq thr_p\frac{L}{thr_{av}}$. Therefore, harvesting cost is $HaC = w\frac{L}{har_{av}}$ and threshing cost is $ThC = w\frac{L}{thr_{av}}$. Now, consider tillage cost to find total labor cost for producing crops in Ethiopia.

## 3.8. Farm land share renting

Farmers rent out farm land when they have labor scarcity. They rent in farm land when they have excess labor. Factors such as availability of oxen feed and house holding capacity control farmers' oxen and workers holding capacity. Thus, farmers decide to hold oxen and workers by assessing the availability of feed for oxen and house holding capacity for oxen and workers. Suppose that a farmers holds $n$ number of oxen and $m$ number of workers. Let $L_f$ be a farm land that a farmer holds. Now a challenge is to decide when to rent out or rent in farm land based on $n$, $m$ and $L_f$.

Suppose that $L_{n,m,p}$ is optimal farm land in hectare, where $n$ is the number of oxen, $m$ is the number of workers and $p$ is the number of tillage days. Let $tl_{av}$ be average tillage ability of one worker or ox per day. Suppose that $f_{av}$ is average tillage frequency. Find $(L_{n,m,p}, p)$ from the following tillage arrangement model $(TAM)$. Let $tl_p$ be tillage period and $k$ be tillage gap. Define $y_{L_{n,m},p} = 3tl_{av}Td(n,m,p) + tl_{av}rs(n,m,p) - f_{av}L_{n,m}$.

$$\begin{aligned} TAM: \quad & \min_{L_{n,m},p} y^2_{L_{n,m},p} \\ \text{Subject to}: \quad & p(n+m)tl_{av} = f_{av}L_{n,m}, \\ & \frac{p}{f_{av}} \geq k, \\ & p \leq tl_p, \\ & L_{n,m}, p \geq 0. \end{aligned}$$

Suppose that $(L^*_{n,m}, p^*)$ be the optimal solution of $TAM$. If $\frac{tl_p - p^*}{f_{av}} < k$, the optimal solution is $(n, m, L^*_{n,m}, p^*)$. But a farmer has fixed farm land $L_f$. If $L_f > L^*_{n,m}$, then a farmer should rent out $L_f - L^*_{n,m}$ farm land provided $\frac{tl_p - p^*}{f_{av}} < k$. If $L_f < L^*_{n,m}$, then a farmer should rent in $-L_f + L^*_{n,m}$ farm land provided $\frac{tl_p - p^*}{f_{av}} < k$. If $L_f = L^*_{n,m}$, then a farmer neither rent out nor rent in farm land provided $\frac{tl_p - p^*}{f_{av}} < k$. Suppose that $\frac{tl_p - p^*}{f_{av}} \geq k$. Then a farmer can use $(n, m, p^*)$ to prepare $L^*_{n,m}$ farm land. This farmer holds $(n, m)$ in period $tl_p - p^*$. Let us consider the following model to determine farm land for period $tl_p - p^*$ using $(n, m)$. The following model is called second period tillage arrangement model $SPTAM$.

$$SPTAM: \quad \min_{L_{n,m}} y^2_{L_{n,m}, tl_p - p^*}$$

$$\text{Subject to}: \quad (tl_p - p^*)(n+m)tl_{av} = f_{av} L_{n,m},$$
$$L_{n,m} \geq 0.$$

Let $L^{\delta}_{n,m} = \frac{(tl_p - p^*)(n+m)tl_{av}}{f_{av}}$ be the optimal solution of $SPTAM$. If $\frac{tl_p - p^*}{f_{av}} \geq k$, then a farmer can prepare $L^{\delta}_{n,m} + L^*_{n,m}$ farm land using $(n, m, tl_p)$. A farmer decides to rent out $L_f - L^*_{n,m} - L^{\delta}_{n,m}$ if $\frac{tl_p - p^*}{f_{av}} \geq k$ and $L_f - L^*_{n,m} - L^{\delta}_{n,m} > 0$. A farmer decides to rent in $-L_f + L^*_{n,m} + L^{\delta}_{n,m}$ if $\frac{tl_p - p^*}{f_{av}} \geq k$ and $L_f - L^*_{n,m} - L^{\delta}_{n,m} < 0$. A farmer decides not to rent out or rent in a farm land if $\frac{tl_p - p^*}{f_{av}} \geq k$ and $L_f - L^*_{n,m} - L^{\delta}_{n,m} = 0$.

## 3.9. Tillage cost

Let $wg$ be daily wage for one ox or worker. Tillage cost is total labor cost less harvesting and threshing cost. Tillage cost depends on number of workers and number of oxen assigned for a given farm land tillage. If $\frac{tl_p - p^*}{f_{av}} < k$, then tillage cost $TiC$ is given by:

$$TiC = wg(n+m)p^*. \tag{18}$$

If $\frac{tl_p - p^*}{f_{av}} \geq k$, then tillage cost $TiC$ is given by:

$$TiC = wg(n+m)tl_p. \tag{19}$$

## 3.10. Farm land cost

Farm land cost is cash rental cost of farm. Farmers know farm land cost for each crop production season using available farm land rental price. Farm land cost varies from place to place and time to time. Let $FL_c$ be farm land cost. Suppose that $fh_c$ is one hectare farm land cost. If $\frac{tl_p - p^*}{f_{av}} < k$, then tillage cost $FL_c$ is given by:

$$FL_c = fh_c L^*_{n,m}. \tag{20}$$

If $\frac{tl_p - p^*}{f_{av}} \geq k$, then tillage cost $FL_c$ is given by:

$$FL_c = fh_c(L^*_{n,m} + L^{\delta}_{n,m}). \tag{21}$$

## 3.11. Inputs cost

Inputs cost is total production cost less the sum of farm land cost and total labor cost. Inputs cost consists of seed cost, pesticide cost, fertilizer cost and weeding cost.

## 3.12. Total labor cost

Clearly total labor cost is the sum of tillage cost, harvesting cost and threshing cost.
If $\frac{tl_p - p^*}{f_{av}} < k$, then tillage cost $TLC$ is given by:

$$TLC = HaC + ThC + TiC = wg\left(\frac{L^*_{n,m}}{har_{av}} + \frac{L^*_{n,m}}{thr_{av}} + (n+m)p^*\right). \tag{22}$$

If $\frac{tl_p - p^*}{f_{av}} \geq k$, then tillage cost $TLC$ is given by:

$$TLC = HaC + ThC + TiC = wg\left(\frac{L^*_{n,m} + L^{\delta}_{n,m}}{har_{av}} + \frac{L^*_{n,m} + L^{\delta}_{n,m}}{thr_{av}} + (n+m)tl_p\right). \tag{23}$$

## 3.13. Total production cost

Total production cost is the sum of total labor cost, farm land cost and inputs cost. Total production cost depends on time and location.

## 3.14. Application of portfolio selection models in agriculture

Let $L_{ti}$ be a plot of land assigned for crop $i$ at time $t$. We consider three scenarios to construct farm land allocation model. In scenario one we assume that there is no seasonal and crop rotation constraints. In scenario two we consider seasonal constraints. Finally, we study farm land allocation model with seasonal and crop rotation constraints in scenario three.

## 3.15. Farm land allocation model without seasonal and crop rotation constraints

Let $p_{ti}$ be unit price of crop $i$ at time $t$. Suppose that $c_{ti}$ $(rev_{ti})$ stands for unit cost (revenue) of crop $i$ at time $t$. Then total revenue and cost as follows. Total revenue is defined by:

$$TR_{t+1} = \sum_{i=1}^{n} p_{(t+1)i} y_{(t+1)i} L_{ti}, \tag{24}$$

where $y_{ti}$ yield of crop $i$ at time $t$. Total cost is defined by:

$$TC_t = \sum_{i=1}^{n} p_{ti} q_{ti} L_{ti}, \tag{25}$$

where $q_{ti}$ quantity of seed $i$ per hectare at time $t$. Clearly, $c_{ti} = p_{ti}q_{ti}$ and $rev_{ti} = p_{ti}y_{ti}$. Now one can define return $r_{t+1}$ by:

$$r_{t+1} = \frac{TR_{t+1} - TC_t}{TC_t} = \frac{\sum\limits_{i=1}^{n} \frac{rev_{(t+1)i} - c_{ti}}{c_{ti}} L_{ti} c_{ti}}{\sum\limits_{i=1}^{n} c_{ti} L_{ti}}. \tag{26}$$

Define return of crop $i$ at time $t$ by:

$$r_{(t+1)i} = \frac{rev_{(t+1)i} - c_{ti}}{c_{ti}}. \tag{27}$$

Weight of crop $i$ at time $t$ is defined by:

$$w_{ti} = \frac{c_{ti} L_{ti}}{\sum\limits_{i=1}^{n} c_{ti} L_{ti}}. \tag{28}$$

Note that $r_{t+1}$ has the following form.

$$r_{t+1} = \sum_{i=1}^{n} w_{ti} r_{(t+1)i}. \tag{29}$$

We know that $\sum\limits_{i=1}^{n} L_{ti} = L_t$. This implies that

$$\sum_{i=1}^{n} \frac{w_{ti}}{c_{ti}} = \frac{L_t}{TPR_t}. \tag{30}$$

One can solve for $w_{ti}$ using portfolio selection models. This scenario works only in absence of seasonal and crop rotation constraints.

### 3.16. Agricultural inputs allocation model

Assume that $w_{ti} = w_i$. Let $(AIAM)$ be agricultural inputs allocation model. Agricultural inputs allocation model without seasonal and crop rotation constraints is given by:

$$\begin{aligned} AIAM: &\quad \max_{w} \frac{mean(r_{t+1}) - r_f}{\sqrt{variance(r_{t+1})}} \\ \text{Subject to}: &\quad \sum_{i=1}^{n} w_i = 1, \\ &\quad \sum_{i=1}^{n} mean\left(\frac{w_i}{c_{ti}}\right) = mean\left(\frac{L_t}{TPR_t}\right), \quad 0 \le w_i \le 1. \end{aligned}$$

### 3.17. Farm land allocation with seasonal constraint

We know that growing period for all crops is not the same. Farmers do not grow some crops in some season because of seasonal factor. Assumption: We assume that there is no crop rotation constraint. We categorize crops in to three groups based on growing period. Season one crops are crops that grow in season one. Season two crops are crops that grow in season two. There are some crops that grow from the beginning of season one up to the end of season two. Define $n_1$ as the number season one crops, $n_2$ as the number season two crops and $n_{12}$ as the number one year growing period crops. Let $L_{ti}^{[1]}$ be a plot of land assigned for season one crop $i$ at time $t$. Suppose that $L_{ti}^{[2]}$ is a plot of land assigned for season two crop $i$ at time $t$. Let $L_{ti}^{[12]}$ be a plot of land assigned for one year growing period crop $i$ at time $t$. Define

1. $L_{ti}^{[k]}$: Farm land allocated for crop $i$ at time $t$ in season $k$,
2. $y_{ti}^{[k]}$: Crop $i$ yield at time $t$ in season $k$,
3. $q_{ti}^{[k]}$: Required crop $i$ seed per hectare at time $t$ in season $k$,
4. $p_{ti}^{[k]}$: Crop $i$ price at time $t$ in season $k$.

Total revenue is defined by:

$$TR_{t+1}^{s} = \sum_{i=1}^{n_1} p_{(t+1)i}^{[1]} y_{(t+1)i}^{[1]} L_{ti}^{[1]} + \sum_{i=1}^{n_2} p_{(t+2)i}^{[2]} y_{(t+2)i}^{[2]} L_{ti}^{[2]} + \sum_{i=1}^{n_{12}} p_{(t+2)i}^{[12]} y_{(t+2)i}^{[12]} L_{ti}^{[12]}, \tag{31}$$

where $y_{ti}$ yield of crop $i$ at time $t$. Total cost is defined by:

$$TC_t^s = \sum_{i=1}^{n_1} p_{ti}^{[1]} q_{ti}^{[1]} L_{ti}^{[1]} + \sum_{i=1}^{n_2} p_{ti}^{[2]} q_{ti}^{[2]} L_{ti}^{[2]} + \sum_{i=1}^{n_{12}} p_{ti}^{[12]} q_{ti}^{[12]} L_{ti}^{[12]}, \tag{32}$$

Define weights as follows:

$$w_i^{[1]} = \frac{p_{ti}^{[1]} q_{ti}^{[1]} L_{ti}^{[1]}}{\sum_{i=1}^{n_1} p_{ti}^{[1]} q_{ti}^{[1]} L_{ti}^{[1]} + \sum_{i=1}^{n_2} p_{ti}^{[2]} q_{ti}^{[2]} L_{ti}^{[2]} + \sum_{i=1}^{n_{12}} p_{ti}^{[12]} q_{ti}^{[12]} L_{ti}^{[12]}}. \tag{33}$$

$$w_i^{[2]} = \frac{p_{ti}^{[2]} q_{ti}^{[2]} L_{ti}^{[2]}}{\sum_{i=1}^{n_1} p_{ti}^{[1]} q_{ti}^{[1]} L_{ti}^{[1]} + \sum_{i=1}^{n_2} p_{ti}^{[2]} q_{ti}^{[2]} L_{ti}^{[2]} + \sum_{i=1}^{n_{12}} p_{ti}^{[12]} q_{ti}^{[12]} L_{ti}^{[12]}}. \tag{34}$$

$$w_i^{[12]} = \frac{p_{ti}^{[12]} q_{ti}^{[12]} L_{ti}^{[12]}}{\sum_{i=1}^{n_1} p_{ti}^{[1]} q_{ti}^{[1]} L_{ti}^{[1]} + \sum_{i=1}^{n_2} p_{ti}^{[2]} q_{ti}^{[2]} L_{ti}^{[2]} + \sum_{i=1}^{n_{12}} p_{ti}^{[12]} q_{ti}^{[12]} L_{ti}^{[12]}}. \tag{35}$$

Define crops return as follows.

$$r_{(t+1)i}^{[1]} = \frac{p_{(t+1)i}^{[1]} y_{(t+1)i}^{[1]} - p_{ti}^{[1]} q_{ti}^{[1]}}{p_{ti}^{[1]} q_{ti}^{[1]}}. \tag{36}$$

$$r^{[2]}_{(t+1)i} = \frac{p^{[2]}_{(t+2)i} y^{[2]}_{(t+2)i} - p^{[2]}_{ti} q^{[2]}_{ti}}{p^{[2]}_{ti} q^{[2]}_{ti}}. \tag{37}$$

$$r^{[12]}_{(t+1)i} = \frac{p^{[12]}_{(t+2)i} y^{[12]}_{(t+2)i} - p^{[12]}_{ti} q^{[12]}_{ti}}{p^{[12]}_{ti} q^{[12]}_{ti}}. \tag{38}$$

Now we define portfolio return as follows.

$$r^{s}_{t+1} = \sum_{i=1}^{n_1} w^{[1]}_i r^{[1]}_{(t+1)i} + \sum_{i=1}^{n_2} w^{[2]}_i r^{[2]}_{(t+1)i} + \sum_{i=1}^{n_{12}} w^{[12]}_i r^{[12]}_{(t+1)i}. \tag{39}$$

Farmers grow season two crops using season one crops farm land. Thus, the same farm land can be used for season one and two crops. One year growing period crops occupy allocated farm land. If a farm has $L^s_t$ farm land for producing crops, then $\sum\limits_{i=1}^{n_1} L^{[1]}_{ti} = \sum\limits_{i=1}^{n_2} L^{[2]}_{ti}$ and $\sum\limits_{i=1}^{n_1} L^{[1]}_{ti} = L^s_t - \sum\limits_{i=1}^{n_{12}} L^{[12]}_{ti}$. This implies that

$$\sum_{i=1}^{n_1} \frac{w^{[1]}_i}{p^{[1]}_{ti} q^{[1]}_{ti}} = \sum_{i=1}^{n_2} \frac{w^{[2]}_i}{p^{[2]}_{ti} q^{[2]}_{ti}} \tag{40}$$

and

$$\sum_{i=1}^{n_1} \frac{w^{[1]}_i}{p^{[1]}_{ti} q^{[1]}_{ti}} + \sum_{i=1}^{n_{12}} \frac{w^{[12]}_i}{p^{[12]}_{ti} q^{[12]}_{ti}} = \frac{L^s_t}{TPC^s_t}. \tag{41}$$

Let $(SAIAM)$ be agricultural inputs allocation model with seasonal constraint. Agricultural inputs allocation model with seasonal constraints is given by:

$$\begin{aligned}
SAIAM: \quad & \max \frac{mean(r^s_{t+1}) - r_f}{\sqrt{variance(r^s_{t+1})}} \\
\text{Subject to}: \quad & \sum_{i=1}^{n_1} w^{[1]}_i + \sum_{i=1}^{n_2} w^{[2]}_i + \sum_{i=1}^{n_{12}} w^{[12]}_i = 1, \\
& \sum_{i=1}^{n_1} mean\left(\frac{1}{p^{[1]}_{ti} q^{[1]}_{ti}}\right) w^{[1]}_i = \sum_{i=1}^{n_2} mean\left(\frac{1}{p^{[2]}_{ti} q^{[2]}_{ti}}\right) w^{[2]}_i, \\
& \sum_{i=1}^{n_1} mean\left(\frac{1}{p^{[1]}_{ti} q^{[1]}_{ti}}\right) w^{[1]}_i + \sum_{i=1}^{n_{12}} mean\left(\frac{1}{p^{[12]}_{ti} q^{[12]}_{ti}}\right) w^{[12]}_i = mean\left(\frac{L^s_t}{TPC^s_t}\right), \quad 0 \le w^{[1]}_i, w^{[2]}_i, w^{[12]}_i \le 1.
\end{aligned}$$

In this research, we introduce seasonal and local markets factors to construct marketing strategy portfolio optimization problem. Let consider two scenarios to build marketing strategy portfolio models. In scenario one, we consider local markets factor to construct marketing strategy optimization model as portfolio selection model. In scenario two, we include both seasonal and local markets factor to build marketing strategy portfolio models. Farmers choose local markets to purchase and sell agricultural products by considering marketing transaction cost. Let $q_{ti}$ be required seed $i$ quantity per hectare. Suppose that there are $n_m$ number of local markets. A farmer want to purchase some portion $h_{tim}$ of $q_{ti}$ from market $m$. Then $\sum\limits_{m=1}^{n_m} h_{tim} = 1$. Assume that a farmer sells $h_{tim}$ portion of yield $y_{(t+1)i}$ at market $m$. Define

1. $L_{ti}$: Farm land allocated for crop $i$ at time $t$,
2. $y_{ti}$: Crop $i$ yield at time $t$,
3. $q_{ti}$: Required crop $i$ seed per hectare at time $t$,
4. $p_{tim}$: Crop $i$ price at time $t$ in market $m$ ,
5. $T_{tim}$: Crop $i$ marketing transaction cost at time $t$ in market $m$.

Define total revenue $TR_{(t+1)}$ and total production cost $TC_t$ as follows.

$$TC_t = \sum_{i=1}^{n}\sum_{m=1}^{n_m} h_{tim} q_{ti} L_{ti} \left(p_{tim} + T_{tim}\right). \tag{42}$$

$$TR_{(t+1)} = \sum_{i=1}^{n}\sum_{m=1}^{n_m} h_{tim} y_{(t+1)i} L_{ti} \left(p_{(t+1)im} - T_{(t+1)im}\right). \tag{43}$$

Define $w_i$ by:

$$w_i = \frac{h_{tim} q_{ti} L_{ti} \left(p_{tim} + T_{tim}\right)}{\sum\limits_{i=1}^{n}\sum\limits_{m=1}^{n_m} h_{tim} q_{ti} L_{ti} \left(p_{tim} + T_{tim}\right)}. \tag{44}$$

Define $f_{(t+1)i}$ by:

$$f_{(t+1)i} = \sum_{m=1}^{n_m} \frac{\left(p_{(t+1)im} - T_{(t+1)im}\right)}{\left(p_{tim} + T_{tim}\right)}. \tag{45}$$

Clearly return $r_{t+1}$ is given by:

$$r_{t+1} = -1 + \sum_{i=1}^{n} \frac{f_{(t+1)i} y_{(t+1)i}}{q_{ti}} w_i. \tag{46}$$

This implies that marketing strategy portfolio is constant mix portfolio. Since $\sum_{i=1}^{n} L_{ti} = L_t$, we obtain

$$\frac{L_t}{TC_t} = \sum_{i=1}^{n} \left( \sum_{m=1}^{n_m} \frac{1}{q_{ti}(p_{tim} + T_{tim})} \right) w_i. \tag{47}$$

Note that

$$h_{tim} = \frac{\frac{1}{(p_{tim}+T_{tim})}}{\sum\limits_{m=1}^{n_m} \frac{1}{(p_{tim}+T_{tim})}}. \tag{48}$$

# 4. Marketing Strategy Optimization Model (MSOM)

Marketing strategy optimization model (MSOM)is given by:

$$\begin{aligned} MSOM: \quad & \max \frac{mean\left(-1 + \sum\limits_{i=1}^{n} \frac{f_{(t+1)i} y_{(t+1)i}}{q_{ti}} w_i\right) - r_f}{\sqrt{variance\left(-1 + \sum\limits_{i=1}^{n} \frac{f_{(t+1)i} y_{(t+1)i}}{q_{ti}} w_i\right)}} \\ \text{Subject to}: \quad & \sum_{i=1}^{n} w_i^1 = 1, \\ & mean\left(\frac{L_t}{TC_t}\right) = \sum_{i=1}^{n} mean\left(\left(\sum_{m=1}^{n_m} \frac{1}{q_{ti}(p_{tim} + T_{tim})}\right)\right) w_i, \quad 0 \le w_i^1 \le 1. \end{aligned}$$

# 5. Marketing Strategy Optimization Model with Seasonal Factor (MSOMSF)

In Ethiopia farmers grow crops in season one, season two and one year growing period crops. Define

1. $L_{ti}^{[km]}$: Farm land allocated for crop $i$ at time $t$ in season $k$ with market factor $m$,
2. $y_{ti}^{[km]}$: Crop $i$ yield at time $t$ in season $k$ with market factor $m$,
3. $q_{ti}^{[km]}$: Required crop $i$ seed per hectare at time $t$ in season $k$ with market factor $m$,
4. $p_{tim}^{[km]}$: Crop $i$ price at time $t$ in season $k$ with market factor $m$,

5. $T_{tim}^{[km]}$: Crop $i$ marketing transaction cost at time $t$ in season $k$ with market factor $m$.

Define

1. $h_{tim}^{[1m]} = \dfrac{\frac{1}{\left(p_{tim}^{[1m]}+T_{tim}^{[1m]}\right)}}{\sum\limits_{m=1}^{n_m}\frac{1}{\left(p_{tim}^{[1m]}+T_{tim}^{[1m]}\right)}}$,

2. $h_{(t+1)im}^{[2m]} = \dfrac{\frac{1}{\left(p_{(t+1)im}^{[2m]}+T_{(t+1)im}^{[2m]}\right)}}{\sum\limits_{m=1}^{n_m}\frac{1}{\left(p_{(t+1)im}^{[2m]}+T_{(t+1)im}^{[2m]}\right)}}$,

3. $h_{tim}^{[12m]} = \dfrac{\frac{1}{\left(p_{tim}^{[12m]}+T_{tim}^{[12m]}\right)}}{\sum\limits_{m=1}^{n_m}\frac{1}{\left(p_{tim}^{[12m]}+T_{tim}^{[12m]}\right)}}$.

Define seasonal total production cost and revenue as follows.

$$TC_t^{[1m]} = \sum_{i=1}^{n_1}\sum_{m=1}^{n_m} h_{tim}^{[1m]} q_{ti}^{[1m]} L_{ti}^{[1m]} \left(p_{tim}^{[1m]} + T_{tim}^{[1m]}\right). \tag{49}$$

$$TC_{t+1}^{[2m]} = \sum_{i=1}^{n_2}\sum_{m=1}^{n_m} h_{(t+1)im}^{[2m]} q_{(t+1)i}^{[2m]} L_{(t+1)i}^{[2m]} \left(p_{(t+1)im}^{[2m]} + T_{(t+1)im}^{[2m]}\right). \tag{50}$$

$$TC_t^{[12m]} = \sum_{i=1}^{n_{12}}\sum_{m=1}^{n_m} h_{tim}^{[12m]} q_{ti}^{[12m]} L_{ti}^{[12m]} \left(p_{tim}^{[12m]} + T_{tim}^{[12m]}\right). \tag{51}$$

$$TR_{t+1}^{[1m]} = \sum_{i=1}^{n_1}\sum_{m=1}^{n_m} h_{tim}^{[1m]} y_{(t+1)i}^{[1m]} L_{ti}^{[1m]} \left(p_{(t+1)im}^{[1m]} + T_{(t+1)im}^{[1m]}\right). \tag{52}$$

$$TR_{t+2}^{[2m]} = \sum_{i=1}^{n_2}\sum_{m=1}^{n_m} h_{(t+1)im}^{[2m]} y_{(t+2)i}^{[2m]} L_{(t+1)i}^{[2m]} \left(p_{(t+2)im}^{[2m]} + T_{(t+1)im}^{[2m]}\right). \tag{53}$$

$$TR_{t+2}^{[12m]} = \sum_{i=1}^{n_{12}}\sum_{m=1}^{n_m} h_{tim}^{[12m]} y_{(t+2)i}^{[12m]} L_{ti}^{[12m]} \left(p_{(t+2)im}^{[12m]} + T_{(t+2)im}^{[12m]}\right). \tag{54}$$

Total production cost with seasonal and market factor is given by:

$$TC_{t+1}^{[sm]} = TC_t^{[1m]} + TC_{t+1}^{[2m]} + TC_t^{[12m]}. \tag{55}$$

Total revenue with seasonal and market factor is given by:

$$TR_{t+2}^{[sm]} = TR_{t+1}^{[1m]} + TR_{t+2}^{[2m]} + TR_{t+2}^{[12m]}. \tag{56}$$

Define weight with seasonal and market factors as follows.

$$w_i^{[1m]} = \frac{h_{tim}^{[1m]} q_{ti}^{[1m]} L_{ti}^{[1m]} \left(p_{tim}^{[1m]} + T_{tim}^{[1m]}\right)}{TC_{t+1}^{[sm]}}. \tag{57}$$

$$w_i^{[2m]} = \frac{h_{(t+1)im}^{[2m]} q_{(t+1)i}^{[2m]} L_{(t+1)i}^{[2m]} \left(p_{(t+1)im}^{[2m]} + T_{(t+1)im}^{[2m]}\right)}{TC_{t+1}^{[sm]}}. \tag{58}$$

$$w_i^{[12m]} = \frac{h_{tim}^{[12m]} q_{ti}^{[12m]} L_{ti}^{[12m]} \left(p_{tim}^{[12m]} + T_{tim}^{[12m]}\right)}{TC_{t+1}^{[sm]}}. \tag{59}$$

Clearly

$$\sum_{i=1}^{n_1} w_i^{[1m]} + \sum_{i=1}^{n_2} w_i^{[2m]} + \sum_{i=1}^{n_{12}} w_i^{[12m]} = 1. \tag{60}$$

Farmers grow season two crops using season one crops farm land. Thus, the same farm land can be used for season one and two crops. One year growing period crops occupy allocated farm land. If a farm has $L_t^{[sm]}$ farm land for producing crops, then $\sum_{i=1}^{n_1} L_{ti}^{[1m]} = \sum_{i=1}^{n_2} L_{(t+1)i}^{[2m]}$ and $\sum_{i=1}^{n_1} L_{ti}^{[1m]} = L_t^{[sm]} - \sum_{i=1}^{n_{12}} L_{ti}^{[12m]}$. This implies that

$$\sum_{i=1}^{n_1}\left(\sum_{m=1}^{n_m}\frac{1}{q_{ti}^{[1m]}(p_{tim}^{[1m]}+T_{tim}^{[1m]})}\right)w_i^{[1m]} = \sum_{i=1}^{n_2}\left(\sum_{m=1}^{n_m}\frac{1}{q_{(t+1)i}^{[2m]}(p_{(t+1)im}^{[2m]}+T_{(t+1)im}^{[2m]})}\right)w_i^{[2m]} \tag{61}$$

and

$$\sum_{i=1}^{n_1}\left(\sum_{m=1}^{n_m}\frac{1}{q_{ti}^{[1m]}(p_{tim}^{[1m]}+T_{tim}^{[1m]})}\right)w_i^{[1m]} + \sum_{i=1}^{n_{12}}\left(\sum_{m=1}^{n_m}\frac{1}{q_{ti}^{[12m]}(p_{tim}^{[12m]}+T_{tim}^{[12m]})}\right)w_i^{[12m]} = \frac{L_t^{[sm]}}{TC_{t+1}^{[sm]}}. \tag{62}$$

Define return of each crop as follows.

$$r_i^{[1m]} = \frac{y_{(t+1)i}^{[1m]}\left(p_{(t+1)im}^{[1m]}+T_{(t+1)im}^{[1m]}\right) - q_{ti}^{[1m]}\left(p_{tim}^{[1m]}+T_{tim}^{[1m]}\right)}{q_{ti}^{[1m]}\left(p_{tim}^{[1m]}+T_{tim}^{[1m]}\right)}. \tag{63}$$

$$r_i^{[2m]} = \frac{y_{(t+2)i}^{[2m]}\left(p_{(t+2)im}^{[2m]}+T_{(t+2)im}^{[2m]}\right) - q_{(t+1)i}^{[2m]}\left(p_{(t+1)im}^{[2m]}+T_{(t+1)im}^{[2m]}\right)}{q_{(t+1)i}^{[2m]}\left(p_{(t+1)im}^{[2m]}+T_{(t+1)im}^{[2m]}\right)}. \tag{64}$$

$$r_i^{[12m]} = \frac{y_{(t+2)i}^{[12m]}\left(p_{(t+2)im}^{[12m]}+T_{(t+2)im}^{[12m]}\right) - q_{ti}^{[12m]}\left(p_{tim}^{[12m]}+T_{tim}^{[12m]}\right)}{q_{ti}^{[12m]}\left(p_{tim}^{[12m]}+T_{tim}^{[12m]}\right)}. \tag{65}$$

Clearly total return $r_{t+2}^{[sm]}$ is given by:

$$r_{t+2}^{[sm]} = \sum_{i=1}^{n_1} r_i^{[1m]}w_i^{[1m]} + \sum_{i=1}^{n_2} r_i^{[2m]}w_i^{[2m]} + \sum_{i=1}^{n_{12}} r_i^{[12m]}w_i^{[12m]}. \tag{66}$$

Marketing strategy optimization model with seasonal factor (MSOMSF) is given by:

$$\begin{aligned}
MSOMSF: \quad & \max \frac{mean(r_{t+2}^{[sm]}) - r_f}{\sqrt{variance(r_{t+2}^{[sm]})}} \\
\text{Subject to}: \quad & \sum_{i=1}^{n_1} w_i^{[1m]} + \sum_{i=1}^{n_2} w_i^{[2m]} + \sum_{i=1}^{n_{12}} w_i^{[12m]} = 1, \\
& \sum_{i=1}^{n_1} mean(\left(\sum_{m=1}^{n_m}\frac{1}{q_{ti}^{[1m]}(p_{tim}^{[1m]}+T_{tim}^{[1m]})}\right))w_i^{[1m]} = \\
& \sum_{i=1}^{n_2} mean(\left(\sum_{m=1}^{n_m}\frac{1}{q_{(t+1)i}^{[2m]}(p_{(t+1)im}^{[2m]}+T_{(t+1)im}^{[2m]})}\right))w_i^{[2m]}, \\
& \sum_{i=1}^{n_1} mean(\left(\sum_{m=1}^{n_m}\frac{1}{q_{ti}^{[1m]}(p_{tim}^{[1m]}+T_{tim}^{[1m]})}\right))w_i^{[1m]} + \\
& \sum_{i=1}^{n_{12}} mean(\left(\sum_{m=1}^{n_m}\frac{1}{q_{ti}^{[12m]}(p_{tim}^{[12m]}+T_{tim}^{[12m]})}\right))w_i^{[12m]} = mean\left(\frac{L_t^{[sm]}}{TC_{t+1}^{[sm]}}\right), \quad 0 \le w_i^{[1m]}, w_i^{[2m]}, w_i^{[12m]} \le 1.
\end{aligned}$$

# 6. Results

The following results follow from Table 1.

1. For every number of oxen $N_O(L)$ there exists corresponding average farm land area $L(N_O)$ such that

$$L(N_O) = \frac{1}{20}\left[N_O(L)^2 + 13N_O(L) + 24\right]. \tag{67}$$

2. For every average farm land area $L(N_O)$ there exists corresponding number of oxen $N_O(L)$ such that

$$N_O(L) = \frac{1}{2}\left[-13 + \sqrt{73 + 80L(N_O)}\right]. \tag{68}$$

Cropping arrangement model $CAM$ has the following solution.

$$(L_{ic}, OL_{ic}, I_{ic}, HL_{ic}, LH_{ic}, TH_{ic}) = \left(\frac{Rev_{ic}}{4}, \frac{Rev_{ic}}{3}, \frac{Rev_{ic}}{12}, \frac{Rev_{ic}}{3}, \frac{Rev_{ic}}{4}, \frac{Rev_{ic}}{12}\right).$$

# 7. Discussion

It is said that there is linear relationship between number of oxen and average harvested area, [1]. However, as described in result section there is quadratic relationship between the number of oxen and average harvested area. Farmers should follow the following steps for producing crops.

- Collect historical data of unit cost and unit revenue for each crops.
- Collect historical data of aggregate farm land and total production cost.
- Categorize crops into three groups based on seasons.
- Collect marketing transaction cost data of crops.
- Solve $MSOMSF$.
- Determine $(L_c, OL_c, I_c, HL_c, LH_c, TH_c)$.

Moreover, this research suggests that farmers can determine revenue for farm land, inputs and labor contribution using share cropping and labor sharing arrangements.

# 8. Conclusion and Recommendation

In this study, we have reviewed literature on share cropping and labor sharing arrangements to determine revenue for farm land, inputs and labor contribution. We have described share cropping and labor sharing arrangements mathematically. Furthermore, we have constructed tillage arrangement model and cropping arrangement model using mathematical description of share cropping and labor sharing arrangements. We have built agricultural inputs allocation model with seasonal constraints to determine total revenue of crops. We have shown that farmers can estimate total tillage cost for a given level of farm land using tillage arrangement model. In this research, we constructed marketing strategy portfolio models subject to seasonal and local markets factor. We assumed that weights are constant for simplicity. This assumption does not matter because we introduced dynamic mix portfolio selection models. Moreover, we transformed agricultural farm land allocation problem into portfolio selection problem.

Finally, we have derived a method for determining revenue for farm land, inputs and labor contribution. We would like to recommend for researchers to do empirical study on determination of revenue for farm land, inputs and labor contribution using share cropping and labor sharing arrangements. Moreover, tillage and reserved functions should be studied further for simplifying mathematical behavior because these functions are not simple to be understood.

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