

New Approach to Find the Maxima and Minima of a Function

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Abstract: In this paper we have introduced a new and effective method to find the maxima and minima of a function. The proposed method is very short compared with existing methods. The proposed method is illustrated by the examples.

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1. Introduction

Absolute Maximum and Absolute Minimum: The most important applications of differential calculus are optimizational problems. In which we are required to find the optimal way of doing something. In many cases these problems can be reduced to finding the maximum or minimum values of a function.

Definition 1.1.

- A function f has absolute maximum at c if $f(c) > f(x)$ for all in the domain D of f .
- A function f has absolute minimum at c if $f(c) \leq f(x)$ for all in the domain D of f .

Local Maximum and Local Minimum: We now consider one of the most important applications of the derivative, here we use the first derivative as an aid in determining the high points or the low points on a curve. These points are called Local Maximum and Local Minimum.

2. Main Results

2.1. Absolute Maximum and Minimum

To find the absolute maximum and minimum values of a continuous function f on a closed interval $[a, b]$;

- (1). Find all of the critical points of f in the interval $[a, b]$.
- (2). Evaluate f at all of the critical numbers in the interval $[a, b]$.
- (3). Evaluate f at the endpoints of the interval, (calculate $f(a)$ and $f(b)$).

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- (4). The largest of the values from steps 2 and 3 is the absolute maximum of the function on the interval $[a, b]$ and the smallest of the values from Steps 2 and 3 is the absolute minimum of the function on the interval $[a, b]$.

or we proceed with following algorithm.

Working Rule to find absolute maximum and minimum

Step 1: Let $y = f(x)$, $a \leq x \leq b$, $f(a) = a$, $f(b) = b$. Find $\frac{dy}{dx}$,

Step 2: Find $\frac{dy}{dx} = 0$, and also find critical numbers.

Step 3: Substituting $f(a)$, $f(b)$ and critical numbers in y .

Step 4: To Find Absolute Maximum. Take the largest value of y then the value is called absolute maximum.

Step 5: To Find Absolute Minimum. Take the smallest value of y then the value is called absolute minimum.

Example 2.1. Find the absolute maximum and minimum values of the function $y = x^3 - 3x^2 + 1$ on $\frac{1}{2} \leq x \leq 4$.

Solution.

$$y = x^3 - 3x^2 + 1 \quad \text{on} \quad \frac{1}{2} \leq x \leq 4 \quad (1)$$

Step 1: Find $\frac{dy}{dx}$

$$\begin{aligned} \frac{dy}{dx} &= 3x^2 - 6x \\ \frac{dy}{dx} &= 0, \quad 3x(x-2) = 0 \\ x &= 0 \quad (\text{or}) \quad x = 2 \end{aligned}$$

Therefore the critical points are $= 0, 2$.

Step 2: Put $x = 0$ in (1), we get $y = 1$ and put $x = 2$ in (1), we get $y = -3$.

Step 3: Find the values of y at end points of $\frac{1}{2} \leq x \leq 4$.

$$\begin{aligned} \text{Put } a &= \frac{1}{2} \text{ in Equation (1)} \\ \Rightarrow y &= \left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 + 1 \\ y &= \frac{1}{8} - \frac{3}{4} + 1 \\ y &= \frac{1-6+8}{8} = \frac{3}{8} \end{aligned}$$

$$\text{Put } y = 0 \text{ in (1), we get } y = 1$$

$$\begin{aligned} \text{Put } b &= 4 \text{ in (1)} \\ \Rightarrow y &= (4)^3 - 3(4)^2 + 1 \\ y &= 64 - 48 + 1 \\ y &= 17 \end{aligned}$$

$$\text{Put } y = 2 \text{ in (1), we get } y = -3$$

Step 4: To find absolute maximum, select the largest value of y . Then the value from $y(0)$, $y(2)$, $y(4)$ and $y(\frac{1}{2})$. Therefore Absolute maximum value of $y = 17$.

Step 5: To find absolute minimum, select the smallest value of y value from $y(0)$, $y(2)$, $y(4)$ and $y(\frac{1}{2})$. Therefore Absolute minimum value of $y = -3$.

□

2.2. Local Maximum and Local Minimum

Working Rule to find local maximum and local minimum

Step 1: $y = f(x)$. Find $\frac{dy}{dx}$.

Step 2: Put $\frac{dy}{dx} = 0$, find critical numbers.

Step 3: Find $\frac{d^2y}{dx^2}$.

Step 4: Substituting critical numbers in $\frac{d^2y}{dx^2}$.

- (i). $\frac{d^2y}{dx^2} < 0$, the critical number is Local Maximum.
- (ii). $\frac{d^2y}{dx^2} > 0$, the critical number is Local Minimum.
- (iii). $\frac{d^2y}{dx^2} = 0$, $\frac{d^3y}{dx^3} \neq 0$, the given conditions are points of inflection.

Step 5: Substitute local maximum, local minimum values in y .

First-Derivative Test: Suppose that a continuous function f has a critical point at p of the type $\frac{df}{dp} = 0$. If $\frac{df}{dp}$ changes sign from negative to positive at p , then f has a local minimum at p . If $\frac{df}{dp}$ changes sign from positive to negative at p , then f has a local maximum at p .

Second-Derivative Test: Let f be a continuous function such that $\frac{df}{dp} = 0$. If $\frac{d^2f}{dp^2} > 0$ then f has a local minimum at p . If $\frac{d^2f}{dp^2} < 0$ then f has a local maximum at p . If $\frac{d^2f}{dp^2} = 0$ then the test fails. In this case,

Example 2.2. Find the local maximum and minimum of using the second derivative test $y = \sqrt{x} - \sqrt[4]{x}$.

Solution.

$$\begin{aligned}
 y &= \sqrt{x} - \sqrt[4]{x} \\
 &= x^{\frac{1}{2}} - x^{\frac{1}{4}} \\
 \frac{dy}{dx} &= \frac{1}{2\sqrt{x}} - \frac{1}{4}x^{-\frac{3}{4}} \\
 &= \frac{1}{2\sqrt{x}} - \frac{1}{4x^{\frac{3}{4}}} \\
 &= \frac{1}{2x^{\frac{1}{2}}} - \frac{1}{4x^{\frac{3}{4}}} \\
 \frac{dy}{dx} &= \frac{4x^{\frac{3}{4}} - 2x^{\frac{1}{2}}}{8x^{\frac{5}{4}}} \\
 \frac{dy}{dx} = 0 &\Rightarrow 4x^{\frac{3}{4}} - 2x^{\frac{1}{2}} = 0 \\
 4x^{\frac{3}{4}} &= 2x^{\frac{1}{2}} \\
 \frac{x^{\frac{3}{4}}}{x^{\frac{1}{2}}} &= \frac{2}{4} \\
 x^{\frac{3}{4} - \frac{1}{2}} &= \frac{1}{2} \\
 x^{\frac{3-2}{4}} &= \frac{1}{2} \\
 x^{\frac{1}{4}} &= \frac{1}{2} \\
 x &= \left(\frac{1}{2}\right)^{\frac{1}{4}} \\
 &= \frac{1}{16}
 \end{aligned}$$

$$\begin{aligned}
\frac{d^2y}{dx^2} &= \frac{1}{4} \left[2x - \frac{1}{2}x^{-\frac{3}{2}} - \left(-\frac{3}{4}x^{-\frac{7}{4}} \right) \right] \\
&= \frac{1}{4} \left[-\frac{x}{x^{\frac{3}{2}}} + \frac{3}{4x^{\frac{7}{4}}} \right] \\
\frac{d^2y}{dx^2} \text{ at } \left(\frac{1}{2} \right)^{\frac{1}{4}} &= \frac{1}{4} \left[-\frac{1}{\left[\frac{1}{16} \right]^{\frac{3}{2}}} + \frac{3}{4 \left[\frac{1}{16} \right]^{\frac{7}{4}}} \right] \\
&= \frac{1}{4} \left[-\frac{1}{\left(\frac{1}{16} \right)^{\frac{3}{2}}} + \frac{3}{4 \left(\frac{1}{16} \right)^{\frac{7}{4}}} \right] \Rightarrow \frac{d^2y}{dx^2} > 0 \\
&= \text{Positive}
\end{aligned}$$

Therefore, y is local minimum at $x = \left(\frac{1}{2} \right)^{\frac{1}{4}} = \frac{1}{16}$. The local minimum value of $y = \frac{1}{16}^{\frac{1}{2}} - \frac{1}{16}^{\frac{1}{4}} = -\frac{1}{4}$. □

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