Int. J. Math. And Appl., 6(4)(2018), 267–270
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

A Study on the Usage of Number Theory for the Formulation of Mathematical Problems

Gaurav Singh Sisodia[1,*]

1 Department of Applied Science & Humanities, Ch. Devi Lal State Institute of Engineering & Technology, Sirsa, Haryana, India.

Abstract: Number theory (or arithmetic or higher arithmetic in more seasoned utilization) is a part of unadulterated science committed principally to the investigation of the integers and integer-esteemed functions. German mathematician Carl Friedrich Gauss said that Arithmetic is the sovereign of technical disciplinesand number theory is the sovereign of science. Number scholars study prime numbers just as the properties of articles made out of integers (for instance, sound numbers) or characterized as speculations of the integers (for instance, algebraic integers). The current paper highlights the usage of number theory in the mathematical problems.

Keywords: Integer, Algebraic, Number.

Accepted on: 19.12.2018

1. Introduction

Integers can be considered either in themselves or as answers for equations (Diophantine geometry). Inquiries in number theory are frequently best comprehended through the investigation of scientific items (for instance, the Riemann zeta function) that encode properties of the integers, primes or other number-theoretic articles in some style (logical number theory). One may likewise contemplate real numbers corresponding to objective numbers, for instance, as approximated by the last mentioned. An algebraic number is any mind boggling number that is an answer for some polynomial equation with discerning coefficients. Fields of algebraic numbers are additionally called algebraic number fields, or right away number fields. Algebraic number theory examines algebraic number fields. Along these lines, logical and algebraic number theory can and do cover: the previous is characterized by its strategies, the last by its objects of study. Number fields are regularly examined as expansions of more modest number fields: a field L is said to be an augmentation of a field K if L contains K. (For instance, the mind boggling numbers C are an augmentation of the reals R, and the reals R are an expansion of the rationals Q). Grouping the potential expansions of a given number field is a troublesome and mostly open issue. Abelian augmentationsthat is, expansions L of K with the end goal that the Galois group Gal (L/K) of L over K is an abelian groupare moderately surely known. Their grouping was the object of the program of class field theory, which was started in the late nineteenth century (incompletely by Kronecker and Eisenstein) and did to a great extent in 1900-1950. A case of a functioning zone of research in algebraic number theory is Iwasawa theory. The Langlands program, one of the fundamental ebb and flow huge scope inquire about plans in science, is now and then depicted as an endeavor to sum up class field theory to non-abelian expansions of number fields. The focal issue of Diophantine geometry is to decide when a Diophantine

* *E-mail: nishasapra9@gmail.com*

equation has arrangements, and in the event that it does, what number of. The methodology taken is to think about the arrangements of an equation as a geometric item. For instance, an equation in two factors characterizes a bend in the plane. All the more for the most part, an equation, or arrangement of equations, in at least two factors characterizes a bend, a surface or some other such item in n-dimensional space. In Diophantine geometry, one asks whether there are any balanced (focuses the entirety of whose directions are rationales) or vital (focuses the entirety of whose directions are integers) on the bend or surface. On the off chance that there are any such focuses, the following stage is to ask what number of there are and how they are appropriated.

2. Usage of Number Theory for the Formulation of Mathematical Problems

On the off chance that x can't be all around approximated, at that point a few equations don't have integer or judicious arrangements. Besides, a few ideas (particularly that of stature) end up being basic both in Diophantine geometry and in the investigation of Diophantine approximations. This inquiry is likewise of uncommon enthusiasm for supernatural number theory: in the event that a number can be preferable approximated over any algebraic number, at that point it is a supernatural number. It is by this contention that and e have been demonstrated to be supernatural.

Diophantine geometry ought not be mistaken for the geometry of numbers, which is an assortment of graphical strategies for addressing certain inquiries in algebraic number theory. Arithmetic geometry, notwithstanding, is a contemporary term for a lot of a similar area as that secured by the term Diophantine geometry. The term arithmetic geometry is seemingly utilized regularly when one wishes to underline the associations with present day algebraic geometry (as in, for example, Faltings' hypothesis) as opposed to strategies in Diophantine approximations. A lot of probabilistic number theory can be viewed as a significant exceptional instance of the investigation of factors that are nearly, yet not exactly, commonly free. For instance, the occasion that an irregular integer among one and a million be distinguishable by two and the occasion that it be distinct by three are practically autonomous, yet not exactly.

It is once in a while said that probabilistic combinatorics utilizes the way that whatever occurs with likelihood more noteworthy than must happen now and again; one may state with equivalent equity that numerous utilizations of probabilistic number theory rely on the way that whatever is abnormal must be uncommon. On the off chance that specific algebraic items (state, discerning or integer answers for specific equations) can be demonstrated to be in the tail of certain reasonably characterized conveyances, it follows that there must be not many of them; this is a solid non-probabilistic explanation following from a probabilistic one.

For a prime number p, Gauss characterized a crude root modulo p to be an integer a whose multiplicative request modulo p will be $p-1$. At the end of the day, a will be a generator of the multiplicative group of non- zero integers modulo p. All the more by and large, for a positive integer n, each integer a coprime to n is with the end goal that a $\varphi(n)$ is 1 modulo n. A crude root modulo n is an integer a with the end goal that $\varphi(n)$ is the littlest $r > 0$ for which a r is 1 modulo n. Gauss additionally indicated that crude roots modulo n exist if, and just if, n is $2, 4$, dad or $2p$ a for some odd prime p.

For example, the crude roots modulo 5 among the integers 1 to 4 are 2 and 3. Their entirety is Q modulo 5. Presently, take a gander at the crude roots modulo I among 1 to 6. These are 3 and 5. Modulo Z, these entirety to 1. Shouldn't something be said about 11? The crude roots here are $2, 6, 7$ and 8 and these give the total 1 modulo 11. What is the example here? Without letting out the mystery, let us proceed to examine the issue for a general prime p. When is an integer modulo p a crude root? As we previously watched, an integer a will be a crude root modulo p accurately when p separates the integer $\Phi_{p-1}(a)$. This implies when the polynomial $\Phi_{p-1}(X)$ is viewed as a polynomial with coefficients integers modulo p, a will

be a root.

Henceforth the total of all the crude roots modulo p is basically the total modulo p of the foundations of Φ_{p-1} modulo p. As we will demonstrate beneath, the above aggregate is $\mu(p-1)$, where $\mu(n)$ is the Mobius function.

3. Discussion

Recently, the theory of group representations of the permutation groups (specifically, the so-called super character theory has been used to re- prove old identities in a quick way and also, to discover new identities. It is convenient to write $\Delta n = \{e2irr\pi/n_n : (r,n) = 1, 1 \leq r \leq n\}$. Then, the set of all n-th roots of unity $\{e2ik\pi/n : 0 \leq k < n\}$ is a union of the disjoint sets Δd as d varies over the divisors of n. This is because an n-th root of unity is a primitive d-th root of unity for a unique divisor d of n. It is also convenient to introduce the 'characteristic' function $\delta k/n$ which has the value 1 when k divides n and the value 0 otherwise. Before stating some properties of the ck(n)'s, let us recall two arithmetic functions which are ubiquitous in situations where elementary number-theoretic counting is involved.

The first one is Euler's to totient function $\varphi(n) = |\{r : 1 \leq r \leq n, (r,n) = 1\}|$. The other arithmetic function is the Mobius function defined by $\mu(1) = 1$, $\mu(n) = (-1)k$ or 0 for $n > 1$ according as to if n is a square-free integer that is a product of k distinct primes or otherwise. The Mobius function keeps tab when we use the principle of inclusion exclusion to do counting.

For any positive integer n, consider the set $\{1, 2, \ldots, 2n\}$ of the first $2n$ positive integers. We claim that this set can be written as the union of n pairs of integers $\{a_i, b_i\}$ $(1 \leq i \leq n)$ such that $a_i + b_i$ is prime! Indeed, this is clear for $n = 1$ as $1 + 2 = 3$ is prime, and we will apply induction on n to prove it in general. Assume that $n > 1$ and that our assertion is valid for every $m < n$.

Now, Bertrand's postulate ensures we have a prime p among the numbers in the set $\{2n + 1, 2n + 2, \ldots, 4n - 1\}$. Writing $p = 2n + r$, we have $r \in \{1, 2, \ldots, 2n - 1\}$. Thus, note that r is odd as p must be an odd prime. If $r > 1$, then by induction hypothesis, the set $\{1, 2, \ldots, r - 1\}$ can be split into pairs $\{a_i, b_i\}$ $(1 \leq i \leq r - 12)$ such that $a_i + b_i$ is prime for each i. Now, $\{r, r + 1, \ldots, 2n\}$ is evidently split into the pairs $\{r, 2n\}$, $\{r + 1, 2n - 1\}$, ... whose sums are all equal to the prime p.

Another very interesting application is the following one.

By refining the above methods, one may prove that for any positive integer k there is a sufficiently large N such that there is a prime between n and $2n - k$ for all $n > N$. Applying this to $k = 11$, Robert Dressler showed in 1972 that every positive integer other than $1, 2, 4, 6, 2$ is a sum of distinct odd primes.

Let $P(X) = a_0 + a_1X + \cdots + a_nX_n$ be an integral polynomial with $n > 0$ and an $6 = 0$. For any integer d, look at the polynomial $P(a_0dX) = a_0(1 + a_1dX + a_0a_2d_2X_2 + \cdots + a_{n-1}0$ and $nX_n)$.

since $Q(X) = 1 + a_1dX + a_0a_2d_2X_2 + \cdots + a_{n-1}0$ and nX_n takes the values $0, 1, -1$ at the most for finitely many values of X, it takes a value $Q(m)6 = 0, 1, -1$ which must then be a multiple of some prime p.

As $Q(m) = 1 \bmod d, p$ is coprime to d. Therefore, for any d, we have shown that there is some m such that $P(a_0d_m)$ is zero modulo p for some prime p coprime to d. Varying d, we have infinitely many such primes p. The set of odd primes modulo which the polynomial $X_2 + 1$ has roots, consists precisely of all primes in the arithmetic progression $4n + 1$.

4. Conclusion

In general, every quadratic polynomial has a corresponding arithmetic progression such that the polynomial has roots modulo each prime in this progression, and modulo no other primes. This follows from the famous quadratic reciprocity law. The first remarkable property cn(k) have is that they are integers. Ramanujan showed that several arithmetic functions (that is,

functions defined from the set of positive integers to the set of complex numbers) have Fourier-like of expansions in terms of the sums; hence, nowadays these expansions are known as Ramanujan expansions. They often yield very pretty elementary number-theoretic identities.

References

[1] P. Morandi, *Field and Galois theory*, Graduate texts in Mathematics 167, Springer-Verlag, (2014).

[2] M. Ram Murty and J. Esmonde, *Problems in Algebraic Number Theory*, Graduate Texts in Mathematics 190, Springer-Verlag, New York, (2015).

[3] M. Artin, *Algebra*, Prentice Hall, (2011.

[4] D. Suryaramana, Resonance, 2(6)(2013).

[5] Ian Stewart, Gauss, Scientific American, (2014).

[6] C. F. Gauss, *Disquisitiones Arithmeticae*, English Edition, Springer, (2015).

[7] J. Gray, *English translation and commentary on Gausss mathematical diary*, Expo. Math., 2(2014).

[8] M. Rosen, Amer. Math. Monthly, (2011).

[9] B. Sury, *Ramanujan's Awesome Sums*, Mathematics Newsletter, 24(2)(2013), 3136.

[10] S. Ramanujan, *On certain trigonometrical sums and their applications in the theory of numbers*, Trans. Cambridge Philos. Soc., 22(13)(2014), 259-276.