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The Hilbert – Reimann Paradox

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Abstract: In this paper we introduced a method to solve the derivation towards Reimann - HiIbert's paradox. We are using the very basic concepts of Reimann - Hilbert's paradox. The basic concepts of the complex analysis and gaussian integrals, to solve the equation like $\infty - \infty = z$, where 'z' is any complex constant and as we know a complex number is also a real number when it's imaginary part has zero as its co-efficient. As we know the quantity infinity is a very hard countability as well as complex mathematical concept. Many ideas that we find intuitive when working with the normal numbers don't work anymore, and instead there are countless apparent paradoxes. However, this paper clears the idea behind it. Hilbert's paradox of the grand hotel is a mathematical paradox named after the German Mathematician David Hilbert. According to the Hilbert's Grand hotel paradox, suppose consider a hotel that has an infinite number of rooms. As a convenience the rooms have numbers, the first room has a number 1, the second has number 2 and so on. If all the rooms are filled, it might appear that no more guests can be take in, as in a hotel with the finite number of rooms. This is where the statement slightly contradicts, now in the Hilbert's hotel a room can always be provided for another guest by moving the guest in room1 to room2, and the guest in room 2 to room 3 and so on. In the general case the case in guest in room number 'n' is moved to room number '$n + 1$'. After all the guests are moved now the room 1 is empty and the new guest can occupy this room. This shows how a new guest can be accommodated in the hotel.

Keywords: Gaussian Integral, Definite integral properties with the infinity limits, Basic mathematical concepts and properties.

1. Introduction

Statement: There is always a room to the new guests during any time, even though the hotel is completely accommodated with the guests.

Mathematically presenting the solution, If the hotel has infinite number of rooms where initially all the rooms are considered to be vacant, now consider the following situations

2. Materials and Methods

In case of infinitely new guests: If we double the number of guests inside the hotel and then we can also double the number of guests inside the hotel, again when all the rooms are full. This is done by shifting each guest to the room number exactly twice the room number he was and shifting into these new rooms, now we can again have infinite number of vacant rooms whose room number is a sequence of odd numbers in the hotel.

If infinite groups of infinite guests come: The people in the present rooms can be moved to the new rooms whose room numbers like the person in the first room is moved to the new room whose number is equal to the first prime number and on generalizing we can say like a person in the room number 'n' can be moved to the room number 'p' where 'p' is the

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nth prime number and again now the people are filled in all the prime numbered rooms and now again there are infinite number of vacant rooms in the hotel thus now again there are infinite rooms.

If infinite vehicles and each vehicle containing infinite number of guests arrive at the same time: The guests in the first room is shifted and all the so that the guests from the first bus are moved to the rooms whose room numbers are obtained by raising there seat number as an exponent to the seat number and by shifting the people in those particular rooms and now these rooms can be accommodated with the new guests.

Prime powers method: Empty the odd numbered rooms by sending the guest in room i to room 2^i then put the first coach's load in rooms 3^n, the second coach's load in rooms 5^n, for coach number c we use the rooms p^n where p is the c^{th} odd prime number. This solution leaves certain rooms empty (which may or may not be useful for the hotel); specifically, all odd numbers that are not prime powers, such as 15 or 847, will no longer be occupied.

3. Results and Discussion

Gaussian Integral: Consider the Gaussian Integral $I = \int\limits_{-\infty}^{\infty} e^{-ax^2} dx$; changing the order of integration

$$I = \int\limits_{-\infty}^{\infty} e^{-ax^2} dx = \int\limits_{-\infty}^{\infty} e^{-ay^2} dy$$

Multiplying the above integrals we get

$$I^2 = \left(\int\limits_{-\infty}^{\infty} e^{-ax^2} dx \right) \left(\int\limits_{-\infty}^{\infty} e^{-ay^2} dy \right)$$
$$I^2 = \int \int e^{-a(x^2+y^2)} dxdy$$

Converting the rectangular co-ordinate system to the polar co-ordinate system using the following identity we get, $\int \int f(x, y)\, dxdy = \int \int f(r\cos\alpha, r\sin\alpha)\, rdrd\alpha$ the integral is defined in any region R

$$I^2 = \int_0^{2\pi} \int_{-\infty}^{\infty} e^{-r^2} rdrd\alpha$$
$$I' = \int_{-\infty}^{\infty} e^{-r^2} rdr$$

Put $r^2 = s \Rightarrow 2rdr = ds \Rightarrow rdr = \frac{1}{2}ds$. Now the integral I reduces to

$$I' = \frac{1}{2}(2) \int_0^{\infty} e^{-s} ds$$
$$I' = \left(-e^{-s}\right)_0^{\infty}$$

Resubstituting the value of 'z' we get $I' = \frac{1}{2}\left(e^{-r^2}\right)$ the limits are from $r = 0$ to $r = \infty$. Now by substituting the limits we get

$$I' = -\left(e^{-\infty} - e^0\right)$$
$$I' = -(0-1)$$
$$I' = 1$$

Now the integral given above becomes

$$I^2 = \int_0^{2\pi} 1 d\alpha$$
$$I^2 = \frac{1}{2}(2\pi)$$
$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Now taking the substitution

$$-x^2 = y^2$$
$$x^2 = -y^2$$

Taking the square root on both the sides we get

$$\sqrt{x^2} = \sqrt{-y^2}$$
$$x = iy$$
$$dx = idy$$
$$i\int_{-\infty}^{\infty} e^{y^2} dy = \sqrt{\pi}$$

Since $i = \sqrt{-1}$ is a complex number, now splitting the above integral into parts we get

$$i\int_{-\infty}^{0} e^{y^2} dy + \ i\int_{0}^{\infty} e^{y^2} dy = \sqrt{\pi}$$

Now differentiating the above equation we get

$$i\int_{-\infty}^{0} \frac{\partial}{\partial y}\left(e^{y^2}\right) dy + i\int_{0}^{\infty} \frac{\partial}{\partial y}\left(e^{y^2}\right) dy = \frac{\partial\left(\sqrt{\pi}\right)}{\partial y}$$

since the term after the equality represents a differentiation of the constant the value of the term becomes zero. The differentiation of the function

$$\frac{\partial(e^{x^2})}{\partial x} = 2xe^{x^2}$$
$$i\int_{-\infty}^{0} 2ye^{y^2} dy + i\int_{0}^{\infty} 2ye^{y^2} dy = 0$$

Now consider the integral formula $\int f'(x)\, e^{f(x)} dx = e^{f(x)} + C$, where c the constant of integration

$$i(e^{y^2})_{-\infty}^{0} + i(e^{y^2})_{0}^{\infty} = 0 \tag{1}$$

The function defined by $f(x) = e^{x^2}$ is the continuously increasing function and as the value of x increase the diverges to infinity, so

$$\lim_{x\to\infty}(e^{x^2}) = \infty$$

Also the function defined here is even function so the value is same whether the limit value tends to 'a' or '$-a$'. So in this contrast

$$\lim_{x\to-\infty}\left(e^{x^2}\right) = \lim_{x\to\infty}(e^{x^2}) = \infty$$

Applying the above conditions in the equation number (1) we get

$$i\left\{e^{0^2}-\lim_{x\to-\infty}\left(e^{x^2}\right)\right\}+i\left\{\lim_{x\to\infty}\left(e^{x^2}\right)-e^{0^2}\right\}=0$$

Simplifying the above expression we get

$$i\,(1-\infty)+i\,(\infty-1)=0$$
$$i\,(1-\infty+\infty-1)=0$$

Cancelling '1' we get

$$i\,(\infty-\infty)=0$$

If we consider a constant complex number k then we can write the term zero on the right hand side as $dk=0$, then the above equation can be written as

$$i\,(\infty-\infty)=dk$$

Integrating on both the sides we get

$$\int i\,(\infty-\infty)\,dz'=\int dk$$
$$i\int(\infty-\infty)\,dz'=k$$

Multiplying by 'i' on both the sides we get

$$i^2\int(\infty-\infty)\,dz'=ik$$
$$-\left[\int\infty dz'-\int\infty dz'\right]=ik$$

Since the integral of infinity represents the area under the curve $y=\infty$, represents the area under the rectangle with infinite length and some random finite breadth, so the integral becomes infinity. Now simplifying the above integral we get

$$-\,(\infty-\infty)=ik+s$$

Where 's' is any arbitrary constant now the final equation reduces as

$$\infty-\infty=s+ik$$

Since s, k are any real constants obtained from integration the right hand side represents a constant complex quantity this can also be any real or any kind of constant quantity in the set of real numbers. So let us denote that as $s+ik=z$, This results in,

$$\infty-\infty=z$$

where z is any complex number. The above complex number z is a complex number represented in the general form $z=s+ik$, where s, k are the real constants now if we consider a complex function defined as $f\,(z)=z=s+ik$ is a complex function in the complex set. The defined function $f\,(z)=z$ is a complex valued function defined in the complex set and $f:C\to C$, where C is a complex space. If we consider any set A such that $A\subset C$ then we can easily note that the function $f\,(z)=z$

is differentiable at any point in the space. The function $f(z) = z$ also consider $z = s + ik$, then according to the Laplace equation if the function $f(z) = z$ is called analytic then it must satisfy the Laplace equation given as

$$f_{xx} + f_{yy} = 0$$

As the complex function is a constant complex number differentiating the function we always get zero and double differentiating the function with respect to any independent variable is always zero. So the function will always satisfy the Laplace equation. Thus, we can conclude that the function is Holomorphic in the complex space and also the function is analytic as it also satisfies the Laplace equation.

4. Small Mathematical Illustration of the Proof

Consider the sum of natural numbers as,

$$1 + 2 + 3 + \cdots + n + \cdots = S$$

Consider the grouping and rearranging of the sums

$$(1 + 3 + 5 + \dots) + (2 + 4 + 6 + \dots) = S$$

Taking '2' as common from the even numbers

$$(1 + 3 + 5 + \dots) + 2(1 + 2 + 3 + \dots) = S$$

Now again by observing the above equation we can write something like

$$\begin{aligned} 1 + 3 + 5 + 7 + 9 + \cdots + 2S &= S \\ 1 + 3 + 5 + 7 + 9 + \dots &= S - 2S \\ 1 + 3 + 5 + 7 + 9 + \dots &= -S \end{aligned}$$

Also, the sum of even natural numbers is

$$2 + 4 + 6 + 8 + \cdots = 2(1 + 2 + 3 + 4 + \dots)$$

This is again 'S' so the sum of even natural numbers is

$$2 + 4 + 6 + 8 + \cdots = 2S$$

Since the sum of natural numbers is a diverging sequence the sum must be diverging to infinity. So,

$$1 + 2 + 3 + 4 + \cdots + n + \cdots = \infty$$

And

$$1 + 3 + 5 + 7 + 9 + \dots = -\infty$$

$$2+4+6+8+\ldots=2(\infty)$$

$$2+4+6+8+\ldots=\infty$$

Adding all the odd numbered sequences and even numbered sequences we can get

$$(1+3+5+\ldots)+(2+4+6+8+\ldots)=\infty-\infty$$

Now concluding again that the sum of the sequence on the left hand side is something again like,'$\infty+k$', it can be written as

$$\infty+k=\infty-\infty$$

Now again sending the infinity on the left hand side to the right hand side we get

$$k=\infty-\infty-\infty$$

Now taking minus as common from the infinities we get something like

$$\infty-\infty+\infty=k$$

Since sum of infinities is again infinity we get

$$\infty-\infty=k$$

5. Conclusion

This paper is concentrated on one of the unsolved mathematical problem called Reimann - paradox, which explains if an event is fulfilled, there is a possibility of adding additional objects to that event. Derivation of this paradox proves the statement by using some simple techniques of mathematics such as Gaussian Integral, etc. Mathematically $\infty-\infty=z$ is proved where 'z' is any complex number.

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