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Functions Associated with ${}^*g\hat{\alpha}$-Closed Sets in Topological Spaces

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Abstract: In this paper, we introduce ${}^*g\hat{\alpha}$-continuous maps and study their relation with various generalized continuous maps. We also discuss certain properties of ${}^*g\hat{\alpha}$-continuous maps. We then introduce ${}^*g\hat{\alpha}$-irresolute maps, strongly ${}^*g\hat{\alpha}$-continuous maps and perfectly ${}^*g\hat{\alpha}$-continuous maps in topological spaces and some properties of these maps are discussed.

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1. Introduction

In 1991 Balachandran [5] introduced and studied g-continuity. Levine [17], Noiri [24] have respectively introduced and studied strong continuous maps and perfectly continuous maps, which are strong form of continuous maps. Devi [10] Ganster and Reilly [15] have introduced $g\alpha$-continuity, αg-continuity, which are weaker than continuity. Recently various types of continuity have been introduced and studies by several topologists. Crossely and Hildebrand [6] introduced and investigated irresolute maps which are stronger than semi-continuous maps but are independent of continuous maps. Since then various forms of strong and weak irresolute maps have been introduced and investigated by many researchers. In our present work, we introduce ${}^*g\hat{\alpha}$-continuous maps and study their relation with various generalized continuous maps. We also investigate certain properties of ${}^*g\hat{\alpha}$-continuous maps. We then introduce ${}^*g\hat{\alpha}$-irresolute maps, strongly ${}^*g\hat{\alpha}$-continuous maps and perfectly ${}^*g\hat{\alpha}$-continuous maps in topological spaces and some properties of these maps are discussed.

2. Preliminaries

Throughout this paper, (X, τ) or X denotes the topological spaces. For a subset A of X, the closure, the interior and the complement of A are denoted by $cl(A)$, $int(A)$ and A^c respectively. We recall some basic definitions that are used in the sequel.

Definition 2.1. *A subset A of a topological space (X, τ) is called*

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(1). α-open [22] if $A \subseteq int(cl(int(A)))$,

(2). semi-open [18] if $A \subseteq cl(int(A))$,

(3). preopen [19] if $A \subseteq int(cl(A))$,

(4). semi-preopen [1] if $A \subseteq cl(int(cl(A)))$.

Moreover, A is said to be α-closed (res. semi-closed, preclosed and semi-preclosed) if $X \backslash A$ is α-open (res. semi-open, preopen and semi-preopen). The collection of all α-open subsets in (X, τ) is denoted by τ^α. The α-closure (res. semi closure, preclosure and semi-preclosure) of a subset A is the smallest α-closed (res. semi-closed, preclosed and semi-preclosed) set containing A and this is denoted by τ^α-$cl(A)$ (res. $scl(A)$, $pcl(A)$ and $spcl(A)$) in the present paper.

Definition 2.2. *A subset A of a topological space (X, τ) is called*

(1). g-closed [16] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open set in (X, τ),

(2). αg-closed [21] if τ^α-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open set in (X, τ),

(3). gs-closed [2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open set in (X, τ),

(4). g^*-closed [26] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a g-open set in (X, τ),

(5). $^*g\alpha$-closed [23] if τ^α-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an g-open set in (X, τ),

(6). gp-closed [4] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open set in (X, τ),

(7). gsp-closed [11] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open set in (X, τ),

(8). gpr-closed [13] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is a regular open set in (X, τ),

(9). $g\alpha$-closed [20] if τ^α-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an α-open set in (X, τ).

Moreover, A is said to be g-open (res. αg-open, gs-open, g^*-open, $^*g\alpha$-open, gp-open, gsp-open, gpr-open and $g\alpha$-open) if $X \backslash A$ is g-closed (res. αg-closed, gs-closed, g^*-closed, $^*g\alpha$-closed, gp-closed, gsp-closed, gpr-closed and $g\alpha$-closed).

Definition 2.3. *A map $f : (X, \tau) \to (Y, \sigma)$ is called*

(1). g-continuous [17] if $f^{-1}(A)$ is g-closed in (X, τ) for every closed set A in (Y, σ).

(2). pre continuous [5] if $f^{-1}(A)$ is pre closed in (X, τ) for every closed set A in (Y, σ).

(3). semi continuous [5] if $f^{-1}(A)$ is semi closed in (X, τ) for every closed set A in (Y, σ).

(4). αg-continuous [8] if $f^{-1}(A)$ is αg-closed in (X, τ) for every closed set A in (Y, σ).

(5). gs-continuous [7] if $f^{-1}(A)$ is gs-closed in (X, τ) for every closed set A in (Y, σ).

(6). g^*-continuous [26] if $f^{-1}(A)$ is g^*-closed in (X, τ) for every closed set A in (Y, σ).

(7). $^*g\alpha$-continuous [23] if $f^{-1}(A)$ is $^*g\alpha$-closed in (X, τ) for every closed set A in (Y, σ).

(8). gp-continuous [3] if $f^{-1}(A)$ is gp-closed in (X, τ) for every closed set A in (Y, σ).

(9). gsp-continuous [11] if $f^{-1}(A)$ is gsp-closed in (X, τ) for every closed set A in (Y, σ).

(10). gpr-continuous [13] if $f^{-1}(A)$ is gpr-closed in (X,τ) for every closed set A in (Y,σ).

(11). $g\alpha$-continuous[8] if $f^{-1}(A)$ is $g\alpha$-closed in (X,τ) for every closed set A in (Y,σ).

Definition 2.4 ([25]). *A subset A of a topological space (X,τ) is called a $^*g\hat{\alpha}$-closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a $^*g\alpha$-open set in (X,τ). The class of $^*g\hat{\alpha}$-closed subsets of (X,τ) is denoted by $^*g\hat{\alpha}C(X,\tau)$.*

3. $^*g\hat{\alpha}$-Continuous Mapping

In this section, we introduce $^*g\hat{\alpha}$-Continuous Mapping and analyze its relationship between various generalised continuous mappings in topological spaces. We also investigate the characterizations of $^*g\hat{\alpha}$-Continuous Mapping in topological spaces.

Definition 3.1. *A map $f : (X,\tau) \to (Y,\sigma)$ is called $^*g\hat{\alpha}$-continuous if $f^{-1}(A)$ is $^*g\hat{\alpha}$-closed in (X,τ) for every closed set A in (Y,σ).*

Proposition 3.2. *Every continuous map is $^*g\hat{\alpha}$-continuous, but not conversely.*

Proof. Let $f : (X,\tau) \to (Y,\sigma)$ be a continuous map. Then for each closed subset A of Y, its pre image $f^{-1}(A)$ is closed in X. Since every closed set is $^*g\hat{\alpha}$-closed, we have $f^{-1}(A)$ is $^*g\hat{\alpha}$-closed in X. Thus f is $^*g\hat{\alpha}$-continuous. □

Example 3.3. *Let $X = \{a,b,c\}$, $\tau = \{\emptyset, \{a\}, \{b,c\}, X$, $Y = \{p,q\}$ and σ be the discrete topology on Y. Define $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = f(b) = p$ and $f(c) = q$, then the function f is $^*g\hat{\alpha}$-continuous but not continuous, because for the open set $A = \{q\}$ in (Y,σ), $f^{-1}(A) = \{c\}$ is not open in (X,τ). Thus the class of all $^*g\hat{\alpha}$-continuous maps properly contains the class of all continuous maps.*

In the next propositions we show that the class of all $^*g\hat{\alpha}$-continuous maps is properly contained in the classes of various generalized continuous maps in topological spaces.

Proposition 3.4.

(1). Every $^*g\hat{\alpha}$-continuous map is g-continuous, but not conversely.

(2). Every $^*g\hat{\alpha}$-continuous map is gs-continuous, but not conversely.

(3). Every $^*g\hat{\alpha}$-continuous map is gp-continuous, but not conversely.

(4). Every $^*g\hat{\alpha}$-continuous map is gsp-continuous, but not conversely.

(5). Every $^*g\hat{\alpha}$-continuous map is gpr-continuous, but not conversely.

(6). Every $^*g\hat{\alpha}$-continuous map is αg-continuous, but not conversely.

Proof. Let $f : (X,\tau) \to (Y,\sigma)$ be a $^*g\hat{\alpha}$- continuous map. Then for each closed subset V of Y, its pre image $f^{-1}(V)$ is $^*g\hat{\alpha}$-closed in X. Since every $^*g\hat{\alpha}$-closed set is g-closed (res. gs, gp, gsp, gpr and αg-closed), we have $f^{-1}(A)$ is g-closed (res. gs, gp, gsp, gpr and αg-closed) in X. Thus f is g-continuous (res. gs, gp, gsp, gpr and αg-continuous). □

Example 3.5.

(1). Let $X = \{a,b,c\}$,$Y = \{a,b\}$, $\tau = \{\emptyset, X, \{a\}$ and σ be the discrete topology on Y. Define $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = f(b) = a$ and $f(c) = b$. Then f is g continuous but not $^*g\hat{\alpha}$-continuous.

(2). Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{b\}, \{a, b\}\}$ and $\sigma = \{\emptyset, X, \{a, c\}, \{c\}\}$. Define $f : (X, \tau) \to (Y, \sigma)$ be the identity map. Then f is αg- continuous and gs-continuous but it is not ${}^*g\hat{\alpha}$-continuous.

(3). Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a, b\}\}$ and $\sigma = \{\emptyset, X, \{a, c\}, \{c\}\}$. Define $f : (X, \tau) \to (Y, \sigma)$ be the identity map. Then f is gp- continuous and gsp-continuous but it is not ${}^*g\hat{\alpha}$-continuous.

(4). Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$ and $\sigma = \{\emptyset, X, \{a, c\}\}$. Define $f : (X, \tau) \to (Y, \sigma)$ be the identity map. Then f is gpr-continuous but not ${}^*g\hat{\alpha}$-continuous.

Remark 3.6. *The following examples shows that ${}^*g\hat{\alpha}$-continuity is independent of pre continuity, semi continuity, $g\alpha$-continuity and α-continuity.*

Example 3.7. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, c\}\}$ and $\sigma = \{\emptyset, Y, \{a\}\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c, f(b) = b$ and $f(c) = a$. Then f is ${}^*g\hat{\alpha}$-continuous but neither semi continuous nor α-continuous.*

Example 3.8. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{a, b\}, \{a, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity map . Then f is $g\alpha$- continuous, pre continuous, α-continuous and semi continuous but not ${}^*g\hat{\alpha}$-continuous.*

Example 3.9. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\{a, c\}\}$ and $Y = \{p, q\}$, $\sigma = \{\emptyset, Y, \{p\}\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = f(b) = p$ and $f(c) = q$. Then f is ${}^*g\hat{\alpha}$-continuous but neither $g\alpha$- continuous nor pre continuous.*

Proposition 3.10. *A map $f : (X, \tau) \to (Y, \sigma)$ is ${}^*g\hat{\alpha}$-continuous if and only if $f^{-1}(A)$ is ${}^*g\hat{\alpha}$-open in (X, τ) for every open set A in (Y, σ).*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be ${}^*g\hat{\alpha}$-continuous and A be an open set in (Y, σ). Then A^c is closed in (Y, σ) and since f is ${}^*g\hat{\alpha}$-continuous, $f^{-1}(A^c)$ is ${}^*g\hat{\alpha}$-closed in (X, τ). But $f^{-1}(A^c) = (f^{-1}(A))^c$ and so $f^{-1}(A)$ is ${}^*g\hat{\alpha}$- open in (X, τ). Conversely, assume that $f^{-1}(A)$ is ${}^*g\hat{\alpha}$-open in (X, τ) for each open set A in (Y, σ). Let B be a closed set in (Y, σ). Then B^c is open in (Y, σ) and by assumption, $f^{-1}(B^c)$ is ${}^*g\hat{\alpha}$-open in (X, τ). Since $f^{-1}(B^c) = (f^{-1}(B^c))$, we have $f^{-1}(B)$ is closed in (X, τ) and so f is ${}^*g\hat{\alpha}$-continuous. □

Proposition 3.11. *The composition of two ${}^*g\hat{\alpha}$-continuous maps need not be ${}^*g\hat{\alpha}$-continuous and this is shown by following example.*

Example 3.12. *Let $X = Y = Z = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a, b\}\}$, $\sigma = \{\emptyset, Y, \{a\}, \{b, c\}\}$ and $\eta = \{\emptyset, Z, \{a, b\}\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = f(b) = a$, $f(c) = c$ and $g : (Y, \sigma) \to (Z, \eta)$ be the identity map. Then, the set $A = \{c\}$ is closed in (Z, η) but $(g \circ f)^{-1}(A) = \{c\}$, which is not ${}^*g\hat{\alpha}$-continuous.*

Proposition 3.13. *Let (X, τ) and (Z, η) be any two topological spaces and (Y, σ) be a $T_{1/2}$-space. Then the composition $g \circ f : (X, \tau) \to (Z, \eta)$ of the ${}^*g\hat{\alpha}$ - continuous map $f : (X, \tau) \to (Y, \sigma)$ and the g-continuous (resp. αg-continuous and gs-continuous) map $g : (Y, \sigma) \to (Z, \eta)$ is ${}^*g\hat{\alpha}$-continuous.*

Proof. Let A be any closed set of (Z, η). Then $g^{-1}(A)$ is closed in (Y, σ), since g is g-continuous and (Y, σ) be a $T_{1/2}$-space. Since $g^{-1}(A)$ is closed in (Y, σ) and f is ${}^*g\hat{\alpha}$-continuous, $f^{-1}(g^{-1}(A))$ is ${}^*g\hat{\alpha}$-closed in (X, τ). But $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ and so $g \circ f$ is ${}^*g\hat{\alpha}$-continuous. □

Proposition 3.14. *If $f : (X, \tau) \to (Y, \sigma)$ is ${}^*g\hat{\alpha}$-continuous and $g : (Y, \sigma) \to (Z, \eta)$ is continuous, then their composition $g \circ f : (X, \tau) \to (Z, \eta)$ is ${}^*g\hat{\alpha}$-continuous.*

Proof. Let A be any closed set in (Z, η). Since $g : (Y, \sigma) \to (Z, \eta)$ is continuous, $g^{-1}(A)$ is closed in (Y, σ). Since $f : (X, \tau) \to (Y, \sigma)$ is ${}^*g\hat{\alpha}$-continuous, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is ${}^*g\hat{\alpha}$-closed in (X, τ) and so $g \circ f$ is ${}^*g\hat{\alpha}$-continuous. □

We have the following proposition for the restriction of a $^*g\hat{\alpha}$-continuous map.

Proposition 3.15. *Let $f:(X,\tau)\to(Y,\sigma)$ be a $^*g\hat{\alpha}$-continuous map and let H be a $^*g\hat{\alpha}$-closed subset of (X,τ). Then the restriction $f_H:(H,\tau_H)\to(Y,\sigma)$ is $^*g\hat{\alpha}$-continuous.*

Proof. Let A be a closed set in (Y,σ). Since f is $^*g\hat{\alpha}$-continuous, $f^{-1}(A)$ is $^*g\hat{\alpha}$-closed in (X,τ). Let $f^{-1}(A)\cap H=H_1$. Then H_1 is $^*g\hat{\alpha}$-closed in (X,τ) ($\because$ the intersection of two $^*g\hat{\alpha}$-closed set is $^*g\hat{\alpha}$-closed). Since $(f_H)^{-1}(A)=f^{-1}(A)\cap H=H_1$. Claim: H_1 is $^*g\hat{\alpha}$-closed in (H,τ_H).
Proof: Let U be any $*g\alpha$-open set of (H,τ_H) such that $H_1\subseteq U$. Since U is $*g\alpha$-open in (H,τ_H), $U=B\cap H$ for some $*g\alpha$ set B in (X,τ). Now, $H_1\subseteq B\cap H$ so $H_1\subseteq B$. Since H_1 is $^*g\hat{\alpha}$-closed in (X,τ), $cl(H_1)\subseteq B$. We have $cl_H(H_1)=cl(H_1)\cap H\subseteq B\cap H=U$ and hence H_1 is $^*g\hat{\alpha}$-closed in (H,τ_H) and f_H is $^*g\hat{\alpha}$-continuous. □

Definition 3.16. *Let $\{A_i:i\in\wedge\}$ be a given cover set of A and $\{f_i\};i\in\wedge$ be a family of maps $f_i:A_i\to B$. We say that the maps are compatible if for every pair $i,j\in\wedge$, we have $f_{i-(A_i\cap A_j)}=f_{j-(A_i\cap A_j)}$. Then we define a map $\bigtriangledown f:A\to B$ as $(\bigtriangledown f)(x)=f_i(x)$ for every $x\in A_i$. This is called the combination of the maps $\{f_i\}_{i\in\wedge}$ which is obtained by pasting $\{f_i\}_{i\in\wedge}$ together with their common domains. If $\wedge=\{1,2,...,k\}$, then this is denoted by $f_1\bigtriangledown f_2\bigtriangledown\cdots\bigtriangledown f_k$.*

Definition 3.17. *Let x be a point of (X,τ) and N be a subset of (X,τ). Then N is called a $^*g\hat{\alpha}$-neighborhood of x in (X,τ), if there exists an $^*g\hat{\alpha}$-open set U of (X,τ) such that $x\in U\subseteq N$.*

Proposition 3.18. *Let A be a subset of (X,τ). Then $x\in^* g\hat{\alpha}-cl(A)$ if and only if for any $^*g\hat{\alpha}$-neighborhood N_x of x in (X,τ), $A\cap N_x\neq\emptyset$.*

Proof. Nessesity : Assume $x\in^* g\hat{\alpha}-cl(A)$. Suppose that there is a $^*g\hat{\alpha}$-neighborhood N of the point x in (X,τ) such that $N\cap A=\emptyset$. Since N is a $^*g\hat{\alpha}$-neighborhood of x in (X,τ), by the above definition, there exists an $^*g\hat{\alpha}$-open set U_x such that $x\in U_x\subseteq W$. Therefore, we have $U_x\cap A=\emptyset$ and so $A\subseteq(U_x)^c$. Since $(U_x)^c$ is $^*g\hat{\alpha}$-closed set containing A and hence $^*g\hat{\alpha}-cl(A)\subseteq(U_x)^c$. Therefore $x\notin^* g\hat{\alpha}-cl(A)$, which is a contradiction.
Sufficiency : Assume for each $^*g\hat{\alpha}$-neighborhood N_x of x in (X,τ), $A\cap N_x\neq\emptyset$. Suppose that $x\notin^* g\hat{\alpha}-cl(A)$. Then there exists a $^*g\hat{\alpha}$-closed set B of (X,τ) such that $A\subseteq B$ and $x\notin B$. Hence $x\in B^c$ and B^c is $^*g\hat{\alpha}$-open in (X,τ) and B^c is a $^*g\hat{\alpha}$-neighborhood of x in (X,τ). But $A\cap B^c=\emptyset$, whish is a contradiction. □

Theorem 3.19 (Characterization of $^*g\hat{\alpha}$-continuous functions)**.** *Let $f:(X,\tau)\to(Y,\sigma)$ be a map between two topological spaces (X,τ) and (Y,σ). Then the following statements are equivalent:*

(1). The function f is $^*g\hat{\alpha}$-continuous‘

(2). The inverse of each open set is $^*g\hat{\alpha}$-open.

(3). For each point x in (X,τ) and each open set A in (Y,σ) with $f(x)\in A$, there is a $^*g\hat{\alpha}$-open set B in (X,τ) such that $x\in B, f(B)\subseteq A$.

(4). The inversee of each closed set is $^*g\hat{\alpha}$-closed.

(5). For each x in (X,τ), the inverse of every neighborhood of $f(x)$ is a $^*g\hat{\alpha}$-neighborhood of x.

(6). For each x in (X,τ) and each neighborhood N of $f(x)$, there is a $^*g\hat{\alpha}$-neighborhood M of x such that $f(M)\subseteq N$.

(7). For each subset W of (X,τ), $f(^*g\hat{\alpha}-cl(W))\subseteq cl(f(W))$.

(8). For each subset D of (Y,σ), $^*g\hat{\alpha}-cl(f^{-1}(D))\subseteq f^{-1}(cl(D))$.

Proof. (1) $\iff$ (2): This follows from Proposition 3.10

(1) $\iff$ (3): Suppose that (3) holds and let A be an open set in (Y,σ) and let $x \in f^{-1}(A)$. Then $f(x) \in A$ and there exists a $^*g\hat{\alpha}$-open set B_x such that $x \in B_x$ and $f(B_x) \subseteq A$. Now, $x \in B_x \subseteq f^{-1}(A)$ and $f^{-1}(A)$ is $^*g\hat{\alpha}$-open in (X,τ) and hence f is $^*g\hat{\alpha}$-continuous.

Conversely, suppose that (1) holds and let $f(x) \in A$. Then $x \in f^{-1}(V)$, since f is $^*g\hat{\alpha}$-continuous. Let $B = f^{-1}(A)$. Then $x \in B$ and $f(B) \subseteq A$.

(2) $\iff$ (4): This one follows from the fact that if V is a subset of (Y,σ), then $f^{-1}(V^c) = (f^{-1}(V))^c$.

(2) $\iff$ (5) : For x in (X,τ), let W be a neighborhood of $f(x)$. Then there exists an open set U in (Y,σ) such that $f(x) \in U \subseteq W$. Consequently, $f^{-1}(U)$ is a $^*g\hat{\alpha}$-open set in (X,τ) and $x \in f^{-1}(U) \subseteq f^{-1}(W)$. Thus $f^{-1}(W)$ is a $^*g\hat{\alpha}$-neighborhood of x.

(5) $\iff$ (6): Let $x \in X$ and let N be a neighborhood of $f(x)$. Then by assumption, $M = f^{-1}(N)$ is a $*g\hat{\alpha}$-neighborhood of x and $f(M) = f(f^{-1}(N)) \subseteq N$.

(6) $\iff$ (3): For x in (X,τ), let N be an open set containing $f(x)$. Then N is a neighborhood of $f(x)$. So by assumption, there exists a $^*g\hat{\alpha}$-neighborhood M of x such that $f(M) \subseteq N$. Thus there exists a $^*g\hat{\alpha}$-open set U in (X,τ) such that $x \in U \subseteq B$ and so $f(U) \subseteq f(M) \subseteq N$.

(7) $\iff$ (4): Suppose (4) holds and let W be a subset of (X,τ). Since $W \subseteq f^{-1}(f(W))$, we have $W \subseteq f^{-1}(cl(f(W)))$. Since $cl(f(W))$ is a closed set in (Y,σ), $f^{-1}(cl(f(W)))$ is a $^*g\hat{\alpha}$-closed set containing W. Consequently, $^*g\hat{\alpha} - cl(W) \subseteq f^{-1}(cl(f(W)))$. Hence $f(^*g\hat{\alpha} - cl(W)) \subseteq f(f^{-1}(cl(f(W)))) \subseteq cl(f(W))$.

Conversely, suppose that (7) holds for any subset W of (X,τ). Let C be a closed subset of (Y,σ). Then $f(^*g\hat{\alpha}-cl(f^{-1}(C))) \subseteq cl(f(f^{-1}(C))) \subseteq cl(C) = C$. That is $^*g\hat{\alpha} - cl(f^{-1}(C)) \subseteq f^{-1}(C)$ and so $f^{-1}(C)$ is $^*g\hat{\alpha}$-closed.

(7) $\iff$ (8): Assume (8) holds and D be any subset of (Y,σ) . Then by replacing W by $f^{-1}(D)$ in (7), we obtain $f(^*g\hat{\alpha} - cl(D)) \subseteq cl(f(f^{-1}(D))) \subseteq cl(D)$. That is $^*g\hat{\alpha} - cl(f^{-1}(D)) \subseteq f^{-1}(cl(D))$.

Conversely, assume that (8) holds. Let $D = f(W)$, where W is a subset of (X,τ). Then $^*g\hat{\alpha} - cl(W) \subseteq^* g\hat{\alpha} - cl(f^{-1}(D)) \subseteq f^{-1}(cl(f(W)))$ and $f(^*g\hat{\alpha} - cl(W)) \subseteq cl(f(W))$. $\square$

4. $^*g\hat{\alpha}$-irresolute Maps, Strongly $^*g\hat{\alpha}$-continuous Maps and Perfectly $^*g\hat{\alpha}$-continuous Maps

In this section, we introduce $^*g\hat{\alpha}$-irresolute maps, strongly $^*g\hat{\alpha}$-continuous maps and perfectly $^*g\hat{\alpha}$-continuous maps in topological spaces and we discuss some properties of these maps.

Definition 4.1. *A map $f : (X,\tau) \to (Y,\sigma)$ is called a $^*g\hat{\alpha}$-irresolute map if the inverse image of every $^*g\hat{\alpha}$-closed set in (Y,σ) is $^*g\hat{\alpha}$ in (X,τ).*

Proposition 4.2. *A function $f : (X,\tau) \to (Y,\sigma)$ is $^*g\hat{\alpha}$-irresolute if and only if the inverse image of every $^*g\hat{\alpha}$-open set in (Y,σ) is $^*g\hat{\alpha}$-open in (X,τ).*

Proof. Let A be any $^*g\hat{\alpha}$-open set in Y. Then A^c is $^*g\hat{\alpha}$-closed in Y. Since f is $^*g\hat{\alpha}$-irresolute, $f^{-1}(A^c)$ is $^*g\hat{\alpha}$-closed in X. But $f^{-1}(A^c) = X \setminus f^{-1}(A)$ and so $f^{-1}(A)$ is $^*g\hat{\alpha}$-open in X.

Conversely, assume that the inverse image of every $^*g\hat{\alpha}$-open set in Y is $^*g\hat{\alpha}$-open in X. Let B $^*g\hat{\alpha}$-closed set in Y. Then B^c is $^*g\hat{\alpha}$-open in Y. By assumption, $f^{-1}(B^c)$ is $^*g\hat{\alpha}$-open in X. But $f^{-1}(B^c) = X \setminus f^{-1}(B)$, $f^{-1}(B)$ is $^*g\hat{\alpha}$-closed in X. Thus f is $^*g\hat{\alpha}$-irresolute. $\square$

Proposition 4.3. *A function $f : (X,\tau) \to (Y,\sigma)$ is $^*g\hat{\alpha}$-irresolute then it is $^*g\hat{\alpha}$-continuous but not conversely.*

Proof. Assume f is $^*g\hat{\alpha}$-irresolute. Let V be a closed set in Y. Since every closed set is $^*g\hat{\alpha}$-closed , V is $^*g\hat{\alpha}$-closed. By assumption f is $^*g\hat{\alpha}$-irresolute, $f^{-1}(V)$ is $^*g\hat{\alpha}$-closed in X. Hence f is $^*g\hat{\alpha}$-continuous. □

Example 4.4. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{a, b\}\}$. Then the identity map on X is $^*g\hat{\alpha}$-continuous but not $^*g\hat{\alpha}$-irresolute, since the inverse image of the $^*g\hat{\alpha}$-closed in (Y, σ) is not $^*g\hat{\alpha}$-closed in (X, τ).*

Proposition 4.5. *Let $f : (X, \tau) \to (Y, \sigma)$ is $^*g\hat{\alpha}$-irresolute and H is $^*g\hat{\alpha}$-closed subset of (X, τ). Then the restriction $f_H : (H, \tau_H) \to (Y, \sigma)$ is $^*g\hat{\alpha}$-irresolute.*

Proof. Let A be any $^*g\hat{\alpha}$-closed subset of (Y, σ). Since f is $^*g\hat{\alpha}$-irresolute, $f^{-1}(A)$ is $^*g\hat{\alpha}$-closed in (X, τ). Define $f^{-1}(A) \cap H = H_1$. Then H_1 is $^*g\hat{\alpha}$-closed in (H, τ_H). But $(f_H)^{-1}(A) = f^{-1}(A) \cap H = H_1$ and hence f_H is $^*g\hat{\alpha}$-irresolute. □

Proposition 4.6. *If $f : (X, \tau) \to (Y, \sigma)$ is bijective, closed then the inverse map $f^{-1} : (Y, \sigma) \to (X, \tau)$ is $^*g\hat{\alpha}$-irresolute.*

Proof. Let A be a $^*g\hat{\alpha}$-closed set in (X, τ). Let $(f^{-1})^{-1}(A) = f(A) \subseteq U$ where U is $g\alpha$ open in (Y, σ). Then $A \subseteq f^{-1}(U)$ holds. Since $f^{-1}(U)$ is $g\alpha$ open in (X, τ) and A is $^*g\hat{\alpha}$-closed in (X, τ), $cl(A) \subseteq f^{-1}(U)$ and hence $f(cl(A)) \subseteq U$. Since f is closed and $cl(A)$ is closed in (X, τ), $f(cl(A))$ is closed in (Y, σ) and so $f(cl(A))$ is $^*g\hat{\alpha}$-closed in (Y, σ). Therefore $cl(f(cl(A))) \subseteq U$ and hence $cl(f(A)) \subseteq U$. Thus $f(A)$ is $^*g\hat{\alpha}$-closed in (Y, σ) and so f^{-1} is $^*g\hat{\alpha}$-irresolute. □

Definition 4.7. *A function $f : (X, \tau) \to (Y, \sigma)$ is called strongly $^*g\hat{\alpha}$-continuous if the inverse image of every $^*g\hat{\alpha}$-open set in (Y, σ) is open in (X, τ).*

Proposition 4.8. *If $f : (X, \tau) \to (Y, \sigma)$ is strongly $^*g\hat{\alpha}$-continuous, then it is continuous but not conversely.*

Proof. Since the function f is strongly $^*g\hat{\alpha}$-continuous, $f^{-1}(A)$ is open in (X, τ), for every $^*g\hat{\alpha}$-open set A in (Y, σ). As every open set is $^*g\hat{\alpha}$-open we have f is continuous. □

Example 4.9. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{a, b\}, Y\}$. Define a map $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = f(b) = a$ and $f(c) = b$. Then f is continuous but not f is not strongly $^*g\hat{\alpha}$-continuous, as the $^*g\hat{\alpha}$-open set in (Y, σ) is not open in (X, τ).*

Proposition 4.10. *If $f : (X, \tau) \to (Y, \sigma)$ is strongly continuous then it is strongly $^*g\hat{\alpha}$-continuous but not conversely.*

Proof. Since the function f is strongly continuous, $f^{-1}(A)$ is open in (X, τ), for every open set A in (Y, σ). As every open set is $^*g\hat{\alpha}$-open we have f is strongly $^*g\hat{\alpha}$-continuous. □

Example 4.11. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{a, b\}\}$. Define $f : (X, \tau) \to (Y, \sigma)$ be the identity map. Then f is strongly $^*g\hat{\alpha}$-continuous, but not strongly continuous.*

Definition 4.12. *A topological space (X, τ) is called a $^*g\hat{\alpha}$-space if every subset in it is called $^*g\hat{\alpha}$-closed.*

Proposition 4.13. *Let (X, τ) be a discrete topological space, (Y, σ) be a $^*g\hat{\alpha}$-space and $f : (X, \tau) \to (Y, \sigma)$ be a map. Then the following are equivalent:*

(1). f is strongly continuous

(2). f is strongly $^*g\hat{\alpha}$-continuous.

Proof. (1) $\implies$ (2): Follows from Proposition 4.10.
(2) $\implies$ (1): Let A be $^*g\hat{\alpha}$-open set in (Y, σ). Since (Y, σ) is a $^*g\hat{\alpha}$-space, A is $^*g\hat{\alpha}$-open subset of (Y, σ) and by given hypothesis, $f^{-1}(A)$ is open in (X, τ). Since (X, τ) is a discrete topological space, $f^{-1}(A)$ is closed in (X, τ). That is, $f^{-1}(A)$ is both open and closed in (X, τ) and hence f is strongly continuous. □

Proposition 4.14. *A function $f : (X, \tau) \to (Y, \sigma)$ is strongly $^*g\hat{\alpha}$-continuous if and only if the inverse image of every $^*g\hat{\alpha}$-closed set in (Y, σ) is closed in (X, τ).*

Proof. Let A be a $^*g\hat{\alpha}$-closed set in (Y, σ). Then A^c is $^*g\hat{\alpha}$-open in (Y, σ) and since f is strongly $^*g\hat{\alpha}$-continuous, $f^{-1}(A^c)$ is open in (X, τ). But $f^{-1}(A^c) = (f^{-1}(A))^c$ and so $f^{-1}(A)$ is $^*g\hat{\alpha}$-closed in (X, τ).
Conversely, assume that $f^{-1}(A)$ is $^*g\hat{\alpha}$-closed in (X, τ) for each $^*g\hat{\alpha}$-open set A in (Y, σ). Let B be a $^*g\hat{\alpha}$-closed set in (Y, σ). Then B^c is $^*g\hat{\alpha}$-open in (Y, σ) and by assumption, $f^{-1}(B^c)$ is open in (X, τ). Since $f^{-1}(B^c) = (f^{-1}(B^c))$, we have $f^{-1}(B)$ is closed in (X, τ) and so f is strongly $^*g\hat{\alpha}$-continuous. □

Proposition 4.15. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be two strong $^*g\hat{\alpha}$-continuous maps. Then their composition $g \circ f : (X, \tau) \to (Z, \eta)$ is strongly $^*g\hat{\alpha}$-continuous.*

Proof. Let A be a $^*g\hat{\alpha}$-open set in (Z, η). Since g is strongly $^*g\hat{\alpha}$-continuous, $g^{-1}(A)$ is open in (Y, σ). Since $g^{-1}(A)$ is open, it is $^*g\hat{\alpha}$-open in (Y, σ). As f is also strongly $^*g\hat{\alpha}$-continuous, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is open in (X, τ) and $g \circ f$ is strongly $^*g\hat{\alpha}$-continuous. □

Proposition 4.16. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be any two functions. Then their composition $g \circ f : (X, \tau) \to (Z, \eta)$ is*

(1). strongly $^*g\hat{\alpha}$-continuous if g is strongly $^*g\hat{\alpha}$-continuous and f is continuous.

(2). $^*g\hat{\alpha}$ -irresolute if g is strongly $^*g\hat{\alpha}$-continuous and f is $^*g\hat{\alpha}$-continuous (f is $^*g\hat{\alpha}$-irresolute).

(3). strongly $^*g\hat{\alpha}$-continuous if g is strongly continuous and f is $^*g\hat{\alpha}$-irresolute.

(4). continuous if g is $^*g\hat{\alpha}$-continuous and f is strongly $^*g\hat{\alpha}$-continuous.

Proof.

(1). Let A be a $^*g\hat{\alpha}$-open set in (Z, η). Since $g : (Y, \sigma) \to (Z, \eta)$ is strongly $^*g\hat{\alpha}$-continuous, $g^{-1}(A)$ is open in (Y, σ). Since $f : (X, \tau) \to (Y, \sigma)$ is continuous, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is open in (X, τ) and so $g \circ f$ is strongly $^*g\hat{\alpha}$-continuous.

(2). Let A be a $^*g\hat{\alpha}$-open set in (Z, η). Since $g : (Y, \sigma) \to (Z, \eta)$ is strongly $^*g\hat{\alpha}$-continuous, $g^{-1}(A)$ is open in (Y, σ). Since $f : (X, \tau) \to (Y, \sigma)$ is $^*g\hat{\alpha}$-continuous, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is $^*g\hat{\alpha}$-open in (X, τ) and so $g \circ f$ is $^*g\hat{\alpha}$-irresolute.

(3). Similar to the proof of 2.

(4). Let A be an open set (Z, η). Since $g : (Y, \sigma) \to (Z, \eta)$ is $^*g\hat{\alpha}$-continuous, $g^{-1}(A)$ is $^*g\hat{\alpha}$-open in (Y, σ). Since $f : (X, \tau) \to (Y, \sigma)$ is strongly $^*g\hat{\alpha}$-continuous, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is open in (X, τ) and so $g \circ f$ is continuous. □

Proposition 4.17. *Let $f : (X, \tau) \to (Y, \sigma)$ be strongly $^*g\hat{\alpha}$-continuous and H is any subset of (X, τ). Then the restriction map $f_H : (H, \tau_H) \to (Y, \sigma)$ is also strongly $^*g\hat{\alpha}$-continuous.*

Proof. Proof is similar to Proposition 4.5. □

Definition 4.18. *A function $f : (X, \tau) \to (Y, \sigma)$ is called perfectly $^*g\hat{\alpha}$-continuous, if the inverse image of every $^*g\hat{\alpha}$-open set in (Y, σ) is both open and closed in (X, τ).*

Proposition 4.19. *Let $f : (X, \tau) \to (Y, \sigma)$ is perfectly $^*g\hat{\alpha}$-continuous. Then it is strongly $^*g\hat{\alpha}$-continuous, but not conversely.*

Proof. Since the function f is perfectly $^*g\hat{\alpha}$-continuous, $f^{-1}(A)$ is both open and closed in (X,τ) for every $^*g\hat{\alpha}$-open set A in (Y,σ). Thus f is strongly $^*g\hat{\alpha}$-continuous. □

Example 4.20. *Let $X = \{a,b,c\} = Y$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$ and $\sigma = \{\emptyset, Y, \{a,b\}\}$. Define $f : (X,\tau) \to (Y,\sigma)$ be the identity map. Then f is strongly $^*g\hat{\alpha}$-continuous but f is not perfectly $^*g\hat{\alpha}$-continuous.*

Proposition 4.21. *Let $f : (X,\tau) \to (Y,\sigma)$ is strongly continuous. Then it is perfectly $^*g\hat{\alpha}$-continuous, but not conversely.*

Proof. Since the function f is strongly continuous, $f^{-1}(A)$ is both open and closed in (X,τ), for every $^*g\hat{\alpha}$-open set A in (Y,σ). Thus f is perfectly $^*g\hat{\alpha}$- continuous. □

Example 4.22. *Let $X = \{a,b,c\} = Y$, $\tau = \{\emptyset, X, \{a\}, \{b,c\}\}$ and $\sigma = \{\emptyset, Y, \{a\}$. Define $f : (X,\tau) \to (Y,\sigma)$ be the identity map. Then f is perfectly $^*g\hat{\alpha}$-continuous but f is strongly continuous.*

Proposition 4.23. *Let (X,τ) be a discrete topological spaace, (Y,σ) be any topological space and $f : (X,\tau) \to (Y,\sigma)$ be a map. Then the following are equivalent:*

(1). f is perfectly $^*g\hat{\alpha}$-continuous.

(2). f is strongly $^*g\hat{\alpha}$-continuous.

Proof. (1) $\Longrightarrow$ (2): Proof follows from Proposition 4.21.
(2) $\Longrightarrow$ (1): Let A be $^*g\hat{\alpha}$-open set in (Y,σ). By given condition, $f^{-1}(A)$ is closed in (X,τ). Hence f is perfectly $^*g\hat{\alpha}$-continuous. □

Proposition 4.24. *Let (X,τ) be a discrete topological space, (Y,σ) be $^*g\hat{\alpha}$-space and $f : (X,\tau) \to (Y,\sigma)$ be a map. Then the following are equivalent:*

(1). f is strongly continuous.

(2). f is strongly $^*g\hat{\alpha}$-continuous.

(3). f is perfectly $^*g\hat{\alpha}$-continuous.

Proof. This proof is a direct consequences of Proposition 4.13 and Proposition 4.23. □

Proposition 4.25. *A function $f : (X,\tau) \to (Y,\sigma)$ is perfectly $^*g\hat{\alpha}$-continuous if and only if the inverse image of every $^*g\hat{\alpha}$-closed set in (Y,σ) is both open and closed in (X,τ).*

Proof. The proof follows from Proposition 3.10. □

Proposition 4.26. *Let $f : (X,\tau) \to (Y,\sigma)$ and $g : (Y,\sigma) \to (Z,\eta)$ be two perfectly $^*g\hat{\alpha}$-continuous maps. Then their composition $g \circ f : (X,\tau) \to (Z,\eta)$ is also perfectly $^*g\hat{\alpha}$-continuous.*

Proof. Let A be a $^*g\hat{\alpha}$-open set in (Z,η). Since g is perfectly $^*g\hat{\alpha}$-continuous, $g^{-1}(A)$ is both open and closed in (Y,σ). Since $g^{-1}(A)$ is open, it is $^*g\hat{\alpha}$-open in (Y,σ). As f is also perfeclty $^*g\hat{\alpha}$-continuous, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is both open and closed in (X,τ) and $g \circ f$ is perfectly $^*g\hat{\alpha}$-continuous. □

Proposition 4.27. *If $f : (X,\tau) \to (Y,\sigma)$ and $g : (Y,\sigma) \to (Z,\eta)$ be two maps, then their composition $g \circ f : (X,\tau) \to (Z,\eta)$ is*

(1). $^*g\hat{\alpha}$-continuous, if g is strongly continuous and f is $^*g\hat{\alpha}$-continuous.

(2). $^*g\hat{\alpha}$-irresolute, if g is perfectly $^*g\hat{\alpha}$-continuous and f is $^*g\hat{\alpha}$-continuous.

(3). $^*g\hat{\alpha}$-irresolute, if g is perfectly $^*g\hat{\alpha}$-continuous and f is $^*g\hat{\alpha}$-irresolute.

(4). strongly $^*g\hat{\alpha}$-continuous, if g is perfectly $^*g\hat{\alpha}$-continuous and f is continuous.

(5). strongly $^*g\hat{\alpha}$-continuous, if g is perfectly $^*g\hat{\alpha}$-continuous and f is strongly $^*g\hat{\alpha}$ continuous.

(6). perfectly $^*g\hat{\alpha}$-continuous, if g is strongly continuous and f is perfectly $^*g\hat{\alpha}$-continuous.

Proof.

(1). Let A be a closed set in (Z,η). Since g is strongly continuous, $g^{-1}(A)$ is both open and closed in (Y,σ). As f is $^*g\hat{\alpha}$-continuous, $f^{-1}(g^{-1}(A)) = (g\circ f)^{-1}(A)$ is $^*g\hat{\alpha}$-closed in (X,τ) and $g\circ f$ is $^*g\hat{\alpha}$-continuous.

(2). Let A be a $^*g\hat{\alpha}$-open set in (Z,η). Since g is perfectly $^*g\hat{\alpha}$-continuous, $g^{-1}(A)$ is both open and closed in (Y,σ). As f is $^*g\hat{\alpha}$-continuous, $f^{-1}(g^{-1}(A)) = (g\circ f)^{-1}(A)$ is $^*g\hat{\alpha}$-open in (X,τ) and $g\circ f$ is $^*g\hat{\alpha}$-irresolute.

(3). Let A be a $^*g\hat{\alpha}$-open set in (Z,η). Since g is perfectly $^*g\hat{\alpha}$-continuous, $g^{-1}(A)$ is both open and closed in (Y,σ). Since $g^{-1}(A)$ is both open and closed, it is $^*g\hat{\alpha}$-open in (Y,σ). As f is $^*g\hat{\alpha}$-irresolute, $f^{-1}(g^{-1}(A)) = (g\circ f)^{-1}(A)$ is $^*g\hat{\alpha}$-open in (X,τ) and $g\circ f$ is $^*g\hat{\alpha}$-irresolute.

(4). Let A be a $^*g\hat{\alpha}$-open set in (Z,η). Since g is perfectly $^*g\hat{\alpha}$-continuous, $g^{-1}(A)$ is both open and closed in (Y,σ). As f is continuous,
$f^{-1}(g^{-1}(A)) = (g\circ f)^{-1}(A)$ is open in (X,τ) and $g\circ f$ is stronlgy $^*g\hat{\alpha}$-continuous.

(5). Let A be a $^*g\hat{\alpha}$-open set in (Z,η). Since g is perfectly $^*g\hat{\alpha}$-continuous, $g^{-1}(A)$ is both open and closed in (Y,σ). Since $g^{-1}(A)$ is both open and closed, it is $^*g\hat{\alpha}$-open in (Y,σ).As f is strongly $^*g\hat{\alpha}$-continuous, $f^{-1}(g^{-1}(A)) = (g\circ f)^{-1}(A)$ is open in (X,τ) and $g\circ f$ is stronlgy $^*g\hat{\alpha}$-continuous.

(6). Let A be a $^*g\hat{\alpha}$-open set in (Z,η). Since g is strongly continuous, $g^{-1}(A)$ is both open and closed in (Y,σ). Since $g^{-1}(A)$ is both open and closed, it is $^*g\hat{\alpha}$-open in (Y,σ). As f is perfectly $^*g\hat{\alpha}$-continuous, $f^{-1}(g^{-1}(A)) = (g\circ f)^{-1}(A)$ is both open and closed in (X,τ) and $g\circ f$ is perfectly $^*g\hat{\alpha}$-continuous. □

Proposition 4.28. *If $f:(X,\tau)\to(Y,\sigma)$ is perfectly $^*g\hat{\alpha}$-continuous and $H\subseteq(X,\tau)$, then the restriction map $f_H:(H,\tau_H)\to(Y,\sigma)$ is also perfectly $^*g\hat{\alpha}$-continuous.*

Proof. The proof is similar to Proposition 4.5. □

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