Int. J. Math. And Appl., 6(3)(2018), 333–339
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

$\alpha g\delta$-Continuous and Irresolute Functions

B. Meera Devi[1,*], R. Vinitha[1,†] and D. K. Nathan[1]

1 PG and Research Department of Mathematics, Sri S.R.N.M.College, Sattur, Tamil Nadu, India.

Abstract: The purpose of this paper is to define a new class of functions called $\alpha g\delta$-continuous functions. We obtain several characterizations and some their properties. Also we investigate its relationship with other types of generalized continuous functions. Further we introduce and study a new class of function namely $\alpha g\delta$-irresolute.

Keywords: $\alpha g\delta$-closed sets, $\alpha g\delta$-continuous and $\alpha g\delta$-irresolute.

Accepted on: 27.07.2018

1. Introduction

Levine [9], Noiri [12], Balachandran [4], Dontchev J and Ganster [5] introduced generalized closed sets, δ-continuity, generalized continuous function and δ-generalized continuous (briefly δg-continuous) and δ-irresolute functions respectively. Sundaram [13] and Veerakumar [14] introduced semi-generalized continuiuty and $\hat{g}$-continuity in topogical spaces. Lellis Thivagar M and Meera Devi B [8] introduced $\delta\hat{g}$-continuity in topological spaces. The aim of this paper is to define a new class of generalized continuous functions called $\alpha g\delta$-continuous function and investigate their relationships to other generalized continuous functions. We further introduce and study a new class of functions namely $\alpha g\delta$-irresolute.

2. Preliminaries

Throughout this paper (X, τ) (or simply X) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, $cl(A)$, $int(A)$ and A^c denote the closure of A, the interior of A and the compliment of A respectively. Let us recall the following definitions, which are useful in the sequel.

Definition 2.1. *A subset A of a space (X, τ) is called a*

(1). semi-open [9] if $A \subseteq cl(int(A))$.

(2). pre-open [10] if $A \subseteq int(cl(A))$.

(3). α-open [4] if $A \subseteq int(cl(int(A)))$.

(4). regular open [9] if $A = int(cl(A))$.

* *E-mail: abmeeradevi@gmail.com*
† *Research Scholar*

The complement of a semi-open (respectively pre-open, α-open, regular open) set is called semi-closed (respectively pre-closed, α-closed, regular closed).

Definition 2.2. *The δ-interior [15] of a subset A of X is the union of all regular open set of X contained in A and is denoted by $Int_\delta(A)$. The subset A is called δ-open [15] if $A = Int_\delta(A)$. The complement of a δ-open is called δ-closed. Alternatively, a set $A \subseteq (X,\tau)$ is called δ-closed [15] if $A = cl_\delta(A)$, where $cl_\delta(A) = \{x \in X : int(cl(U)) \cap A \neq \emptyset, U \in \tau \text{ and } x \in U\}$.*

Definition 2.3. *A subset A of (X,τ) is called*

(1). semi-generalized closed (briefly sg-closed) set [3] $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open set in (X,τ).

(2). generalized semi-closed (briefly gs-closed) set [2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X,τ).

(3). α-generalized closed (briefly αg-closed) set [4] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X,τ).

(4). generalized α-closed (briefly $g\alpha$-closed) set [4] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open set in (X,τ).

(5). δ-generalized closed (briefly δg-closed) set [5] if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X,τ).

(6). $\hat{g}$-closed set [14] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open set in (X,τ).

(7). α-$\hat{g}$-closed (briefly $\alpha\hat{g}$-closed) set [1] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open set in (X,τ).

(8). $\delta\hat{g}$-closed set [7] if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open set in (X,τ).

(9). $\alpha g\delta$-closed set [11] if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open set in (X,τ).

The complement of a sg-closed (respectively gs-closed, αg-closed, $g\alpha$-closed, δg-closed, $\hat{g}$-closed, $\alpha\hat{g}$-closed, $\delta\hat{g}$-closed and $\alpha g\delta$-closed) set is called sg-open (respectively gs-open, αg-open, $g\alpha$-open, δg-open, $\hat{g}$-open, $\alpha\hat{g}$-open, $\delta\hat{g}$-open and $\alpha g\delta$-open).

Definition 2.4. *Recall that a function $f : (X,\tau) \to (Y,\sigma)$ is called*

(1). semi-continuous [9] if $f^{-1}(V)$ is semi-closed in (X,τ) for every closed set V of (Y,σ).

(2). pre-continuous [10] if $f^{-1}(V)$ is pre-closed in (X,τ) for every closed set V of (Y,σ).

(3). g-continuous [4] if $f^{-1}(V)$ is g-closed in (X,τ) for every closed set V of (Y,σ).

(4). sg-continuous [13] if $f^{-1}(V)$ is sg-closed in (X,τ) for every closed set V of (Y,σ).

(5). gs-continuous [4] if $f^{-1}(V)$ is gs-closed in (X,τ) for every closed set V of (Y,σ).

(6). $g\alpha$-continuous [4] if $f^{-1}(V)$ is $g\alpha$-closed in (X,τ) for every closed set V of (Y,σ).

(7). super-continuous [12] if $f^{-1}(V)$ is δ-open in (X,τ) for every open set V of (Y,σ).

(8). $\hat{g}$-continuous [14] if $f^{-1}(V)$ is $\hat{g}$-closed in (X,τ) for every closed set V of (Y,σ).

(9). δ-continuous [12] if $f^{-1}(V)$ is δ-open in (X,τ) for every δ-open set V of (Y,σ).

(10). open map [6] if $f(V)$ is open in (Y,σ) for every open set V in (X,τ)

(11). δ-closed [12] if $f(V)$ is δ-closed in (Y,σ) for every δ-closed set V of (X,τ).

(12). δg-continuous [5] if $f^{-1}(V)$ is δg-closed in (X,τ) for every closed set V of (Y,σ).

(13). $\delta\hat{g}$-continuous [8] if $f^{-1}(V)$ is $\delta\hat{g}$-closed in (X,τ) for every closed set V of (Y,σ).

3. $\alpha g\delta$-continuous and $\alpha g\delta$-irresolute Functions

We introduce the following definition.

Definition 3.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is called $\alpha g\delta$-continuous if $f^{-1}(V)$ is $\alpha g\delta$-closed in (X, τ) for every closed set V of (Y, σ).*

Example 3.2. *Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\emptyset, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{b\}, \{a, b\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$, $f(b) = a$ and $f(c) = b$. Clearly f is $\alpha g\delta$-continuous.*

Definition 3.3. *A function $f : (X, \tau) \to (Y, \sigma)$ is called $\alpha g\delta$-irreasolute if $f^{-1}(V)$ is $\alpha g\delta$-closed in (X, τ) for every $\alpha g\delta$-closed set V of (Y, σ).*

Example 3.4. *Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{b\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Clearly f is $\alpha g\delta$-irresolute.*

Theorem 3.5. *Every $\alpha g\delta$-continuous function is δg-continuous.*

Proof. It is true that every $\alpha g\delta$-closed set is δg-closed. □

Remark 3.6. *The converse of the above theorem is not true in general as shown in the following example.*

Example 3.7. *Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\emptyset, \{c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a, c\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is not $\alpha g\delta$-continuous function because $\{b\}$ is closed in (Y, σ) but $f^{-1}(\{b\}) = \{b\}$ is not $\alpha g\delta$-closed in (X, τ). However f is δg-continuous.*

Theorem 3.8. *Every $\alpha g\delta$-continuous function is gs-continuous.*

Proof. It is true that every αg-closed set is gs-closed. □

Remark 3.9. *The converse of the above theorem need not be true as shown in the following example shows.*

Example 3.10. *Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\emptyset, \{a, c\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then f is gs-continuous. But f is not $\alpha g\delta$-continuous function. Since $\{b\}$ is closed in (Y, σ), $f^{-1}(\{b\}) = \{b\}$ is not $\alpha g\delta$-closed in (X, τ).*

Theorem 3.11. *Every $\alpha g\delta$-continuous function is sg-continuous.*

Proof. It is true that every $\alpha g\delta$-closed set is sg-closed. □

Remark 3.12. *The converse of the above theorem need not be true as shown in the following example.*

Example 3.13. *Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\emptyset, \{a\}, \{a, c\}, X\}$ and $\sigma = \{\emptyset, \{c\}, \{b, c\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is sg-continuous. But f is not $\alpha g\delta$-continuous function. Since $\{a\}$ is closed in (Y, σ), $f^{-1}(\{a\}) = \{b\}$ is not $\alpha g\delta$-closed in (X, τ).*

Theorem 3.14. *Every $\alpha g\delta$-continuous function is g-continuous.*

Proof. It is true that every $\alpha g\delta$-closed set is g-closed. □

Remark 3.15. *The converse of the above theorem need not be true as shown in the following example.*

Example 3.16. *Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{b\}, \{a, b\}, Y\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then f is g-continuous. But f is not $\alpha g\delta$-continuous function. Since $\{c\}$ is closed in (Y, σ), $f^{-1}(\{c\}) = \{c\}$ is not $\alpha g\delta$-closed in (X, τ).*

Theorem 3.17. *Every $\alpha g\delta$-continuous function is $g\alpha$-continuous.*

Proof. It is true that every $\alpha g\delta$-closed set is $g\alpha$-closed. □

Remark 3.18. *The converse of the above theorem need not be true as shown in the following example.*

Example 3.19. *Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a, b\}, Y\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a$, $f(b) = c$ and $f(c) = b$. Then f is not $\alpha g\delta$-continuous function because $\{c\}$ is closed in (Y, σ) but $f^{-1}(\{c\}) = \{b\}$ is not $\alpha g\delta$-closed in (X, τ). However f is $g\alpha$-continuous.*

Theorem 3.20. *Every $\alpha g\delta$-continuous function is $\hat{g}$-continuous.*

Proof. It is true that every $\alpha g\delta$-closed set is $\hat{g}$-closed. □

Remark 3.21. *The converse of the above theorem is not true in general as shown in the following example.*

Example 3.22. *Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\emptyset, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{c\}, \{a, c\}, Y\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a$, $f(b) = c$ and $f(c) = b$. Then f is not $\alpha g\delta$-continuous function because $\{b\}$ is closed in (Y, σ) but $f^{-1}(\{b\}) = \{c\}$ is not $\alpha g\delta$-closed in (X, τ). However f is $\hat{g}$-continuous.*

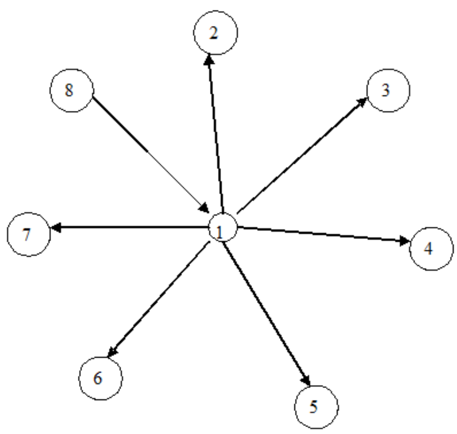
Theorem 3.23. *Every super continuous function is $\alpha g\delta$-continuous.*

Proof. It is true that every δ-closed set is $\alpha g\delta$-closed. □

Remark 3.24. *The converse of the above theorem need not be true as shown in the following example.*

Example 3.25. *Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\emptyset, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{b\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is not super continuous function because $\{b\}$ is open in (Y, σ) but $f^{-1}(\{b\}) = \{b\}$ is not δ-open in (X, τ). However f is $\alpha g\delta$-continuous.*

Remark 3.26. *The following diagram shows that the relationships of $\alpha g\delta$-continuous function with other known existing continuous functions. $A \rightarrow B$ represents A implies B but not conversely.*



1. $\alpha g\delta$-continuous 2. δg-continuous 3. gs-continuous 4. sg-continuous 5. g-continuous 6. $g\alpha$-continuous 7. $\hat{g}$-continuous 8. super continuous.

Remark 3.27. *The following examples shows that $\alpha g\delta$-continuity is independent of semi-continuity, pre-continuity, α-continuity and $\delta\hat{g}$-continuity.*

Example 3.28. *Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a, b\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then f is semi-continuous function. But f is not $\alpha g\delta$-continuous. Since for the closed set $\{c\}$ of (Y, σ), $f^{-1}(\{c\}) = \{a\}$ is not $\alpha g\delta$-closed in (X, τ).*

Example 3.29. *Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a, b\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = a$, $f(b) = c$ and $f(c) = b$. Then f is pre-continuous function. But f is not $\alpha g\delta$-continuous. Since for the closed set $\{c\}$ of (Y, σ), $f^{-1}(\{c\}) = \{b\}$ is not $\alpha g\delta$-closed in (X, τ).*

Example 3.30. *Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{b\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = a$, $f(b) = c$ and $f(c) = b$. Then f is $\delta\hat{g}$-continuous function. But f is not $\alpha g\delta$-continuous. Since $\{a, c\}$ is closed in (Y, σ), $f^{-1}(\{a, c\}) = \{a, b\}$ is not $\alpha g\delta$-closed in (X, τ).*

Example 3.31. *Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\emptyset, \{b\}, \{a, b\}, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{b, c\}, Y\}$. Define the function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = c$ and $f(c) = a$. Then f is α-continuous. But f is not $\alpha g\delta$-continuous. Since $\{a\}$ is closed in (Y, σ), $f^{-1}(\{a\}) = \{c\}$ is not $\alpha g\delta$-closed set in (X, τ).*

Example 3.32. *Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{a, b\}, Y\}$. Define the function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then f is neither semi-continuous nor α-continuous function. Moreover, it is not $\delta\hat{g}$-continuous. However f is $\alpha g\delta$-continuous.*

Example 3.33. *Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{b\}, Y\}$. Define the function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then f is not pre-continuous because $\{a, c\}$ is closed in (Y, σ) but $f^{-1}(\{a, c\}) = \{b, c\}$ is not pre-closed in (X, τ). However f is $\alpha g\delta$-continuous.*

4. Properties

Theorem 4.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is $\alpha g\delta$-continuous if and only if $f^{-1}(U)$ is $\alpha g\delta$-open in (X, τ) for every open set U in (Y, σ).*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an $\alpha g\delta$-continuous function and U be an open set in (Y, σ). Then $f^{-1}(U^c)$ is $\alpha g\delta$-closed set in (X, τ). But $f^{-1}(U^c) = [f^{-1}(U)]^c$ and hence $f^{-1}(U)$ is $\alpha g\delta$-open in (X, τ). Conversely, U^c is closed in (Y, σ). Then U is open in (Y, σ). By hypothesis, $f^{-1}(U)$ is $\alpha g\delta$-open in (X, τ). Hence $[f^{-1}(U)]^c$ is $\alpha g\delta$-closed in (X, τ). But $[f^{-1}(U)]^c = f^{-1}(U^c)$. Therefore $f^{-1}(U^c)$ is $\alpha g\delta$-closed in (X, τ). Thus f is $\alpha g\delta$-continuous. $\square$

Remark 4.2. *The composition of two $\alpha g\delta$-continuous functions need not be $\alpha g\delta$-continuous as the following example shows.*

Example 4.3. *0 Let $X = \{a, b, c\} = Y = Z$ with topologies $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$, $\sigma = \{\emptyset, \{b\}, Y\}$ and $\eta = \{\varnothing, \{a\}, Z\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = c$ and $f(c) = a$ and let $g : (Y, \sigma) \to (Z, \eta)$ be the identity function. Clearly f and g are $\alpha g\delta$-continuous functions. But $g \circ f : (X, \tau) \to (Z, \eta)$ is not an $\alpha g\delta$-continuous function because $(g \circ f)^{-1}(\{b, c\}) = f^{-1}(g^{-1}(\{b, c\})) = f^{-1}(\{b, c\}) = \{a, b\}$ is not an $\alpha g\delta$-closed set of (X, τ), where $\{b, c\}$ is a closed set of (Z, η).*

Theorem 4.4. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be two functions. Then*

(1). $g \circ f : (X, \tau) \to (Z, \eta)$ is $\alpha g\delta$-continuous, if g is continuous and f is $\alpha g\delta$-continuous.

(2). $g \circ f : (X,\tau) \to (Z,\eta)$ is $\alpha g\delta$-irresolute, if g is $\alpha g\delta$-irresolute and f is $\alpha g\delta$-irresolute.

(3). $g \circ f : (X,\tau) \to (Z,\eta)$ is $\alpha g\delta$-continuous, if g is $\alpha g\delta$-continuous and f is $\alpha g\delta$-irresolute.

Proof.

(1). Let F be closed set in (Z,η). Then $g^{-1}(F)$ is closed in (Y,σ). Since f is $\alpha g\delta$-continuous, $f^{-1}(g^{-1}(F))$ is $\alpha g\delta$-closed in (X,τ). But $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$. Hence $(g \circ f)^{-1}(F)$ is $\alpha g\delta$-closed of (X,τ). Thus $g \circ f$ is $\alpha g\delta$-continuous function.

(2). Follows from the definition.

(3). Let F be any closed set in (Z,η). Then $g^{-1}(F)$ is $\alpha g\delta$-closed in (Y,σ). Since f is $\alpha g\delta$-irresolute, $f^{-1}(g^{-1}(F))$ is $\alpha g\delta$-closed set of (X,τ). But $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$. Then $(g \circ f)^{-1}(F)$ is $\alpha g\delta$-closed in (X,τ). Hence $g \circ f$ is $\alpha g\delta$-continuous function. □

Theorem 4.5. *Let $f : (X,\tau) \to (Y,\sigma)$ be continuous and δ-closed. Then for every $\alpha g\delta$-closed subset A of (X,τ), f(A) is $\alpha g\delta$-closed in (Y,σ).*

Proof. Let A be $\alpha g\delta$-closed in (X,τ). Let $f(A) \subseteq O$ where O is open in (Y,σ). Since $A \subseteq f^{-1}(O)$ is open in (X,τ), $f^{-1}(O)$ is α-open in (X,τ). Since A is $\alpha g\delta$-closed and since $f^{-1}(O)$ is α-open in (X,τ), $cl_\delta(A) \subseteq f^{-1}(O)$. Thus $f(cl_\delta(A)) \subseteq O$. Hence $cl_\delta(f(A)) \subseteq cl_\delta(f(cl_\delta(A))) = f(cl_\delta(A)) \subseteq O$, since f is δ-closed. Hence $f(A)$ is $\alpha g\delta$-closed in (Y,σ). □

Remark 4.6. *$\alpha g\delta$-continuity and $\alpha g\delta$-irresoluteness are independent notions as seen in the following examples.*

Example 4.7. *Let $X = \{a,b,c\} = Y$ with topologies $\tau = \{\emptyset, \{b\}, \{c\}, \{b,c\}, \{a,c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b,c\}, Y\}$. Define $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = b$, $f(b) = c$ and $f(c) = a$. Then f is $\alpha g\delta$-continuous but it is not $\alpha g\delta$-irresolute function because $f^{-1}(\{c\}) = \{b\}$ is not $\alpha g\delta$-closed in (X,τ), where $\{c\}$ is $\alpha g\delta$-closed in (Y,σ).*

Example 4.8. *Let $X = \{a,b,c\} = Y$ with topologies $\tau = \{\emptyset, \{b\}, \{c\}, \{b,c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a,b\}, \{a,c\}, Y\}$. Define $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then f is $\alpha g\delta$-irresolute but it is not $\alpha g\delta$-continuity because $f^{-1}(\{c\}) = \{c\}$ is not $\alpha g\delta$-closed in (X,τ), where $\{c\}$ is closed in (Y,σ).*

5. Application

Definition 5.1. *A space (X,τ) is called $T_{\alpha g\delta}$-space if every $\alpha g\delta$-closed set in it is δ-closed.*

Theorem 5.2. *Let $f : (X,\tau) \to (Y,\sigma)$ be $\alpha g\delta$-irresolute. Then f is δ-continuous if (X,τ) is $T_{\alpha g\delta}$-space.*

Proof. Let V be a δ-closed subset of (Y,σ). Every δ-closed set is $\alpha g\delta$-closed and hence V is $\alpha g\delta$-closed in (Y,σ). Since f is $\alpha g\delta$-irresolute, $f^{-1}(V)$ is $\alpha g\delta$-closed in (X,τ). Since X is $T_{\alpha g\delta}$, $f^{-1}(V)$ is δ-closed in (X,τ). Thus f is δ-continuous. □

Theorem 5.3. *If (Y,σ) is $T_{\alpha g\delta}$-space, then the composition of two $\alpha g\delta$-continuous functions is also $\alpha g\delta$-continuous function.*

Proof. Let $f : (X,\tau) \to (Y,\sigma)$ and $g : (Y,\sigma) \to (Z,\eta)$ be two $\alpha g\delta$-continuous functions. Let G be any closed set in (Z,η). Then $g^{-1}(G)$ is $\alpha g\delta$-closed in (Y,σ). Since (Y,σ) is $T_{\alpha g\delta}$, $g^{-1}(G)$ is closed in (Y,σ). Since f is $\alpha g\delta$-continuous, $f^{-1}(g^{-1}(G))$ is $\alpha g\delta$-closed in (X,τ). But $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$. Then $(g \circ f)^{-1}(G)$ is $\alpha g\delta$-closed set of (X,τ). Hence is $\alpha g\delta$-continuous function. □

Theorem 5.4. *Let $f : (X, \tau) \to (Y, \sigma)$ be onto, $\alpha g\delta$-irresolute and δ-closed. If (X, τ) is a $T_{\alpha g\delta}$-space, then (Y, σ) is also a $T_{\alpha g\delta}$-space.*

Proof. Let V be a $\alpha g\delta$-closed subset of (Y, σ). Since f is $\alpha g\delta$-irresolute, $f^{-1}(V)$ is $\alpha g\delta$-closed set in (X, τ). Since is $T_{\alpha g\delta}$, $f^{-1}(V)$ is δ-closed in (X, τ). Since f is surjective, V is δ-closed in (Y, σ). Hence is $T_{\alpha g\delta}$-space. □

Theorem 5.5. *If $f : (X, \tau) \to (Y, \sigma)$ is bijection, open and $\alpha g\delta$-continuous, then f is $\alpha g\delta$-irresolute.*

Proof. Let V be $\alpha g\delta$-closed in (Y, σ) and let $f^{-1}(V) \subseteq U$ where U is open in (X, τ). Since f is open, $f(U)$ is open in (Y, σ). Every open set is α-open and hence $f(U)$ is α-open. Clearly $V \subseteq f(U)$. Then $cl_\delta(V) \subseteq f(U)$ and thus $f^{-1}(cl_\delta(V)) \subseteq U$. Since f is $\alpha g\delta$-continuous and since $cl_\delta(V)$ is a closed subset of (Y, σ), $cl_\delta(f^{-1}(V)) \subseteq cl_\delta(f^{-1}(cl_\delta(V))) = f^{-1}(cl_\delta(V)) \subseteq U$. U is open and hence α-open in (X, τ). Thus we have $cl_\delta(f^{-1}(V)) \subseteq U$ whenever $f^{-1}(V) \subseteq U$ and U is α-open set in (X, τ). This shows that $f^{-1}(V)$ is $\alpha g\delta$-closed in (X, τ). Hence f is $\alpha g\delta$-irresolute. □

References

[1] M. E. Abd El-Monsef, S. Rose Mary and M. Lellis Thivagar, *On $\alpha\hat{g}$-closed sets in topological spaces*, Assiut University Journal of Mathematics and Computer Science, 36(1)(2007), 43-51.

[2] S. P. Arya and T. Nour, *Characterizations of S-normal spaces*, Indian J. Pure. Appl. Math., 29(8)(1990), 717-719.

[3] P. Bhattacharya and B. K. Lahiri, *Semi-generalized closed sets in topology*, Indian J. Math., 29(1987), 375-382.

[4] R. Devi, K. Balachandran and H. Maki, *On generalized α-continuous functions and α-generalized continuous functions*, Far East J. Math. Sci., Special Volume, Part I(1997), 1-15.

[5] J. Dontchev and M. Ganster, *On δ-generalized closed sets and $T_{3/4}$-spaces*, Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 17(1996), 15-31.

[6] Lee and M. John, *Introduction to Smooth Manifolds*, Graduate Texts in Math-ematics. 218. Springer Science and Business Media, (2003).

[7] M. Lellis Thivagar, B. Meeradevi and E. Hatir, *$\delta\hat{g}$-closed sets in topological spaces*, Gen. Math. Notes, 1(2)(2010), 17-25.

[8] M. Lellis Thivagar and B. Meera Devi, *Some New Class of Generalized Continuous Functions*, General Mathematics Notes, 1(2)(2010), 17-25.

[9] N. Levine, *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly, 70(1963), 36-41.

[10] A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Debb, *On pre-continuous and weak pre-continuous mappings*, Proc. Math. and Phys. Soc., 55(1982), 47-53.

[11] B. Meera Devi and R. Vinitha, *$\alpha g\delta$-closed sets in topological spaces*, Global Journal For Research Analysis, 6(7)(2017), 345-347.

[12] T. Noiri, *Super continuity and some strong forms of continuity*, Indian J. Pure Appl. Math., 15(3)(1984), 241-250.

[13] P. Sundaram , H. Maki and K. Balachandran, *Semi-generalized continuous maps and semi-$T_{1/2}$-spaces*, Bull. Fukuoka Univ. Ed. Part III, 40(1991), 33-40.

[14] M.K.R.S. Veerakumar, *$\hat{g}$-closed sets in topological spaces*, Bull. Allah. Math. Soc, 18(2003), 99-112.

[15] N. V. Velicko, *H-closed topological spaces*, Amer. Math. Soc. Transl., 78(1968), 103-118.