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A View on Contra-Fr-B-Continuous M-Set Functions in Multiset Topological Spaces

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Abstract: In this paper, the concepts of Fr-B-open M-sets, contra-Fr-B-continuous M-set functions are studied with necessary examples. In this connection, the concepts of Fr-B-connected M-spaces, Fr-B-normal M-spaces, ultra normal M-spaces, strongly S-closed M-spaces and Fr-B-compact M-spaces are studied. Finally, the applications of contra-Fr-B-continuous M-set functions on various M-spaces are discussed.

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1. Introduction and Preliminaries

There are many occasions, however, when one encounters collections of non-distinct objects, in such situations the term 'multiset' is used instead of 'set'. The notion of M-topological spaces and the concept of open M-sets were introduced by Girish and sunil Jacob John [5]. In this paper topologies on multisets were provided and they could be useful for measuring the similarities and dissimilarities between the universes of the objects which are multisets. Moreover, topologies on multisets can be associated to IC-bags or nk-bags introduced by Chakrabarthy [2] with the help of rough set theory. The association of rough set theory and topologies on multisets through bags with interval counts [2] could be used to develop theoretical study of covering based rough sets with respect to universe as multisets. Dontchev [3] introduced a new class of mappings called contra-continuity. Jafari and Noiri [4] exhibited and studied among others a new weaker form of this class of mappings called contra-α-continuous and contra-precontinuous mappings. The concept of B-set was studied by Indira and Rekha [6]. In this paper, the concepts of Fr-B open sets, contra-Fr-B-continuous M-set functions are studied with necessary examples. In this connection, the concepts of Fr-B-connected M-spaces, Fr-B-normal M-spaces, ultra normal M-spaces, strongly S-closed M-spaces and Fr-B-compact M-spaces are studied. Finally, the applications of contra-Fr-B-continuous M-set functions on various M-spaces are discussed. Throughout this paper X denote a non-empty set, $M \in [X]^W$ and $C_M : X \to W$ where W is the set of all whole numbers.

Definition 1.1 ([5])**.** *Let $M \in [X]^w$ and $\tau \subseteq P^*(M)$. Then τ is called a Multiset topology of M if τ satisfies the following properties.*

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(1). The M-set M and the empty M-set ϕ are in τ.

(2). The M-set union of the elements of any sub collection of τ is in τ.

(3). The M-set intersection of the elements of any finite sub collection of τ is in τ.

Definition 1.2 ([5])**.** *Given a sub M-set A of an M-topological space M in $[X]^w$, the interior of A is defined as the M-set union of all open M-sets contained in A and is denoted by $Int(A)$ i.e., $Int(A) = \bigcup\{G \subseteq M : \ G$ is an open M-set and $G \subseteq A\}$ and $C_{Int(A)}(x) = \max\{C_G(x) : G \subseteq A, G \in \tau\}$.*

Definition 1.3 ([5])**.** *Given a sub M-set A of an M-topological space M in $[X]^w$, the closure of A is defined as the M-set intersection of all closed M-sets containing A and is denoted by $Cl(A)$ i.e., $Cl(A) = \bigcap\{K \subseteq M : \ K$ is a closed M-set and $A \subseteq K\}$ and $C_{Cl(A)}(x) = \min\{C_K(x) : A \subseteq K, K \in \tau^c\}$.*

Definition 1.4 ([10])**.** *Let (M, τ) be an M-topological space. If for every two M-singletons $\{k_1/x_1\}, \{k_2/x_2\} \subseteq M$ such that $x_1 \neq x_2$, then there exist $G,\ H \in \tau$ such that $\{k_1/x_1\} \subseteq G$, $\{k_2/x_2\} \subseteq H$ and $G \cap H = \phi$. Hence (M, τ) is a Hausdorff M-space.*

Definition 1.5 ([10])**.** *Let (M, τ) be an M-topological space. If for all $F_1, F_2 \in \tau^C$ such that $F_1 \cap F_2 = \phi$, there exist $G, H \in \tau$ such that $F_1 \subseteq G$, $F_2 \subseteq H$ and $G \cap H = \phi$. Hence (M, τ) is a normal M-space.*

2. Contra Fr-B-Continuous M-Set Functions

Definition 2.1. *Let (M, τ) be an M-topological space. The frontier of any sub M-set A of M is defined as if $Fr(A) = Cl(A) \cap Cl(A^C)$ with $C_{Fr(A)}(x) = \min\{C_{Cl(A)}(x), C^c_{Cl(A}(x)\}$ for all $x \in X$.*

Definition 2.2. *Let (M, τ) be an M-topological space. Any sub M-set A of M is called a t-open M-set if $Int(A) = Int(Cl(A))$ with $C_{Int(A)}(x) = C_{Int(Cl(A))}(x)$, for all $x \in X$.*

Definition 2.3. *Let (M, τ) be an M-topological space and a sub M-set P of M is called a B-open M-set if $P = Q \cap R$ with $C_P(x) = \min\{C_Q(x), C_R(x)\}$, for all $x \in X$ where Q is an open M-set and R is a t-open M-set.*

Definition 2.4. *Let (M, τ) be an M-topological space. Any sub M-set A of M is called a Fr-B-open M-set if $Fr(A) \subseteq P$ with $C_{Fr(A)}(x) \leq C_P(x)$, whenever $A \subseteq P$ with $C_A(x) \leq C_P(x)$ and P is a B-open M-set, for all $x \in X$. The complement of a Fr-B-open M-set is said to be a Fr-B-closed M-set.*

Example 2.5. *Let $X = \{a, b, c\}$, $W = 2$. Let $M = \{1/a, 2/b, 1/c\}$. Let $\tau = \{M, \phi, \{1/a\}, \{2/b\}, \{1/a, 2/b\}\}$. Then $\tau^c = \{\phi, M, \{2/b, 1/c\}, \{1/a, 1/c\}, \{1/c\}\}$. Let $A = \{1/c\}$ be a sub M-set of M. Let $P = \{1/c\}$ where P is B-open M-sets. Now $Fr(A) = \{1/c\} = P$ with $C_{Fr(A)}(x) = C_P(x)$ for all $x \in X$, whenever $A = P$. Therefore A is Fr-B-open M-set.*

Definition 2.6. *Let (M, τ) be an M-topological space. For any sub M-set A of M in $[X]^W$, the Fr-B-interior of A is defined as the M-set union of all Fr-B-open M-sets contained in A and is denoted by Fr-B-Int(A) i.e., $Fr - B - Int(A) = \bigcup\{G : \ $ each $G \subseteq M$ is a Fr-B-open M-set and $G \subseteq A\}$ with $C_{Fr-B-Int(A)}(x) = \max\{C_G(x) : G \subseteq A,\ $ each G is an Fr-B-open M-set$\}$, for all $x \in X$.*

Definition 2.7. *Let (M, τ) be an M-topological space. For any sub M-set A of an M-topological space M in $[X]^W$, the Fr-B-closure of A is defined as the M-set intersection of all closed M-sets containing A and is denoted by Fr-B-Cl(A) i.e., $Fr - B - Cl(A) = \bigcap\{K : \ $ each $K \subseteq M$ is an Fr-B-closed M-set and $A \subseteq K\}$ with $C_{Fr-B-Cl(A)}(x) = \min\{C_k(x) : A \subseteq K,\ $ each K is an Fr-B-closed M-set$\}$, for all $x \in X$.*

Definition 2.8. *Let (M, τ) and (N, σ) be any two M-topological spaces. Any M-set function $f : (M, \tau) \to (N, \sigma)$ is said to be a continuous M-set function if $f^{-1}(V)$ is an open M-set in (M, τ) for every open M-set V in (N, σ).*

Definition 2.9. *Let (M, τ) and (N, σ) be any two M-topological spaces. Any M-set function $f : (M, \tau) \to (N, \sigma)$ is called a Fr-B-continuous M-set function if $f^{-1}(V)$ is a Fr-B-open M-set in (M, τ) for every open M-set V of (N, σ).*

Example 2.10. *Let $X = \{a, b, c\}$, $W_1 = 2$, $Y = \{x, y, z\}$, and $W_2 = 1$. Let $M = \{1/a, 2/b, 1/c\}$ and $N = \{1/x, 1/y, 1/z\}$ be two M-sets. Let $\tau = \{M, \phi, \{1/a\}, \{2/b\}, \{1/a, 2/b\}\}$. $\sigma = \{N, \phi, \{1/z\}\}$ be two M-topologies on M and N respectively. Then (M, τ) and (N, σ) are M-topological spaces. The Fr-B-open M-sets are M, ϕ, $\{1/c\}$, $\{1/a, 1/c\}$, $\{2/b, 1/c\}$, $\{1/b, 1/c\}$, $\{1/a, 1/b, 1/c\}$. Let the M-set function $f : (M, \tau) \to (N, \sigma)$ be given by $f = \{(1/a, 1/x)/1, (2/b, 1/y)/2, (1/c, 1/z)/1\}$. For the open M-set $A = \{1/z\}$ in (N, σ), $f^{-1}(A) = \{1/c\}$ and is a Fr-B-open M-set in (M, τ). Trivially $f^{-1}(\phi) = \phi$ and $f^{-1}(N) = M$ are Fr-B-open M-sets in (M, τ). Thus the inverse image of every open M-set of (N, σ) is a Fr-B-open M-set in (M, τ). Hence f is a Fr-B-continuous M-set function.*

Definition 2.11. *Let (M, τ) and (N, σ) be any two M-topological spaces. Any M-set function $f : (M, \tau) \to (N, \sigma)$ is called a contra continuous M-set function if $f^{-1}(V)$ is a closed M-set(respectively open M-set) in (M, τ) for every open M-set (respectively closed M-set) V of (N, σ).*

Definition 2.12. *Let (M, τ) and (N, σ) be any two M-topological spaces. A function $f : (M, \tau) \to (N, \sigma)$ is called a contra Fr-B-continuous M-set function if $f^{-1}(V)$ is a Fr-B-closed M-set (respectively Fr-B-open M-set) in (M, τ) for every open M-set (respectively closed M-set) V in (N, σ).*

Example 2.13. *Let $X = \{a, b, c\}$, $W_1 = 2$, $Y = \{x, y, z\}$, and $W_2 = 1$. Let $M = \{1/a, 2/b, 1/c\}$ and $N = \{1/x, 1/y, 1/z\}$ be two M-sets. $\tau = \{M, \phi, \{1/a\}, \{2/b\}, \{1/a, 2/b\}\}$ and $\sigma = \{N, \phi, \{1/x\}, \{1/y\}, \{1/x, 1/y\}\}$ be two M-topologies on M and N respectively. Then (M, τ) and (N, σ) are M-topological spaces. The Fr-B-closed M-sets are M, ϕ, $\{1/a, 2/b\}$, $\{2/b\}$, $\{1/a\}$, $\{1/a, 1/b\}$, $\{1/b\}$. Let the M-set function $f : M \to N$ be given by $f = \{(1/a, 1/x)/1, (2/b, 1/y)/2, (1/c, 1/z)/1\}$. For the open M-set $A = \{1/x\}$ in (N, σ), $f^{-1}(A) = \{1/a\}$ is a Fr-B-closed M-set in (M, τ). For the open M-set $A = \{1/y\}$ in (N, σ), $f^{-1}(A) = \{2/b\}$ is a Fr-B-closed M-set in (M, τ). For the open M-set $A = \{1/x, 1/y\}$ in (N, σ), $f^{-1}(A) = \{1/a, 1/b\}$ is a Fr-B-closed M-set in (M, τ). Trivially $f^{-1}(\phi) = \phi$ and $f^{-1}(N) = M$ are a Fr-B-closed M-sets in (M, τ). Thus the inverse image of every open M-set of (N, σ) is a Fr-B-closed M-set in (M, τ). Hence, f is a contra Fr-B-continuous M-set function.*

Proposition 2.14. *Let (M, τ) and (N, σ) be any two M-topological spaces. For any M-set function $f : (M, \tau) \to (N, \sigma)$, the following statements are equivalent:*

(1). f is a contra Fr-B-continuous M-set function.

(2). $f(Fr - B - Cl(A)) \subseteq Clf(A)$, for each $A \subseteq M$ with $C_{f(Fr-B-cl(A))}(x) \leq C_{Clf(A)}(x)$, for all $x \in X$.

(3). $Fr - B - Clf^{-1}(B) \subseteq f^{-1}(Cl(B))$, for each $B \subseteq N$ with $C_{Fr-BCl(f^{-1}(B))}(x) \leq C_{f^{-1}(Cl(B))}(x)$, for all $x \in X$.

(4). $f^{-1}(Int(B)) \subseteq Fr - B - Int(f^{-1}(B))$, for each $B \subseteq N$ with $C_{f^{-1}(Int(B))}(x) \leq C_{Fr-B-Int(f^{-1}(B))}(x)$, for all $x \in X$.

Remark 2.15. *The composition of two contra-Fr-B-continuous M-set functions need not be a contra-Fr-B-continuous M-set function as shown in the following Example.*

Example 2.16. *Let $X = \{x, y, z\}$, $W_1 = 2$, $Y = \{a, b, c\}$, $W_2 = 1$ and $Z = \{d, e\}$, $W_3 = 1$. Let $M = \{2/x, 2/y, 1/z\}$, $N = \{1/a, 1/b, 1/c\}$ and $P = \{1/d, 1/e\}$ be M-sets. Let $\tau = \{M, \phi, \{2/x\}, \{2/y\}, \{2/x, 2/y\}\}$, $\sigma = \{N, \phi, \{1/a\}, \{1/a, 1/c\}\}$ and $\eta = \{P, \phi, \{1/d\}, \{1/e\}\}$ be M-topologies on M, N and P respectively. Now the Fr-B-open M-sets of (M, τ) are M,*

ϕ, $\{1/z\}$, $\{2/x, 1/z\}$, $\{2/y, 1/z\}$, $\{1/x, 2/y, 1/z\}$, $\{2/x, 1/y, 1/z\}$, $\{1/x, 1/y, 1/z\}$, $\{1/y, 1/z\}$ *and the Fr-B-open M-sets of* (N, σ) *are* N, ϕ, $\{1/b\}$, $\{1/b, 1/c\}$, $\{1/a, 1/b\}$ *and the closed M-sets of* (P, η) *are* P, ϕ, $\{1/d\}$, $\{1/e\}$. *Let the M-sets functions* $f : (M, \tau) \to (N, \sigma)$ *and* $g : (N, \sigma) \to (P, \eta)$ *be defined as* $f = \{(2/x, 1/b)/2, (2/y, 1/c)/2, (1/z, 1/a)/1\}$ *and* $g = \{(1/a, 1/d)/1, (1/b, 1/d)/1, (1/c, 1/e)/1\}$. *Since the functions f and g are contra-Fr-B-continuous M-set functions. But their composition is not a contra-Fr-B-continuous M-set functions, since for the open M-set* $U = \{1/d\}$ *in* (P, η), $(g \circ f)^{-1}(U) = \{2/x, 1/z\}$, *which is not Fr-B-closed M-set in* (M, τ). *Also for the closed M-set* $V = \{1/e\}$ *in* (P, η), $(g \circ f)^{-1}(V) = \{2/y\}$, *which is not Fr-B-open M-set in* (M, τ). *Hence, the composition of two contra-Fr-B-continuous M-set functions need not be a contra-Fr-B-continuous M-set function.*

Proposition 2.17. *Let* (M, τ), (N, σ) *and* (P, γ) *be any three M-topological spaces. If a function* $f : (M, \tau) \to (N, \sigma)$ *is a surjective Fr-B-continuous M-set function and* $g : (N, \sigma) \to (P, \gamma)$ *is a contra continuous M-set function then* $g \circ f : (M, \tau) \to (P, \gamma)$ *is a contra Fr-B-continuous M-set function.*

Proof. Let A be an open M-set in (P, γ). Since g is a contra continuous M-set function, $g^{-1}(A)$ is a closed M-set in (N, σ). Since f is a surjective Fr-B-continuous M-set function, $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is Fr-B-closed M-set in (M, τ). Therefore $g \circ f$ is a contra Fr-B-continuous M-set function. □

Definition 2.18. *Let* (M, τ) *be an M-topological space. A sub M-set A of M is called a clopen M-set if it is both open M-set and closed M-set in* (M, τ).

Definition 2.19. *Let* (M, τ) *be an M-topological space. A sub M-set A of M is called a Fr-B-clopen M-set if it is both Fr-B-open M-set and Fr-B-closed M-set in* (M, τ).

Definition 2.20. *Let* (M, τ) *and* (N, σ) *be any two M-topological spaces. Any M-set function* $f : (M, \tau) \to (N, \sigma)$ *is called a perfectly continuous M-set function if* $f^{-1}(V)$ *is a clopen M-set in* (M, τ) *for every open M-set V in* (N, σ).

Definition 2.21. *Let* (M, τ) *and* (N, σ) *be any two M-topological spaces. Any M-set function* $f : (M, \tau) \to (N, \sigma)$ *is called a perfectly Fr-B-continuous M-set function if* $f^{-1}(V)$ *is a clopen M-set in* (M, τ) *for every Fr-B-open M-set V in* (N, σ).

Proposition 2.22. *Let* (M, τ), (N, σ) *and* (P, η) *be any three M-topological spaces. If* $f : (M, \tau) \to (N, \sigma)$ *is a perfectly-Fr-B-continuous M-set function and* $g : (N, \sigma) \to (P, \eta)$ *is a contra-Fr-B-continuous M-set function, then* $g \circ f : (M, \tau) \to (P, \eta)$ *is a perfectly continuous M-set function.*

Proof. Let U be any open M-set in (P, η). Since g is a contra-Fr-B-continuous M-set function, then $g^{-1}(U)$ is a Fr-B-closed M-set in (N, σ) and since f is a perfectly-Fr-B-continuous M-set function, then $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is clopen in (M, τ). Therefore $g \circ f$ is a perfectly continuous M-set function. □

Definition 2.23. *Let* (M, τ) *and* (N, σ) *be any two M-topological spaces. Any M-set function* $f : (M, \tau) \to (N, \sigma)$ *is said to be an irresolute M-set function if* $f^{-1}(V)$ *is an open M-set (respectively closed M-set) in* (M, τ) *for every open M-set (respectively closed M-set) V of* (N, σ).

Definition 2.24. *Let* (M, τ) *and* (N, σ) *be any two M-topological spaces. Any function* $f : (M, \tau) \to (N, \sigma)$ *is said to be a Fr-B-irresolute M-set function, if* $f^{-1}(A)$ *is a Fr-B-open M-set (respectively Fr-B-closed M-set) in* (M, τ) *for every Fr-B-open M-set (respectively Fr-B-closed M-set) A of* (N, σ).

Proposition 2.25. *Let* (M, τ), (N, σ) *and* (P, η) *be any three M-topological spaces. If* $f : (M, \tau) \to (N, \sigma)$ *is an Fr-B-irresolute M-set function and* $g : (N, \sigma) \to (P, \eta)$ *is a contra-Fr-B-continuous M-set function. Then* $g \circ f : (M, \tau) \to (P, \eta)$ *is contra-Fr-B-continuous M-set function.*

Proof. Let A be any open M-set in (P, η). Since g is a contra-Fr-B continuous M-set function then $g^{-1}(A)$ is an Fr-B-closed M-set in (N, σ) and since f is an Fr-B-irresolute M-set function then $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is a Fr-B-closed M-set in (M, τ). Therefore, $g \circ f$ is a contra-Fr-B-continuous M-set function. □

Definition 2.26. *Let (M, τ) be an M-topological space. If for each pair of distinct points m/x and m/y in M, there exists Fr-B-open M-sets A and B containing m/x and m/y respectively, such that $m/x \notin B$ and $m/y \notin A$, then (M, τ) is said to be an $Fr - B - MT_1$ space.*

Definition 2.27. *Let (M, τ) be an M-topological space. If for each pair of distinct points m/x and m/y in M, there exist Fr-B-open M-sets A and B containing m/x and m/y respectively, such that $m/x \notin B$ and $m/y \notin A$ and $A \cap B = \phi$, with $C_{A \cap B}(x) = 0$, then (M, τ) is said to be a $Fr - B - MT_2$ space.*

Definition 2.28. *Let (M, τ) be an M-topological space. If for each pair of distinct points m/x and m/y in M, there exist disjoint open M-sets U and V containing m/x and m/y respectively, such that $Cl(U) \cap Cl(V) = \phi$ with $C_{Cl(U) \cap Cl(V)} = 0$, then (M, τ) is said to be an Urysohn M-space.*

Proposition 2.29. *Let (M, τ) and (N, σ) be any two M-topological spaces. If*

(1). for each pair of distinct points m/x and m/y in M, there exist an M-set function f of (M, τ) into (N, σ) such that $f(m/x) \neq f(m/y)$;

(2). (N, σ) is an Urysohn M-space and

(3). f is a contra Fr-B-continuous M-set function at m/x and m/y then (M, τ) is a $Fr - B - MT_2$-space.

3. Applications of Contra-Fr-B-Continuous M-Set Functions

Definition 3.1. *An M-topological space (M, τ) is called a Fr-B-M $T_{1/2}$ Space if every Fr-B-closed M-set (respectively, Fr-B-open M-set) is a closed M-set (respectively, open M-set).*

Proposition 3.2. *Let (M, τ) and (N, σ) be any two M-topological spaces. If (M, τ) is an Fr-B-M $T_{1/2}$ Space, then every contra Fr-B-continuous M-set function $f : (M, \tau) \to (N, \sigma)$ is a contra continuous M-set function.*

Proof. Let A be an open M-set of (N, σ). Since f is contra Fr-B-continuous, $f^{-1}(A)$ is a Fr-B-closed M-set in (M, τ). Since (M, τ) is a Fr-B-M $T_{1/2}$ Space, every Fr-B-closed M-set is a closed M-set. Hence $f^{-1}(A)$ is a closed M-set. Therefore, f is contra continuous M-set function. □

Definition 3.3. *An M-topological space (M, τ) is called a connected M-space, if M is not the union of two disjoint non-empty open M-sets.*

Definition 3.4. *An M-topological space (M, τ) is called a Fr-B-connected M-space, if M is not the union of two disjoint non-empty Fr-B-open M-sets.*

Example 3.5. *Let $X = \{a, b\}$, $W = 2$ and $M = \{2/a, 2/b\}$, $t = \{M, \phi, \{2/a\}, \{1/a\}, \{2/b\}, \{1/a, 2/b\}\}$. Clearly t is an M-topology and the ordered pair (M, τ) is an M-topological space. The Fr-B-open M-sets $\{M, \phi, \{2/a\}, \{1/a\}, \{2/b\}, \{1/b\}, \{2/a, 1/b\}, \{1/a, 1/b\}, \{1/a, 2/b\}\}$. Let $U = \{1/a\}$ and $V = \{2/b\}$, which are disjoint non-empty Fr-B-open sub M-sets of M such that $M \neq U \cup V$ with $C_M(x) \neq \max\{C_U(x), C_V(x)\}$, for all $x \in X$. Hence, (M, τ) is a Fr-B-connected M-space.*

Remark 3.6.

(1). An M-topological space (M, τ) is connected if and only if the only sub M-sets of M that are both open M-set and closed M-set in (M, τ) are the empty M-set and M itself.

(2). Similarly, an M-topological space (M, τ) is Fr-B-connected if and only if Fr-B-clopen M-sets are empty M-set and M itself.

Proposition 3.7. *A contra-Fr-B-continuous image of a Fr-B-connected M-space is connected.*

Proposition 3.8. *Let (M, τ) and (N, σ) be any two M-topological spaces. If $f : (M, \tau) \to (N, \sigma)$ is a contra Fr-B-continuous injection and (N, σ) is Hausdorff M-space, then (M, τ) is a Fr-B-M T_1 space.*

Definition 3.9. *An M-topological space (M, τ) is said to be an ultra Hausdorff M-space, if for each pair of distinct points m/x and m/y in M, there exist clopen M-sets A and B containing m/x and m/y respectively such that $A \cap B = \phi$ with $C_A \cap B(x) = \min\{C_A(x), C_B(x)\}$, for all $x \in X$.*

Proposition 3.10. *Let (M, τ) and (N, σ) be any two M-topological spaces. If $f : (M, \tau) \to (N, \sigma)$ is a contra Fr-B-continuous injection, and (N, σ) is an ultra Hausdorff M-space, then (M, τ) is a Fr-B-M T_2-space.*

Proof. Let m/x and m/y be any two distinct points of M. Since (N, σ) is an ultra Hausdorff M-space and $f(m/x) \neq f(m/y)$, there exist clopen M-sets A and B containing $f(m/x)$ and $f(m/y)$ respectively such that $A \cap B = \phi$ with $C_A \cap B(x) = \min\{C_A(x), C_B(x)\}$, for all $x \in X$. Since f is contra-Fr-B-continuous, then $f^{-1}(A)$ and $f^{-1}(B)$ are Fr-B-closed M-sets such that $f^{-1}(A) \cap f^{-1}(B) = \phi$ with $C_{f^{-1}(A) \cap f^{-1}(B)}(x) = C_\phi(x) = 0$. Also $x \in {}^m f^{-1}(A)$ and $y \in {}^m f^{-1}(B)$. Hence, M is a Fr-B-M T_2-space. □

Definition 3.11. *An M-topological space (M, τ) is said to be a Fr-B-normal M-space, if for each pair A, B of disjoint closed M-sets of M, there exist disjoint Fr-B-open M-sets containing A and B respectively.*

Definition 3.12. *Any M-topological space (M, τ) is said to be an ultra normal M-space, if each pair A, B of disjoint closed M-sets of M, there exist disjoint clopen M-sets U and V containing A and B respectively.*

Proposition 3.13. *Let (M, τ) and (N, σ) be any two M-topological spaces. If $f : (M, \tau) \to (N, \sigma)$ is a contra Fr-B-continuous closed injection and (N, σ) is an ultra normal, then (M, τ) is a Fr-B-normal.*

Proof. Let A and B be disjoint closed sub M-sets of M. Since f is a closed injection, $f(A)$ and $f(B)$ are disjoint and closed in N. Since (N, σ) is ultra normal, $f(A)$ and $f(B)$ are separated by disjoint clopen M-sets U and V respectively. Since f is a contra Fr-B-continuous M-set function, $A \subseteq f^{-1}(U)$ with $C_A(x) \leq C_{f^{-1}(U)}(x)$ and $B \subseteq f^{-1}(V)$ with $C_B(x) \leq C_{f^{-1}(U)}(x)$ are Fr-B-open M-sets such that $f^{-1}(U) \cap f^{-1}(V) = \phi$ with $C_{f^{-1}(U) \cap f^{-1}(V)}(x) = C_\phi(x) = 0$. Hence (M, τ) is Fr-B-normal. □

Definition 3.14. *Let (M, τ) be an M-topological space. A collection $\mathcal{C}$ of subM-sets of M is said to cover M, or to be a covering of M, if the union of elements of $\mathcal{C}$ is equal to M i.e., $\bigcup_{i \in I} A_i = M$ where I is an indexed set with max $\{C_{A_i}(x), i \in I$ where each $A_i \subseteq M\} = C_M(x)$, for all x in X. Such a cover of M is said to be an open (respectively preopen, regular open, Fr-B-open) covering of M if each $A_i \in \mathcal{C}$ is an open (respectively preopen, regular open, Fr-B-open) M-set of (M, τ).*

Definition 3.15. *Let (M, τ) be an M-topological space. A cover $\mathcal{C}$ of M is said to be a closed (respectively regular closed, preclosed, Fr-B-closed) cover of M if $\bigcup_{i \in I} A_i = M$ with max $\{C_{A_i}(x), i \in I$ where each $A_i \subseteq M\} = C_M(x)$, for all x in X, where each $A_i \in \mathcal{C}$ is a closed (respectively regular closed, preclosed, Fr-B-closed) M-set with $\max\{C_{A_i}(x), i \in I\} = C_M(x)$, for all x in X.*

Definition 3.16. *An M-topological space (M,τ) is said to be a strongly S-closed M-space if every closed cover of M has a finite subcover i.e., for any collection $\{A_i \subseteq M, i \in I\}$ of closed M-sets with $\bigcup_{i\in I} A_i = M$ and $\max\{C_{A_i}(x), i \in I\} = C_M(x)$, for all x in X, there exists a finite subset J of I such that $\bigcup_{i\in J} A_i = M$ with $\max\{C_{A_i}(x), i \in J\} = C_M(x)$, for all x in X.*

Definition 3.17. *An M-topological space (M,τ) is said to be a compact M-space if every open covering of M contains a finite sub collection that also covers M i.e., for any collection $\{A_i \subseteq M, i \in I\}$ of open M-sets with $\bigcup_{i\in I} A_i = M$ and $\max\{C_{A_i}(x), i \in I\} = C_M(x)$, for all x in X, there exists a finite subset J of I such that $\bigcup_{i\in J} A_i = M$ with $\max\{C_{A_i}(x), i \in J\} = C_M(x)$, for all x in X.*

Definition 3.18. *An M-topological space (M,τ) is said to be a Fr-B-compact M-space if every Fr-B-open cover of M has a finite subcover i.e., for any collection $\{A_i \subseteq M, i \in I\}$ of Fr-B-open M-sets with $\bigcup_{i\in I} A_i = M$ and $\max\{C_{A_i}(x), i \in I\} = C_M(x)$, for all x in X, there exists a finite subset J of I such that $\bigcup_{i\in J} A_i = M$ with $\max\{C_{A_i}(x), i \in J\} = C_M(x)$, for all x in X.*

Example 3.19. *Let $X = \{a,b\}$, $W = 2$ and $M = \{2/a, 2/b\}$, $\tau = \{M, \phi, \{2/a\}, \{1/a\}, \{2/b\}, \{1/a, 2/b\}\}$. Clearly τ is an M-topology and the ordered pair (M,τ) is an M-topological space. The Fr-B-open covering $\{M, \phi, \{2/a\}, \{1/a\}, \{2/b\}, \{1/b\}, \{2/a, 1/b\}, \{1/a, 1/b\}, \{1/a, 2/b\}\}$ of M contains a finite subcollection $\{\phi, \{1/a, 2/b\}, \{2/a, 1/b\}, \{2/a\}\}$ that also cover M. Hence, (M,τ) is a Fr-B-compact M-space.*

Proposition 3.20. *A contra-Fr-B-continuous image of a Fr-B-compact M-space is strongly S-closed.*

Definition 3.21. *An M-topological space (M,τ) is said to be strongly Fr-B-closed if every Fr-B-closed cover of M has a finite subcover i.e., for any collection $\{A_i \subseteq M, i \in I\}$ of Fr-B-closed M-sets, such $\bigcup_{i\in I} A_i = M$ with $\max\{C_{A_i}(x), i \in I\} = C_M(x)$, for all x in X, there exist a finite subset J of I such that $\bigcup_{i\in J} A_i = M$ with $\max\{C_{A_i}(x), i \in J\} = C_M(x)$, for all x in X.*

Example 3.22. *Let $X = \{a,b\}$, $W = 3$ and $M = \{2/a, 3/b\}$, $\tau = \{M, \phi, \{1/a\}, \{2/b\}, \{1/a, 2/b\}\}$. Clearly τ is an M-topology and the ordered pair (M,τ) is an M-topological space. The Fr-B-closed covering $\{M, \phi, \{3/b\}, \{2/a\}, \{2/a, 1/b\}, \{1/a\}, \{2/a, 2/b\}, \{1/a, 1/b\}, \{1/a, 2/b\}, \{2/b\}, \{1/b\}\}$ of M contains a finite subcollection $\{\phi, \{2/a, 2/b\}, \{3/b\}, \{2/a, 1/b\}\}$ that also cover M. Hence, (M,τ) is a strongly Fr-B-closed M-space.*

Proposition 3.23. *A contra-Fr-B-continuous image of a strongly Fr-B-closed M-space is compact.*

Proof. Let (M,τ) and (N,σ) be any two M-topological spaces. Suppose that $f : (M,\tau) \to (N,\sigma)$ is a contra-Fr-B-continuous surjection. Let $\{A_i \in \sigma, i \in I,$ where I is an indexed set$\}$ be any open cover of (N,σ). Then $\bigcup_{i\in I} A_i = M$ with $\max\{C_{A_i}(x), i \in I\} = C_M(x)$, for all x in X. Since f is a contra-Fr-B-continuous M-set function, then $\{f^{-1}(A_i),\ i \in I\}$ is an Fr-B-closed cover of (M,τ), then $\bigcup_{i\in I} f^{-1}(A_i) = M$. Since (M,τ) is strongly-Fr-B-closed, then there exist a finite subset J of I such that $M = \bigcup_{i\in J} f^{-1}(A_i)$ with $C_M(x) = \max\{C_{f^{-1}(A_i)}, i \in J,$ each $f^{-1}(A_i) \subseteq M$ is a Fr-B-closed M-set $\}$, for all $x \in X$. Thus, $f(M) = N = \bigcup_{i\in J} A_i$, each A_i is an open M-set with $C_N(x) = \max\{C_{A_i}(x), i \in J,$ each A_i is an open M-set $\}$, for all $x \in X$. Hence (N,σ) is compact. □

Proposition 3.24. *Let (M,τ) and (N,σ) be any two M-topological spaces. If $f : (M,\tau) \to (N,\sigma)$ is a Fr-B-irresolute surjective M-set function and (M,τ) is strongly Fr-B-closed then (N, σ) is strongly Fr-B-closed.*

Proof. Suppose that $f : (M,\tau) \to (N,\sigma)$ is a Fr-B-irresolute surjection. Let $\{A_i \subseteq N, i \in I\}$ be any Fr-B-closed cover of (N,σ). Since f is a Fr-B-irresolute M-set function, $\{f^{-1}(A_i), i \in I\}$ is a Fr-B-closed cover of (M,τ) i.e., $\bigcup_{i\in I} f^{-1}(A_i) = M$. Since (M,τ) is strongly Fr-B-closed, there exists a finite subset J of I such that $M = \bigcup_{i\in J} f^{-1}(A_i)$ with $C_M(x) =$

$\max\{C_{f^{-1}(A_i)}, i \in J$, each $f^{-1}(A_i)$ is a Fr-B-closed M-set$\}$, for all $x \in X$. Thus, $f(M) = N = f\left(\bigcup_{i \in J}\{f^{-1}(A_i), i \in J\}\right) = \bigcup_{i \in J} A_i$, each $A_i \subseteq N$ is a Fr-B-closed M-set with $C_N(x) = \max\{C_{A_i}, i \in J$, each A_i is a Fr-B-closed M-set$\}$, for all $x \in X$. Hence (N, σ) is strongly Fr-B-closed. $\square$

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References

[1] S. S. Benchalli and Umadevi I Neeli, *Semi-totally continuous functions in topological spaces*, International Mathematical Forum, 6(10)(2011), 476-492.

[2] Chakrabarty, Kankana and Ioan Despi, *n^k-bags*, Int. J. Intell. Syst., 22(2)(2007), 223-236.

[3] J. Dontchev, *Contra-Continuous functions and strongly S-closed spaces*, Int. J. Math. Sci., 19(1996), 303-310.

[4] J. Dontchev and T. Noiri, *Contra-SemiContinuous functions*, Math. Panonical, 10(1999), 159-168.

[5] K. P Girish and Sunil Jacob John, *On Multiset Topologies*, Computer and Mathematics with Applications, (2012), 37-52.

[6] T. Indira and K. Rekha, *Applications of *b-open Sets and **b-open Sets in Topological Spaces* Annals of Pure and Applied Mathematics, 1(1)(2012), 44-56.

[7] R. James Munkres, *Topology*, Prentice Hall of India Private Limited, New Delhi.

[8] J. Mahanta and D. Das, *Semi Compactness In Multiset Topology*, Arvix : 1403.5642v2[math. GM] 21 Nov 2014.

[9] Miguel Caldas, Saeid Jafari and Govindappa Navalagi, *More on Λ-closed sets in topological Spaces*, Revista Columbiana de Mathematics, 41(2)(2007), 355-369.

[10] Sobhy El-Sheikh, Rajab Omar and Mahamoud Raafat, *Separation Axioms On Multiset Topological Space*, Journal of New Theory, 7(2015), 11-21.