Int. J. Math. And Appl., 6(3)(2018), 399–402
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

Applications of Partial Differential Equations

Gaurav Singh Sisodia[1] and Shyam Sunder[1,*]

1 Department of Applied Science & Humanities, Ch. Devi Lal State Institute of Engineering & Technology, Sirsa, Haryana, India.

Abstract: The hypothesis of partial differential equations (neighborhood and worldwide nearness, consistency, dispersing hypothesis) is inconceivable and has been focused comprehensively by various makers. Only, the procedures became so far limit to differential issues with basic data in issue, fundamentally because of the critical imagined by different arrangements in the examination of partial differential directors. For a case of results and a lovely preface to the field, we propose the partial differential equations. Right now, focus on the arrangement of partial differential equation. Generally speaking, differential data in an adjustment space is more unpleasant than some random one of every a fragmentary potential space and this low-consistency is charming when in doubt. The proposition of partial differential equations were exhibited by Feichtinger during the 80s and have attested themselves recently as the right spaces in partial differential examination. The current paper highlights the functions used for the solution of partial differential equation.

Keywords: Function, Solution, Equation, Partial, Differential.

 Accepted on: 12.09.2018

1. Introduction

The collaboration of partial differential equation with molecule features in a couple of estimations is of both physical and scientific interests. Such arrangements have been found in the numerical reenactments of the full Partial differential equation framework with quick Kerr (or cubic) nonlinearity in two space estimations (2D). They are short partial arrangements that spread without essentially changing shapes over a long detachment and have only a couple differential equations under their envelopes. They have been found significant as information transporters in correspondence, as essentialness sources, switches and method of reasoning entry ways in optical devices. In one space estimation (1D), the partial differential framework displaying equations inciting in nonlinear equation surrenders partial differential equations as cautious arrangements, in any case called the solitons. Or maybe, differential arrangements are progressively solid. The supposed arrangements are of various scale equations with specific stage/bundle respectability and sufficiency components. Regardless of the way that partial numerical propagations of the full partial differential equation framework are moving, asymptotic gauge is essential for investigation in a couple of room estimations. The gauge of partial differential equation framework has been comprehensively analyzed. Long equations are all around approximated through envelope surmise by the partial differential equations. A connection between's partial differential equation arrangements and those of an expanded equations in like manner exhibited that the gauge works reasonably well on short stable partial differential framework. Scientific investigation on the authenticity of partial differential equation estimation of numerical issues has been finished. In any case, in 2D, the envelope gauge with the partial differential equation isolates, considering the way that fundamental breakdown of the partial differential equation occurs in restricted time. On the other hand, due to the trademark physical segment or partial

* *E-mail:*

differential equation, this framework itself consistently continues fine past the issue. One case is the semi-customary two level dissipationless partial differential equation framework where smooth arrangements proceed until the finish of time. It is consequently an extraordinarily interesting request how to modify the partial differential equation to get the correct answers for demonstrating the inducing and joint effort of numeric framework.

2. Applications of Partial Differential Equations

In this study, we consider the Spline method concerning the partial differential equations:

$$\begin{cases} i\partial_t u = \sqrt{1-\Delta}u + F(u) \text{ in } \mathbb{R}^n \times \mathbb{R}, n \geq 3 \\ u(0) = \varphi \end{cases} \tag{1}$$

$$\begin{cases} \partial_t^2 u + (1-\Delta)u = F(u) \text{ in } \mathbb{R}^n \times \mathbb{R}, n \geq 3 \\ u(0) = \varphi_1, \quad \partial_t u(0) = \varphi_2 \end{cases} \tag{2}$$

The nonlinear part $F(u)$ is of Spline type such that $F(u) = V_\gamma(u)u$ where

$$V_\gamma(u)(x) = \lambda\left(|\cdot|^{-\gamma} * |u|^2\right)(x) = \lambda \int_{\mathbb{R}^n} \frac{|u(y)|^2}{|x-y|^\gamma} dy$$

Here A is a non-zero real number and γ is a positive number less than the space dimension n. The equations (1) and (2) can be rewritten in the form of the integral equations

$$u(t) = U(t)\varphi - i\int_0^t U\left(t-t'\right) F(u)\left(t'\right) dt' \tag{3}$$

$$u(t) = (\cos t\omega)\varphi_1 + \omega^{-1}(\sin t\omega)\varphi_2 - \int_0^t \omega^{-1}\left(\sin\left(t-t'\right)\omega\right) F(u)dt' \tag{4}$$

where $\omega = \sqrt{1-\Delta}$ and the associated unitary group $U(t)$ is realized by the transform as

$$U(t)\varphi = \left(e^{-it\omega}\varphi\right)(x) \equiv \frac{1}{(2\pi)^n}\int_{\mathbb{R}^n} e^{ix\cdot\xi} e^{-it\sqrt{1+|\xi|^2}} \hat{\varphi}(\xi)d\xi$$

Where $\widehat{g}$ denotes the Fourier transform of g defined by

$$\widehat{g}(\xi) = \int_{\mathbb{R}^n} e^{-ix\cdot\xi} g(x)dx$$

The operators $\cos t\omega$ and $\sin t\omega$ are defined by replacing $e^{-it\sqrt{1+|\xi|^2}}$ with $\cos(t\sqrt{1+|\xi|^2})$ and $\sin(t\sqrt{1+|\xi|^2})$, respectively. If the solution u of (1) or (3) has a decay at infinity and smoothness, it satisfies two conservation laws:

$$\|u(t)\|_{L^2} = \|\varphi\|_{L^2},$$

$$E_1(u) \equiv K_1(u) + V(u) = E_1(\varphi)$$

$$K(u) = \frac{1}{2}\langle\sqrt{1-\Delta}u, u\rangle, \quad V(u) = \frac{1}{4}\langle F(u), u\rangle$$

where $\langle,\rangle$ is the complex inner product in L^2. Also the solution of (2) or (4) or satisfies the conservation law:

$$E_2\left(u, \partial_t u\right) \equiv K_2\left(u, \partial_t u\right) + V(u) = E_2\left(\varphi_1, \varphi_2\right)$$

$$K_2\left(u, \partial_t u\right)=\frac{1}{2}\left(\langle \partial_t u, \partial_t u\rangle+\langle\sqrt{1-\Delta} u, \sqrt{1-\Delta} u\rangle\right) \tag{5}$$

The main concern of this study is to establish the global well-posedness and scattering of radial solutions of the equations (1) and (2). The study of the global well-posedness (GWP) and scattering for the semi-relativistic equation (1) has not been long before. In (E. Lenzmann) GWP was considered with a three dimensional Coulomb type potential which corresponds to $\gamma=1$. The first and second authors of the present study showed GWP for $0<\gamma \leq 1$ if $n \geq 2$ and $0<\gamma<1$ if $n=1$ for $0<\gamma<\frac{2n}{n+1}$ if $n \geq 2$, and small data scattering for $\gamma>2$ if $n \geq 3$. In this study we tried to fill the gap $1<\gamma \leq 2$ for GWP under the assumption of radial symmetry. For further study like blowup of solutions, solitary waves, mean field limit problem for semi-relativistic equation, see the references. The first result is on the GWP for radial solutions of (3)

Theorem 2.1. *Let $1<\gamma<\frac{2}{3}$ for $n=3$ and $1<\gamma<2$ for $n \geq 4$.*

Let $\varphi \in H^{\frac{1}{2}}$ be radially symmetric and assume that $\|\varphi\|_{L^2 \text{ is}}$ sufficiently small if $\lambda<0$. Then there exists a unique radial solution $u \in C_b H^{\frac{1}{2}}$ such that $|x|^{-1} u \in L_{loc}^2 L^2$ of (3) satisfying the energy and L^2 conservations (5). We mean H_2^s by H^s and $\dot{H}_2^s$ by $\dot{H}^s$. Hereafter, the space $L_T^q(B)$ denotes $L^q(-T, T; B)$ for $T>0$ and $\|\cdot\|_{L_T^q B}$ its norm for some Banach space B. If $T=\infty$, we use $L^q(B)$ for $L^q(\mathbb{R}; B)$ with norm $\|\cdot\|_{L^q B}$, $1 \leq q \leq \infty$, we also denote $v \in L_T^q(B)$ for all $T<\infty$ by $v \in L_{loc}^q(B)$. The next result is on the small data scattering of radial solutions of (3) for $n \geq 4$.

Theorem 2.2. *Let $\frac{3}{2}<\gamma<2$ for $n=3$ and $\frac{3}{2}<\gamma \leq 2$ for $n \geq 4$. Then there is a real number S and ε such that*

$$\frac{1}{2}<s<\frac{\gamma}{2}, \quad 0<\varepsilon<\min \left(\frac{\gamma}{2}-s, s-\frac{1}{2}\right), \quad 1+s-\varepsilon<\gamma<1+s+\varepsilon \tag{6}$$

For fixed such s and ε, let $(\varphi_1, \varphi_2) \in D_{s+\varepsilon, s+\varepsilon} \times D_{s+\varepsilon-1, s+\varepsilon}$ be radially symmetric data. Then if $\|\varphi_1\|_{D_{2+\varepsilon, \alpha+\varepsilon}}+\|\varphi_2\|_{D_{2+\varepsilon-1, ..+\epsilon}}$ is sufficiently small, then there exists a unique radial solution $u \in C_b H^{s-\frac{1}{2}+\varepsilon} \cap L^2 W_{s, \varepsilon}$ to (4). Moreover, there exist radial functions $\varphi_1^{\pm} \in H^{s-\frac{1}{2}+\epsilon}$ and $\varphi_2^{\pm} \in H^{s-\frac{3}{2}+\varepsilon}$ such that

$$\left\|u(t)-u^{\pm}(t)\right\|_{H^{\infty-\frac{1}{2}+c}} \rightarrow 0 \text{ as } t \rightarrow \pm \infty$$

where $u^{\pm}$ is the solutions to the Cauchy problem

$$\left\{\begin{array}{l}
\partial_t^2 u^{\pm}+(1-\Delta) u^{\pm}=0 \\
u^{\pm}(0)=\varphi_1^{\pm}, \partial_t u^{\pm}(0)=\varphi_2^{\pm}
\end{array}\right. \tag{7}$$

In the definition of initial data space $D\alpha, \beta$ the space $L^{\frac{2n}{n+2-2\beta}}$ can be slightly weakened by the homogeneous Sobolev space $\dot{H}^{-(1-\beta)}$. In fact, $L^{\frac{2n}{n+2-2\beta}} \hookrightarrow \dot{H}^{-(1-\beta)}$ for $0<\beta<1$. Let $\tilde{D}_{\alpha, \beta}$ be the weakened space $H^{\alpha-\frac{1}{2}} \cap \dot{H}^{-(1-\beta)}$. Then one can easily show that the solution $u \in C_b\left(\mathbb{R}; \dot{H}^{-(1-(s-\varepsilon))}\right)$ and then the existence of scattering operator of (2) on a small neighborhood of the origin in $\tilde{D}_{s+\varepsilon, s-\varepsilon} \times \tilde{D}_{s+\varepsilon-1, s-\varepsilon}$. For details see Remark below.
The lower bound $\frac{3}{2}$ of γ is caused by the condition which follows $|x|^{-a}$ and the L^2 estimate of Bessel from the relation between the weight function such that

$$\int_0^{\infty} r^{1-2a}\left|J_{\frac{n-2}{2}}(r)\right|^2 dr<\infty$$

For the finiteness, the assumption $\frac{1}{2}<a<\frac{n}{2}$ is inevitable because $J_{\frac{n-2}{2}}(r)=O\left(r^{\frac{n-2}{2}}\right)$ as $r \rightarrow 0$ and $\frac{J_{\frac{n-2}{2}}(r)}{2}=O\left(r^{-\frac{1}{2}}\right)$ as $r \rightarrow \infty$. For more explicit formula, see the identity below. Hence for the present it seems hard to improve the range of γ for the small data scattering. From the perspective of negative result for the scattering, it will be very interesting to show the scattering up to the value of γ greater than 1.

3. Conclusion

In the last few years another numerical technique has been increasingly used to solve mathematical models in engineering research, the spline Method. The spline Method has a few distinct advantages over the Finite Element and Finite Difference Methods. The advantage over the Finite Difference Method is that the spline Collocation Method provides a piecewise-continuous, closed form solution. An advantage over the Finite Element Method is that the spline collocation method procedure is simpler and easy to apply many problems involving differential equations. Our experimental results nicely confirm the excellent numerical approximation properties of Spline and their unique combination of high computational efficiency and low memory consumption, thereby showing huge improvements over standard finite-element methods.

References

[1] I. Podlubny, *Fractional Differential Equations: An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their Solution and Some of Their Applications, Mathematics in Science and Engineering*, Vol. 198. Academic Press, San Diego, (2015).

[2] K. S. Miller and B. Ross, *An Introduction to the Fractional Calculus and Fractional Differential Equations*, (2013).

[3] Q. Wang, *Numerical solutions for fractional KdVBurgers equation by Adomian decomposition method*, Appl. Math. Comput., 182(2)(2016), 10481055.

[4] M. Zurigat, S. Momani and A. Alawneh, *Analytical approximate solutions of systems of fractional algebraicdifferential equations by homotopy analysis method*, Comput. Math. Appl., 59(3)(2010), 12271235.

[5] V. Turut and N. Gzel, *Multivariate Pade approximation for solving nonlinear partial differential equations of fractional order*, Abstr. Appl. Anal., 2013(2013), Article ID 746401.

[6] J. Liu and G. Hou, *Numerical solutions of the space-and time-fractional coupled Burgers equations by generalized differential transform method*, Appl. Math. Comput., 217(16)(2011), 70017008.

[7] N. A. Khan, N.-U. Khan, M. Ayaz, A. Mahmood and N. Fatima, *Numerical study of time-fractional fourth-order differential equations with variable coefficients*, J. King Saud Univ. Sci., 23(1)(2015), 9198.

[8] M. Abbas, A. A. Majid, A. I. M. Ismail and A. Rashid, *The application of cubic trigonometric B-spline to the numerical solution of the hyperbolic problems*, Appl. Math. Comput., 239(2014), 7488.

[9] M. Javidi and B. Ahmad, *Numerical solution of fourth-order time-fractional partial differential equations with variable coefficients*, J. Appl. Anal. Comput., 5(1)(2015), 5263.

[10] A. R. Kanth and K. Aruna, *Solution of fractional third-order dispersive partial differential equations*, Egypt. J. Basic Appl. Sci., 2(3)(2015), 190199.