International Journal of Mathematics And its Applications
Volume 3, Issue 1 (2015), 99–108.
ISSN: 2347-1557

On Alpha Generalized Semi Closed Mappings in Intuitionistic Fuzzy Topological Space

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Abstract : In this paper we study the concepts of intuitionistic fuzzy alpha generalized semi-closed mappings in intuitionistic fuzzy topological space. We also study various properties and relations between the other existing intuitionistic fuzzy open and closed mappings.

Keywords : Intuitionistic fuzzy topology, Intuitionistic fuzzy α-generalized semi closed mapping, Intuitionistic fuzzy α-generalized semi open mapping.

AMS Subject Classification: 54A02, 54A40, 54A99, 03F55.

1 Introduction

Hakeem A.Othman and S.Latha[6] have introduced the concept of fuzzy α-open mapping. Intuitionistic fuzzy closed mapping was introduced by Jeon, Jun and Park [5] in 2005. In this paper, we study the concepts of Intuitionistic fuzzy alpha generalized semi-closed mappings as an extension of our work done in the paper [7]. We studied some of the basic properties and also some characterizations and preservation theorems with the help of intuitionistic fuzzy αga $T_{1/2}$ space.

2 Preliminaries

Definition 2.1 ([1]). *Let X be a non empty fixed set. An intuitionistic fuzzy set(IFS in short) A in X is an object having the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$$

where the function $\mu_A(x) : X \to [0,1]$ denotes the degree of membership(namely $\mu_A(x)$) and the function $\nu_A(x) : X \to [0,1]$ denotes the degree of non-membership(namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. IFS(X) denote the set of all intuitionistic fuzzy sets in X.

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Definition 2.2 ([1]). *Let A and B be IFSs of the form* $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ *and* $B=\{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}$. *Then*

(i) $A \subseteq B$ *if and only if* $\mu_A(x) \leq \mu_B(x)$ *and* $\nu_A(x) \geq \nu_B(x)$ *for all* $x \in X$,

(ii) $A = B$ *if and only if* $A \subseteq B$ *and* $B \subseteq A$,

(iii) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in X\}$,

(iv) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\}$,

(v) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\}$.

For the sake of simplicity, we shall use the notation $A=\langle x, \mu_A, \nu_A\rangle$ *instead of* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$.

Definition 2.3 ([1]). *The intuitionistic fuzzy sets* $0_\sim = \{\langle x, 0, 1\rangle : x \in X\}$ *and* $1_\sim = \{\langle x, 1, 0\rangle : x \in X\}$ *are the empty set and the whole set of X respectively.*

Definition 2.4 ([3]). *An intuitionistic fuzzy topology (IFT in short) on X is a family* τ *of IFSs in X satisfying the following axioms.*

(i) $0_\sim, 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$,

(iii) $\cup G_i \in \tau$ *for any family* $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair (X, τ) *is called an intuitionistic fuzzy topological space(IFTS in short) and any IFS in* τ *is known as an intuitionistic fuzzy open set(IFOS in short) in X. The complement* A^c *of an IFOS A in an IFTS* (X, τ) *is called an intuitionistic fuzzy closed set(IFCS in short) in X.*

Definition 2.5 ([3]). *Let* (X, τ) *be an IFTS and* $A = \langle x, \mu_A, \nu_A\rangle$ *be an IFS in X. Then the intuitionistic fuzzy interior and the intuitionistic fuzzy closure are defined as follows:*

(i) $int(A) = \cup\{G \mid G$ *is an IFOS in X and* $G \subseteq A\}$,

(ii) $cl(A) = \cap\{K \mid K$ *is an IFCS in X and* $A \subseteq K\}$.

Note that for any IFS A in (X, τ), *we have* $cl(A^c) = (int(A))^c$ *and* $int(A^c) = (cl(A))^c$.

Proposition 2.6 ([3]). *For any IFSs A and B in* (X, τ), *we have*

(i) $int(A) \subseteq A$,

(ii) $A \subseteq cl(A)$,

(iii) $A \subseteq B \Rightarrow int(A) \subseteq int(B)$ *and* $cl(A) \subseteq cl(B)$,

(iv) $int(int(A)) = int(A)$,

(v) $cl(cl(A)) = cl(A)$,

(vi) $cl(A \cup B) = cl(A) \cup cl(B)$,

(vii) $int(A \cap B) = int(A) \cap int(B)$.

Proposition 2.7 ([3]). *For any IFS A in* (X, τ), *we have*

(i) $int(0_\sim) = 0_\sim$ *and* $cl(0_\sim) = 0_\sim$,

(ii) $int(1_\sim) = 1_\sim$ *and* $cl(1_\sim) = 1_\sim$,

(iii) $(int(A))^c = cl(A^c)$,

(iv) $(cl(A))^c = int(A^c)$.

Definition 2.8. *An IFS* $A = \langle x, \mu_A, \nu_A \rangle$ *in an IFTS* (X, τ) *is said to be an*

(i) intuitionistic fuzzy regular closed set(IFRCS in short) if $A = cl(int(A))$ *[3],*

(ii) intuitionistic fuzzy α*-closed set(IF*α*CS in short) if* $cl(int(cl(A))) \subseteq A$ *[5],*

(iii) intuitionistic fuzzy semiclosed set(IFSCS in short) if $int(cl(A)) \subseteq A$ *[3],*

(iv) intuitionistic fuzzy preclosed set(IFPCS in short) if $cl(int(A)) \subseteq A$ *[3].*

Definition 2.9 ([9]). *Let A be an IFS of an IFTS* (X, τ). *Then*

(i) $\alpha cl(A) = \cap\{K \mid K$ *is an IF*α*CS in X and* $A \subseteq K\}$,

(ii) $\alpha int(A) = \cup\{K \mid K$ *is an IF*α*OS in X and* $K \subseteq A\}$.

Definition 2.10. *An IFS A of an IFTS* (X, τ) *is an*

(i) intuitionistic fuzzy generalized closed set(IFGCS in short) if $cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is an IFOS in X [11],*

(ii) intuitionistic fuzzy alpha generalized closed set(IFα*GCS in short) if* $\alpha cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is an IFOS in X [9],*

(iii) intuitionistic fuzzy generalized semiclosed set(IFGSCS in short) if $scl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is an IFOS in X [10],*

(iv) intuitionitic fuzzy alpha generalized semi-closed set(IFα*GSCS in short) if* $\alpha cl(A) \subseteq U$ *whenever A* $\subseteq U$ *and U is an IFSOS in X [7].*

The complements of the above mentioned intuitionistic fuzzy closed sets are called their respective intuitionistic fuzzy open sets.

Definition 2.11. *Let f be a mapping from an IFTS* (X, τ) *into an IFTS* (Y, σ). *Then f is said to be an*

(i) intuitionistic fuzzy closed mapping (IF closed map in short) if f(A) is an IFCS in (Y, σ) *for every IFCS A of* (X, τ) *[8],*

(ii) intuitionistic fuzzy α*-closed mapping (IF*α *closed map in short) if f(A) is an IF*α*CS in* (Y, σ) *for every IFCS A of* (X, τ) *[8],*

(iii) *intuitionistic fuzzy pre closed mapping (IFP closed map in short) if f(A) is an IFPCS in (Y, σ) for every IFCS A of (X, τ) [8],*

(iv) *intuitionistic fuzzy generalized semi-closed mapping (IFGS closed map in short) if f(A) is an IFGSC in (Y, σ) for every IFCS A of (X, τ) [8],*

(v) *intuitionistic fuzzy α generalized closed mapping (IFαG closed map in short) if f(A) is an IFαGCS in (Y, σ) for every IFCS A of (X, τ) [9].*

Definition 2.12 ([7]). *An IFTS (X, τ) is said to be an*

(i) *intuitionistic fuzzy $\alpha gaT_{1/2}$(in short IF$_{\alpha ga}T_{1/2}$)space if every IFαGSCS in X is an IFCS in X,*

(ii) *intuitionistic fuzzy $\alpha gbT_{1/2}$(in short IF$_{\alpha gb}T_{1/2}$)space if every IFαGSCS in X is an IFGCS in X,*

(iii) *intuitionistic fuzzy $\alpha gcT_{1/2}$(in short IF$_{\alpha gc}T_{1/2}$)space if every IFαGSCS in X is an IFGSCS in X.*

3 Intuitionistic Fuzzy α-generalized Semi Closed Mappings

In this section we introduce intuitionistic fuzzy α-generalized semi closed mapping and study some of its properties.

Definition 3.1. *A mapping f:(X, τ) $\rightarrow$ (Y, σ) is called an intuitionistic fuzzy α-generalized semi closed(IFαGS closed in short) if for every IFCS A of (X, τ), f(A) is an IFαGSCS in (Y, σ).*

Example 3.2. *Let X = $\{a, b\}$, Y = $\{u, v\}$, G_1 = $\langle x, (0.8, 0.9), (0.2, 0.1)\rangle$ and G_2 = $\langle y, (0.7, 0.8), (0.3, 0.2)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\rightarrow$ (Y, σ) by f(a) = u and f(b) = v. Then f is an IFαGS closed mapping.*

Definition 3.3. *A mapping f:(X, τ) $\rightarrow$ (Y, σ) is called an intuitionistic fuzzy α-generalized semi open(IFαGS open in short) if for every IFOS A of (X, τ), f(A) is an IFαGSOS in (Y, σ).*

Example 3.4. *Let X = $\{a, b\}$, Y = $\{u, v\}$, G_1 = $\langle x, (0.9, 0.9), (0.1, 0.1)\rangle$ and G_2 = $\langle y, (0.6, 0.9), (0.3, 0.1)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\rightarrow$ (Y, σ) by f(a) = u and f(b) = v. Then f is an IFαGS open mapping.*

Theorem 3.5. *Every IF closed mapping is an IFαGS closed mapping but not conversely.*

Proof. Let f:(X, τ) $\rightarrow$ (Y, σ) be an IF closed mapping. Let A be an IFCS in X. Since f is an IF closed mapping, f(A) is an IFCS in Y. Since every IFCS is an IFαGSCS, f(A) is an IFαGSCS in Y. Hence f is an IFαGS closed mapping. □

Example 3.6. *IFαGS closed mapping $\nrightarrow$ IF closed mapping*
Let X = $\{a, b\}$, Y = $\{u, v\}$, G_1 = $\langle x, (0.2, 0.2), (0.8, 0.8)\rangle$ and G_2 = $\langle y, (0.2, 0.3), (0.7, 0.6)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\rightarrow$ (Y, σ) by f(a) = u and f(b) = v. Since the IFS $G_1{}'$ = $\langle x, (0.8, 0.8), (0.2, 0.2)\rangle$ is IFCS in X,but f($G_1{}'$) = $\langle y, (0.8, 0.8), (0.2, 0.2)\rangle$ is not an IFCS in Y. Therefore f is an IFαGS closed mapping but not an IF closed mapping.

Theorem 3.7. *Every IFα closed mapping is an IFαGS closed mapping but not conversely.*

Proof. Let f:(X, τ) $\to$ (Y, σ) be an IFα closed mapping. Let A be an IFCS in X. Then by hypothesis f(A) is an IFαCS in Y. Since every IFαCS is an IFαGSCS, f(A) is an IFαGSCS in Y. Hence f is an IFαGS closed mapping. □

Example 3.8. *IFαGS closed mapping $\nrightarrow$ IFα closed mapping*
Let X = $\{a, b\}$, Y = $\{u, v\}$, $G_1 = \langle x, (0.1, 0.2), (0.9, 0.8)\rangle$ and $G_2 = \langle y, (0.1, 0.3), (0.8, 0.6)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Since the IFS $G_1{}' = \langle y, (0.9, 0.8), (0.1, 0.2)\rangle$ is IFCS in X, but f($G_1{}'$) is not an IFαCS in Y. Therefore f is an IFαGS closed mapping but not an IFα closed mapping.

Remark 3.9. *IFG closed mapping and IFαGS closed mapping are independent of each other.*

Example 3.10. *IFαGS closed mapping $\nrightarrow$ IFG closed mapping.*
Let X = $\{a, b\}$, Y = $\{u, v\}$, $G_1 = \langle x, (0.6, 0.8), (0.3, 0.2)\rangle$ and $G_2 = \langle y, (0.4, 0.7), (0.5, 0.3)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Then f is IFαGS closed mapping but not IFG closed mapping. Since $G_1{}' = \langle x, (0.3, 0.2), (0.6, 0.8)\rangle$ is IFCS in X but f($G_1{}'$) = $\langle$ y, (0.3, 0.2), (0.6, 0.8)$\rangle$ is not an IFGCS in Y.

Example 3.11. *IFG closed mapping $\nrightarrow$ IFαGS closed mapping.*
Let X = $\{a, b\}$, Y = $\{u, v\}$, $G_1 = \langle x, (0.3, 0.1), (0.7, 0.9)\rangle$ and $G_2 = \langle y, (0.6, 0.8), (0.4, 0.2) \rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f:$ (X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Then f is IFG closed mapping but not an IFαGS closed mapping. Since $G_1{}' = \langle x, (0.7, 0.9), (0.3, 0.1)\rangle$ is IFCS in X but f($G_1{}'$) = $\langle$y, (0.7, 0.9), (0.3, 0.1)$\rangle$ is not IFαGSCS in Y.

Theorem 3.12. *Every IFαGS closed mapping is an IFGS closed mapping but not conversely.*

Proof. Let f:(X, τ) $\to$ (Y, σ) be an IFαGS closed mapping. Let A be an IFCS in X. Then by hypothesis f(A) is an IFαGSCS in Y. Since every IFαGSCS is an IFGSCS, f(A) is an IFGSCS in Y. Hence f is an IFGS closed mapping. □

Example 3.13. *IFGS closed mapping $\nrightarrow$ IFαGS closed mapping.*
Let X = $\{a, b\}$, Y = $\{u, v\}$, $G_1 = \langle x, (0.2, 0), (0.8, 0.8)\rangle$ and $G_2 = \langle y, (0.7, 0.2), (0.3, 0.1) \rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Since the IFS $G_1{}' = \langle x, (0.8, 0.8), (0.2, 0)\rangle$ is IFCS in X but f($G_1{}'$) is not IFαGSCS in Y. Therefore f is an IFGS closed mapping but not an IFαGS closed mapping.

Remark 3.14. *IFP closed mappings and IFαGS closed mappings are independent of each other.*

Example 3.15. *IFP closed mapping $\nrightarrow$ IFαGS closed mapping*
Let X = $\{a, b\}$, Y = $\{u, v\}$, $G_1 = \langle x, (0.7, 0.8), (0.2, 0.1)\rangle$ and $G_2 = \langle y, (0.4, 0.3), (0.6, 0.5)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Then f is an IFP closed mapping but not IFαGS closed mapping. Since $G_1{}' = \langle x, (0.2, 0.1), (0.7, 0.8)\rangle$ is IFCS in X but f($G_1{}'$) is IFPCS but not an IFαGSCS in Y.

Example 3.16. *IFαGS closed mapping $\nrightarrow$ IFP closed mapping*
Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.4, 0.5), (0.6, 0.5)\rangle$ and $G_2 = \langle y, (0.3, 0.4), (0.7, 0.6)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f:(X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Since the IFS $A = \langle x, (0.6, 0.5), (0.4, 0.5)\rangle$ is IFCS in X but f(A) is IFαGSCS in Y but not IFPCS in Y. Therefore f is an IFαGS closed mapping but not an IFP closed mapping.

Theorem 3.17. *Let $f:(X, \tau) \to (Y, \sigma)$ be a mapping and let f(A) be an IFRCS in Y for every IFCS A in X. Then f is an IFαGS closed mapping.*

Proof. Let A be an IFCS in X. Then f(A) is an IFRCS in Y. Since every IFRCS is an IFαGSCS, f(A) is an IFαGSCS in Y. Hence f is an IFαGS closed mapping. □

Theorem 3.18. *Every IFαGS closed mapping is an IFαG closed mapping.*

Proof. Let $f:(X, \tau) \to (Y, \sigma)$ be an IFαGS closed mapping. Let A be an IFCS in X. Since f is an IFαGS closed mapping, f(A) is an IFαGSCS in Y. Since every IFαGSCS is an IFαGCS, f(A) is an IFαGCS in Y. Hence f is an IFαG closed mapping. □

Example 3.19. *IFαG closed mapping $\nrightarrow$ IFαGS closed mapping*
Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.6, 0.5), (0.3, 0.4)\rangle$ and $G_2 = \langle y, (0.1, 0.3), (0.7, 0.6) \rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f:(X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Since the IFS $A = \langle y, (0.3, 0.4), (0.6, 0.5)\rangle$ is IFCS in X, f(A) is IFαGCS in Y but not IFαGSCS in Y. Therefore f is an IFαG closed mapping but not an IFαGS closed mapping.

Remark 3.20. *We obtain the following diagram from the results we discussed above.*

$$\begin{tikzcd}
& \text{IF}\alpha\text{G Closed Map} & \\
\text{IF Closed Map} \arrow[ur] \arrow[r] \arrow[dr] & \text{IF}\alpha\text{ Closed Map} \arrow[u] \arrow[r] \arrow[d] & \text{IF}\alpha\text{GS Closed Map} \arrow[ul] \arrow[dl] \\
& \text{IFGS Closed Map} &
\end{tikzcd}$$

None of the reverse implications are not true.

Theorem 3.21. *Let $f:(X, \tau) \to (Y, \sigma)$ be an IFαGS closed mapping. Then f is an IF closed mapping if Y is an $IF_{\alpha ga}T_{1/2}$ space.*

Proof. Let A be an IFCS in X. Then f(A) is an IFαGSCS in Y, by hypothesis. Since Y is an $\text{IF}_{\alpha ga}\text{T}_{1/2}$ space, f(A) is an IFCS in Y. Hence f is an IF closed mapping. □

Theorem 3.22. *Let $f:(X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if Y is an $IF_{\alpha ga}T_{1/2}$ space.*

(i) f is an IFαGS open mapping.

(ii) If A is an IFOS in X then f(A) is an IFαGSOS in Y.

(iii) f(int(A)) $\subseteq$ int(cl(int(f(A)))) for every IFS A in X.

Proof.
(i) $\Rightarrow$ (ii): is obviously true.
(ii) $\Rightarrow$ (iii): Let A be any IFS in X. Then int(A) is an IFOS in X. Then f(int(A) is an IFαGSOS in Y. Since Y is an IF$_{\alpha ga}$T$_{1/2}$ space, f(int(A)) is an IFOS in Y. Therefore f(int(A)) = int(f(int(A)))$\subseteq$ int(cl(int(f(A)))).
(iii) $\Rightarrow$ (i): Let A be an IFOS in X. By hypothesis, f(int(A)) $\subseteq$ int(cl(int(f(A)))). This implies f(A) $\subseteq$ int(cl(int(f(A)))). Hence f(A) is an IFαOS in Y. Since every IFαOS is an IFαGSOS, f(A) is an IFαGSOS in X. Hence f is an IFαGS open mapping. □

Theorem 3.23. *Let f:(X, τ) $\to$ (Y, σ) be an IFαGS closed mapping. Then f is an IFG closed mapping if Y is an IF$_{\alpha gb}$T$_{1/2}$ space.*

Proof. Let A be an IFCS in X. Then f(A) is an IFαGSCS in Y, by hypothesis. Since Y is an IF$_{\alpha gb}$T$_{1/2}$ space, f(A) is an IFGCS in Y. Hence f is an IFG closed mapping. □

Theorem 3.24. *Let f:(X, τ) $\to$ (Y, σ) be an IF closed mapping and g:(Y, σ) $\to$ (Z,η) is an IFαGS closed mapping. Then g$\circ$f : (X, τ) $\to$ (Z,η) is an IFαGS closed mapping.*

Proof. Let A be an IFCS in X. Then f(A) is an IFCS in Y, by hypothesis. Since g is an IFαGS closed mapping, g(f(A)) is an IFαGSCS in Z. Hence g$\circ$f is an IFαGS closed mapping. □

Theorem 3.25. *If f : X $\to$ Y is a mapping, then the following are equivalent if Y is an IF$_{\alpha ga}$T$_{1/2}$ space:*

(i) f is an IFαGS closed mapping.

(ii) f(int(A)) $\subseteq$ αint(f(A)) for each IFS A of X.

(iii) int(f^{-1}(B)) $\subseteq$ f^{-1}(αint(B)) for every IFS B of Y.

Proof. (i) $\Rightarrow$ (ii): Let f be an IFαGS closed mapping. Let A be any IFS in X. Then int(A) is an IFOS in X. By hypothesis, f(int(A)) is an IFαGSOS in Y. Since Y is an IF$_{\alpha ga}$T$_{1/2}$ space, f(int(A)) is an IFOS in Y. We know that every IFOS is an IFαOS, f(int(A)) is an IFαOS in Y. Therefore αint(f(int(A)) = f(int(A)). Hence f(int(A)) = αint(f(int(A)) $\subseteq$ αint(f(A)).
(ii) $\Rightarrow$ (iii): Let B be any IFS in Y. Then f^{-1}(B) is an IFS in X. By hypothesis, f(int(f^{-1}(B)) $\subseteq$ αint(f(f^{-1}(B)) $\subseteq$ αint(B). Therefore int(f^{-1}(B)) $\subseteq$ f^{-1}(αint(B)).
(iii) $\Rightarrow$ (i): Let us assume that A be an IFOS in X. Then int(A) = A and f(A) is an IFS in Y. Then int(f^{-1}f(A)) $\subseteq$ f^{-1}(αint(f(A))), by hypothesis. Now A = int(A) $\subseteq$ int(f^{-1}f(A)) $\subseteq$ f^{-1}(αint(f(A))). Therefore f(A) $\subseteq$ f(f^{-1}(αint(f(A)))) $\subseteq$ αint(f(A)) $\subseteq$ f(A).αint(f(A)) = f(A) is an IFαOS in Y. Hence f(A) is an IFαGSOS in Y. Therefore f is an IFαGS closed mapping. □

Theorem 3.26. *If f:(X, τ) $\to$ (Y, σ) be an IFαGS closed mapping and Y is an IF$_{\alpha gc}$T$_{1/2}$ space, then f is an IFGS closed mapping.*

Proof. Let f:(X, τ) $\rightarrow$ (Y, σ) be a mapping and let A be an IFCS in X. Then by hypothesis f(A) is an IFαGSCS in Y. Since Y is an IF$_{\alpha gc}$T$_{1/2}$ space, f(A) is an IFGSCS in Y. This implies f is an IFGS closed mapping. $\square$

Theorem 3.27. *A mapping f:X $\rightarrow$ Y is an IFαGS open mapping if f(αint(A)) $\subseteq$ αint(f(A)) for every A $\subseteq$ X.*

Proof. Let A be an IFOS in X. Then int(A) = A. Now f(A) = f(int(A)) $\subseteq$ f(αint(A)) $\subseteq$ αint(f(A)), by hypothesis. But αint(f(A)) $\subseteq$ f(A). Hence αint(f(A)) = f(A). That is f(A) is an IFαOS in X. This implies f(A) is an IFαGSOS in X. Hence f is an IFαGS open mapping. $\square$

Theorem 3.28. *Let f:(X, τ) $\rightarrow$ (Y, σ) be a mapping. Then the following conditions are equivalent if Y is an IF$_{\alpha ga}$T$_{1/2}$ space.*

(i) f is an IFαGS closed mapping.

(ii) cl(int(cl((f(A))) $\subseteq$ f(cl(A))) for every IFS A in X.

Proof. (i) $\Rightarrow$ (ii): Let A be an IFS in X. Then cl(A) is an IFCS in X. By hypothesis, f(cl(A)) is an IFαGSCS in Y. Since Y is an IF$_{\alpha ga}$T$_{1/2}$ space, f(cl(A)) is an IFCS in Y. Therefore cl(f(cl(A))) = f(cl(A)). Now clearly cl(int(cl(f(A)))) $\subseteq$ cl(f(cl(A))) = f(cl(A)). Hence cl(int(cl(f(A)))) $\subseteq$ f(cl(A)).
(ii) $\Rightarrow$ (i): Let A be an IFCS in X. By hypothesis cl(int(cl(f(A)))) $\subseteq$ f(cl(A)) = f(A). This implies f(A) is an IFαCS in Y and hence f(A) is an IFαGSCS in Y. That is f is an IFαGS closed mapping. $\square$

Definition 3.29. *A mapping f:X $\rightarrow$ Y is said to be an intuitionistic fuzzy i-alpha generalized semi closed mapping(IFiαGS closed mapping in short) if f(A) is an IFαGSCS in Y for every IFαGSCS A in X.*

Example 3.30. *Let X = {a, b}, Y = {u, v}, G_1 = $\langle$x, (0.6, 0.7), (0.4, 0.3)$\rangle$ and G_2 = $\langle$y, (0.5, 0.6), (0.5, 0.4)$\rangle$. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\rightarrow$ (Y, σ) by f(a) = u and f(b) = v. Then f is an IFiαGS closed mapping.*

Theorem 3.31. *Every IFiαGS closed mapping is an IFαGS closed mapping but not conversely.*

Proof. Assume that the mapping f:X $\rightarrow$ Y be an IFiαGS closed mapping. Let A be an IFCS in X. Then A is an IFαGSCS in X. By hypothesis, f(A) is an IFαGSCS in Y. Hence f is an IFαGS closed mapping. $\square$

Example 3.32. *IFαGS closed mapping $\nrightarrow$ IFiαGS closed mapping.*
Let X = {a, b}, Y = {u, v}, G_1 = $\langle$x, (0.3, 0.2), (0.7, 0.8)$\rangle$ and G_2 = $\langle$y, (0.6, 0.7), (0.4, 0.3)$\rangle$. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\rightarrow$ (Y, σ) by f(a) = u and f(b) = v. Since the IFS A = $\langle$x, (0.8, 0.9), (0.2, 0.1)$\rangle$ is an IFαGSCS in X but f(A) $\langle$y, (0.8, 0.9), (0.2, 0.1)$\rangle$ is not an IFαGSCS in Y.Therefore f is an IFαGS closed mapping but not an IFiαGS closed mapping.

Theorem 3.33. *If f:X $\rightarrow$ Y be an bijective mapping then the following are equivalent:*

(i) f is an IFiαGS closed mapping.

(ii) f(A) is an IFαGSCS in Y for every IFαGSCS A in X.

(iii) f(A) is an IFαGSOS in Y for every IFαGSOS A in X.

Proof. (i) $\Rightarrow$ (ii): It is obviously true.
(ii) $\Rightarrow$ (iii): Let A be an IFαGSOS in X. Then A^c is an IFαGSCS in X. By hypothesis, f(A^c) is an IFαGSCS in Y. That is f(A)^c is an IFαGSCS in Y. That is f(A)^c is an IFαGSCS in Y. Hence f(A) is an IFαGSOS in Y. (iii) $\Rightarrow$ (i): Let A be an IFαGSCS in X.Then A^c is an IFαGSOS in X. By hypothesis, f(A^c) is an IFαGSOS in Y. Hence f(A) is an IFαGSCS in Y. Thus f is an IFiαGS closed mapping. □

Theorem 3.34. *If f:X $\rightarrow$ Y be a mapping where X and Y are IF$_{\alpha ga}T_{1/2}$ space, then the following are equivalent:*

(i) f is an IFiαGS closed mapping.

(ii) f(A) is an IFαGSOS in Y for every IFαGSOS A in X.

(iii) f(αint(B)) $\subseteq$ αint(f(B)) for every IFS B in X.

(iv) αcl(f(B)) $\subseteq$ f(αcl(B)) for every IFS B in X.

Proof. (i) $\Rightarrow$ (ii): It is obviously true. (ii) $\Rightarrow$ (iii): Let B be any IFS in X. Since αint(B) is an IFαOS, it is an IFαGSOS in X. Then by hypothesis, f(αint(B)) is an IFαGSOS in Y. Since Y is an IF$_{\alpha ga}$T$_{1/2}$ space and every IFOS is an IFαOS, f(αint(B)) is an IFαOS in Y. Therefore f(αint(B)) $\subseteq$ αint(f(αint(B))) $\subseteq$ αint(f(B)). (iii) $\Rightarrow$ (iv): It can be proved by taking complement in (3). (iv) $\Rightarrow$ (i): Let A be an IFαGSCS in X. By hypothesis, αcl(f(A)) $\subseteq$ f(αcl(A)). Since X is an IF$_{\alpha ga}$T$_{1/2}$ space and every IFCS is an IFαCS, A is an IFαCS in X. Therefore αcl(f(A)) $\subseteq$ f(αcl(A)) = f(A) $\subseteq$ αcl(f(A)). Hence f(A) is an IFαCS in Y.This implies f(A) is an IFαGSCS in Y. Thus f is an IFiαGS closed mapping. □

4 Conclusion

In this paper we have introduced intuitionistic fuzzy alpha generalized semi-closed mappings and studied some of its basic properties. Also we have studied the relationship between intuitionistic fuzzy open and some of the intuitionistic fuzzy closed mappings already exist.

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