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Two Phases of the Hess Algebraic Decomposition Method Utilized for Watermarking System

Areej M. Abduldaim[1,*]

1 University of Technology, Applied Sciences Department, Mathematics and Computer Applications, Baghdad 10001, Iraq.

Abstract: Some mathematical areas are related to or to be part of computer science. In particular, linear algebra represents a significant tool for increasing the security of digital watermarking systems. On the other hand, with the evolution of social networks, the necessitate of providing information security methods becomes increasingly significant. Therefore, decomposition methods should be studied and analyzed for presenting more secure and appropriate systems. In this paper, since the images represent the most common and widely utilized visual formats, watermarking systems of images relying on algebraic decomposition methods have been proposed. Two phases of the algebraic Hessenberg decomposition are implemented on the original images to show the impact of the matrix decomposition in the embedding process. These two phases are used in two systems, the first one depended on the algebraic Hessenberg decomposition method only and the second used the DWT in addition. The obtained results have been evaluated to show the difference between the utilization of the decomposition method or not.

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Keywords: Hessemberge decomposition method, Watermarking, Discrete wavelet transform (DWT), Upper Hessenberg matrix.

1. Introduction

Since the invention of the computer, mathematics has been the cornerstone of its formation. With the development of computer science, mathematics became an integral part of it, if not the core of this science. As it is well known that linear algebra is a field in mathematics that interest in vectors and matrices, for this reason, it became an important instrument in mathematical security according to the ways that a function can be adjusted to create new results like transformations and matrix decomposition methods. Furthermore, linear algebra methods can extract image features in watermarking systems. All the above help everyone to protect their own resources that are shared on the web after the internet became ubiquitous and digitizing devices such as digital cameras, mobiles and scanners become more available. As an effective system to save digital copyright, digital watermarking systems have been evolving for many years [1–3].

The embedding of the watermark into the digital cover image, video or audio, etc technique to save intellectual property rights is called watermarking system. Without losing generality, matrix decomposition methods are the base of some watermarking systems. Liu et al. in [4] used eigenvalue decomposition (EVD) to introduces a color digital image watermarking method relying on image correction. in the first, the eigenvalues of the pixel of blocks have been found for the color cover image utilizing EVD. After that, to embed the watermark, the eigenvalues absolute sum is modified via the variable quantization method. A hybrid blind digital image watermarking using DCT, DWT, and SVD together is proposed by Begum et

* E-mail: 100044@uotechnology.edu.iq

al. in [5]. The Arnold function is implemented first to hide the watermark in the cover image. Second, before applying SVD, the DCT is performed to the watermark and the cover image followed by DWT. Zhang in [6] used matrix decomposition and Gyrator transform to suggest an image watermarking scheme. Moreover, the author introduced a modern algorithm for obtaining the singular value of an image matrix utilizing the SVD and QR decomposition methods. In particular, a blind symmetric watermarking technique is proposed by Dhar in [7] employing the QR decomposition method and the fan-beam transform (FBT) on color images.

On the other hand, the Hessenberg decomposition method is studied with a lot of other tools [8–13]. Among them, in 2020 [8], a blind symmetric audio watermarking system is introduced by Dhar et al. depending on the Hessenberg decomposition method and parametric Slant-Hadamard transformation. While a heterodoxy blind color images watermarking technique is given in [9] by Abodena and Agoyi relying basically on the Hessenberg decomposition, fast Walsh Hadamard transforms, and the discrete wavelet transform. Su and Chen in [10] utilized the upper Hessenberg matrix of the Hessenberg decomposition method to analyze the image features in a proposed blind color image watermarking algorithm. In addition, to develop the security of this technique, the authors used the Arnold transform. While the Hash pseudo-random method is utilized to improve the robustness of the algorithm.

In this paper, two watermarking systems are proposed based on two phases of the algebraic Hessemberd decomposition method. The first system depends on using the algebraic Hess method only and the second system uses DWT to improve the results obtained from the first system.

The organization of the work is as the following: Section 2 is devoted to explaining the basics of the Hessemberd decomposition method algebraically. Sections 3 and 4 focus on the methodology and the build of the watermarking systems giving all the algorithms. In section 5, the results are obtained for the two systems using several attacks. Finally, in section 6 the conclusion was raised.

2. Hessenberg matrix Decomposition Method

Hessenberg method is introduced in [14] to be a partition any matrix into a product of three matrices as in the form

$$A = \mathcal{P}\mathcal{H}\mathcal{P}^T \tag{1}$$

For illustration, if A is a matrix of size 4×4, so

$$A = \begin{pmatrix} \kappa_{11} & \kappa_{12} & \kappa_{13} & \kappa_{14} \\ \kappa_{21} & \kappa_{22} & \kappa_{23} & \kappa_{24} \\ \kappa_{31} & \kappa_{32} & \kappa_{33} & \kappa_{34} \\ \kappa_{41} & \kappa_{42} & \kappa_{43} & \kappa_{44} \end{pmatrix}$$

$$A = \begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} & \tau_{14} \\ \tau_{21} & \tau_{22} & \tau_{23} & \tau_{24} \\ \tau_{31} & \tau_{32} & \tau_{33} & \tau_{34} \\ \tau_{41} & \tau_{42} & \tau_{43} & \tau_{44} \end{pmatrix} \times \begin{pmatrix} \upsilon_{11} & \upsilon_{12} & \upsilon_{13} & \upsilon_{14} \\ \upsilon_{21} & \upsilon_{22} & \upsilon_{23} & \upsilon_{24} \\ 0 & \upsilon_{32} & \upsilon_{33} & \upsilon_{34} \\ 0 & 0 & \upsilon_{43} & \upsilon_{44} \end{pmatrix} \times \begin{pmatrix} \tau_{11} & \tau_{21} & \tau_{31} & \tau_{41} \\ \tau_{12} & \tau_{22} & \tau_{32} & \tau_{42} \\ \tau_{13} & \tau_{23} & \tau_{33} & \tau_{43} \\ \tau_{14} & \tau_{24} & \tau_{34} & \tau_{44} \end{pmatrix} \tag{2}$$

where the middle matrix H is an upper Hessenberg matrix and the matrix P is orthogonal. obtained by Householder matrices. The Householder matrix Q is an orthogonal matrix of the form

$$\mathcal{Q} = (I_n - uu^T)/u^T u \tag{3}$$

where $u \neq 0$ is a vector in R^n and I_n is the $n \times n$ identity matrix. If A is an $n \times n$ matrix, then it needs n-2 steps. Thus, the Hessenberg decomposition method is found as:

$$P = (\mathcal{Q}_1\mathcal{Q}_2...\mathcal{Q}_{n-3}\mathcal{Q}_{n-2})^T A(\mathcal{Q}_1\mathcal{Q}_2...\mathcal{Q}_{n-3}\mathcal{Q}_{n-2}) \tag{4}$$

$$\mathcal{H} = \mathcal{P}^T A\mathcal{P} \tag{5}$$

where $\mathcal{P} = \mathcal{Q}_1\mathcal{Q}_2...\mathcal{Q}_{n-3}\mathcal{Q}_{n-2}$, to find A again:

$$\mathcal{P}\mathcal{H}\mathcal{P}^T = A \tag{6}$$

3. Two Phases of the Algebraic Hessenberg Decomposition Method for Watermarking System

In this section, a watermarking system is proposed using the Hessenberg matrix decomposition method without using any other transform.

3.1. The Embedding Algorithm Using Hess Method

In the embedding algorithm, the Hessenberg matrix decomposition method is used twice the second period is applied on the $\mathcal{H}$ matrix.

Algorithm 1: The Embedding Algorithm Using 2-Phases Hess Method.

The Input is the Cover Image (I)**, The Watermark** (W).

The Output is the Watermarked Image (WI).

Begin

Step 1: Read the cover image I of size $n \times n$. Convert I to a grayscale type.

Step 2: Partition the cover image I into 8×8 non-overlapping blocks.

Step 3: Implement the Hess method to all blocks depending on Equation (1) to obtain the $\mathcal{H}$ matrix for each block.

Step 4: Implement the Hess method to all blocks of the $\mathcal{H}$ matrix depending on Equation (1).

Step 5: Read the watermark W of size $m \times m$. Convert W to a binary type BW.

Step 6: Embed the BW bits in the matrices $\mathcal{H}$ produced from step 4 by:

$$\mathcal{H}(1,6) = \mathcal{H}(1,6) - \mathcal{H}(1,6)(\text{mod S}) \ + T1 \text{ if } W(i,j) = 1 \tag{7}$$

$$\mathcal{H}(1,6) = \mathcal{H}(1,6) - \mathcal{H}(1,6)(\text{mod S}) \ + T2 \text{ if } W(i,j) = 0, \tag{8}$$

where $0 \leq i \leq 64, 0 \leq j \leq 64$, S is the scaling factor that has authority over the trade-off between the properties of the watermarking system (robustness and imperceptibility), $S = 19, T1 = 0.75 * S, T2 = 0.25 * S$, that set if the watermark bit is 0 or 1.

Step 7: Implement the Hess method to reverse the operation and obtain a watermarked image.

End.

The term (mod) used in step 6 is referred to the modular arithmetic for integers.

The following is the map to illustrate the embedding algorithm:

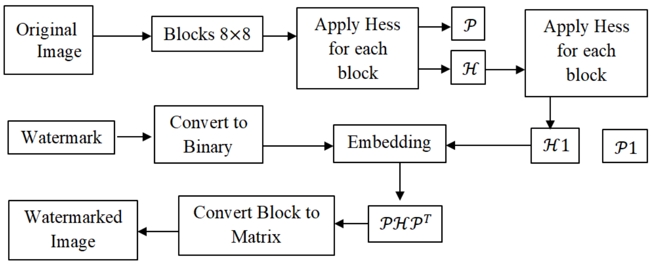

Figure 1: The Flowchart of Embedding W Using 2-Phases Hess Method.

3.2. The Extraction Algorithm Using Hess Method

To complete the proposed watermarking system, the extraction algorithm is given below:

Algorithm 2: The Extraction Algorithm Using 2-Phases Hess Method.

The Input is the Watermarked Image (WI).

The Output is the Watermark (W).

Begin

Step 1: Read the WI of size $n \times n$. Convert WI to a grayscale type.

Step 2: Partition the IW into 8×8 non-overlapping blocks.

Step 3: Implement the Hess method to all blocks.

Step 4: Implement the Hess method to all blocks of the $\mathcal{H}$ matrix.

Step 5: Extract the BW from the matrices $\mathcal{H}$ produced from step 4.

$$W(i,j) = 0 \text{ if } \mathcal{H}(1,6) \pmod{\text{S}} < ave \tag{9}$$

$$W(i,j) = 1 \text{ if } \mathcal{H}(1,6) \pmod{\text{S}} \geq ave \tag{10}$$

where ave is the average of the first column elements in each block.

End.

The following is the map to illustrate the extraction algorithm:

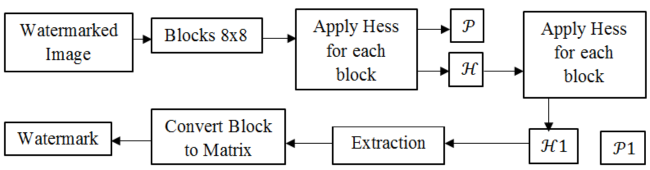

Figure 2: The Flowchart of Extracting W Using 2-Phases Hess Method.

4. Two Phases Hessenberg Matrix Decomposition Watermarking System Using DWT

To attempt to improve the results obtained from the use of the Hess decomposition method only, in this section, a watermarking system is proposed using the Hess method and DWT.

4.1. The Embedding Algorithm Using Hess Method and DWT

In the embedding algorithm, the Hess decomposition method is used twice the second period is applied on the $\mathcal{H}$ matrix.

Algorithm 3: The Embedding Algorithm Using 2-Phases Hess Method and DWT.

The Input is the Cover Image (I)**, The Watermark** (W).

The Output is the Watermarked Image (WI).

Begin

Step 1: Read the cover image I of size $n \times n$. Convert I to a grayscale type.

Step 2: Decompose the cover image using one level of DWT into the sub-bands $\{LL, LH, HL, HH\}$.

Step 3: Partition the LH band into 4×4 non-overlapping blocks.

Step 4: Implement the logistic function to switch locations of all blocks.

Step 5: Implement the 2-Phases Hess method to each block.

Step 6: Read W of size $m \times m$. Convert W to a binary type BW.

Step 7: Embed the BW bits in the matrices $\mathcal{H}$ produced from step 6 by:

$$\mathcal{H}(1,1) = \mathcal{H}(1,1) - \mathcal{H}(1,1)(\text{mod S}) \ + T1 \text{ if } w(i,j) = 1 \tag{11}$$

$$\mathcal{H}(1,1) = \mathcal{H}(1,1) - \mathcal{H}(1,1)(\text{mod S}) \ + T2 \text{ if } w(i,j) = 0 \tag{12}$$

where $S = 19$ is the scaling factor that has authority over the trade-off between the properties of the watermarking system (robustness and imperceptibility), where $T1 = 0.75 * S, T2 = 0.25 * S$ that sets whether the bit of the watermark is 1 or 0.

Step 8: Implement the Hess method, DWT, and the logistic function to reverse the operation and obtain a watermarked image.

End.

4.2. The Extraction Algorithm Using Hess Method and DWT

To complete the proposed watermarking system, the extraction algorithm is given below:

Algorithm 4: The Extraction Algorithm Using 2-Phases Hess Method and DWT.

The Input is the Watermarked Image (WI)**.**

The Output is the Watermark (W).

Begin

Step 1: Read the WI of size $n \times n$. Convert WI to a grayscale type.

Step 2: Decompose WI using 1-level DWT into the sub-bands $\{LL, LH, HL, HH\}$.

Step 3: Partition the LH band into 4×4 non-overlapping blocks.

Step 4: Implement the logistic function to all blocks to restore the authentic position of all blocks.

Step 5: Implement the 2-Phases Hess method to each block to obtain the matrices $\mathcal{H}$.

Step 6: Extract the BW from the matrices $\mathcal{H}$ produced from step 4 by:

$$w(i,j) = 0 \text{ if } \mathcal{H}(1,1)(\text{mod S}) > (T1 + T2)/2 \tag{13}$$

$$w(i,j) = 1 \text{ if } \mathcal{H}(1,1)(\text{mod S}) \leq (T1 + T2)/2 \tag{14}$$

End.

The following is the map to illustrate the embedding algorithm:

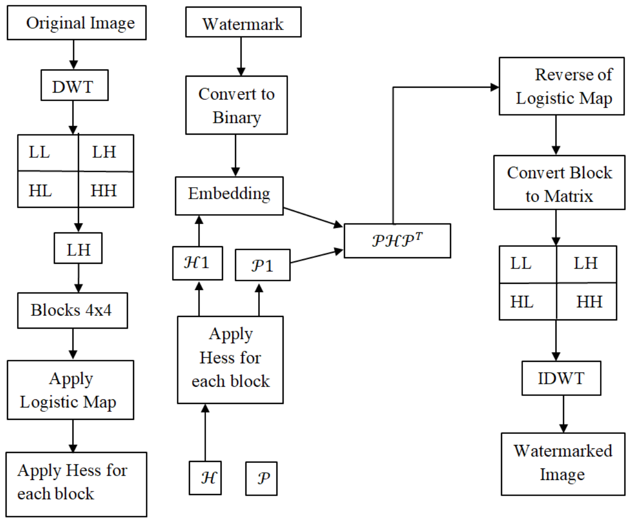

Figure 3: The Flowchart of Embedding W Using 2-Phases Hess Method and DWT.

The following is the map to illustrate the extraction algorithm:

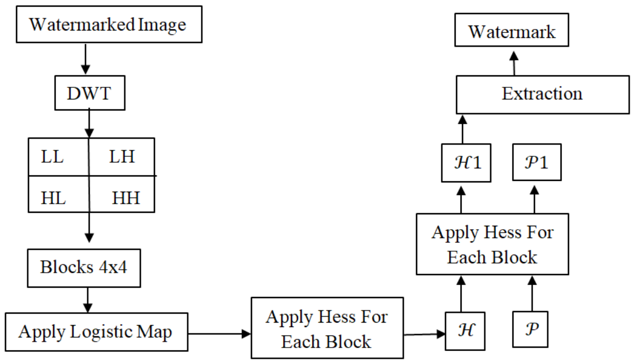

Figure 4: The Flowchart of Extracting W Using 2-Phases Hess Method and DWT.

5. Experimental Results

In this section, the results of the two proposed systems are given.

5.1. The Results of Section 3

Table 1 gives the PSNR and the NC values using the 2-Phases Hess method before using attacks on images.

Image	Image1	Image2	Image3	Image4
PSNR	46.802	48.447	47.149	46.874
NC	1	1	1	1

Table 1: PSNR-NC Values for Images Using 2-Phases Hess Method.

The following figure shows the embedded and the extracted images and extracted watermark (EW):

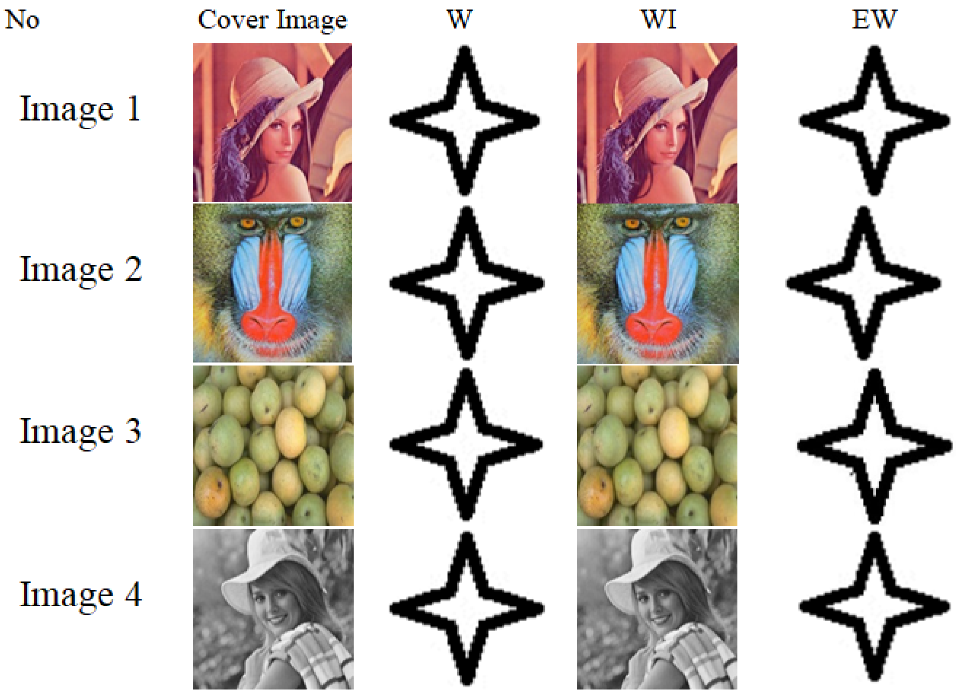

Figure 5: The Embedding and the Extraction of W before Attacks Using 2-Phases Hess Method.

The four used images are attacked by 6 attacks to test the properties of the watermarking system (robustness and imperceptibility) as follows:

Attacks	Image1		Image2		Image3		Image4	
	PSNR	NC	PSNR	NC	PSNR	NC	PSNR	NC
Salt and Pepper %1	26.917	0.879	27.248	0.890	26.076	0.890	26.975	0.887
Adjust Image	22.142	0.709	19.389	0.646	23.542	0.721	21.393	0.745
JPEG Compression	58.579	0.725	58.509	0.797	58.554	0.730	58.534	0.759
Gaussian Noise	37.679	0.644	37.642	0.632	37.664	0.637	37.662	0.633
Speckle Noise	35.649	0.632	35.500	0.629	34.689	0.638	34.963	0.644
Histogram Equalization	19.050	0.641	16.475	0.635	15.932	0.625	18.149	0.648

Table 2: PSNR-NC Values for Images After Attacks Using 2-Phases Hess Method.

The following figure shows the embedded and the extracted images after attacks:

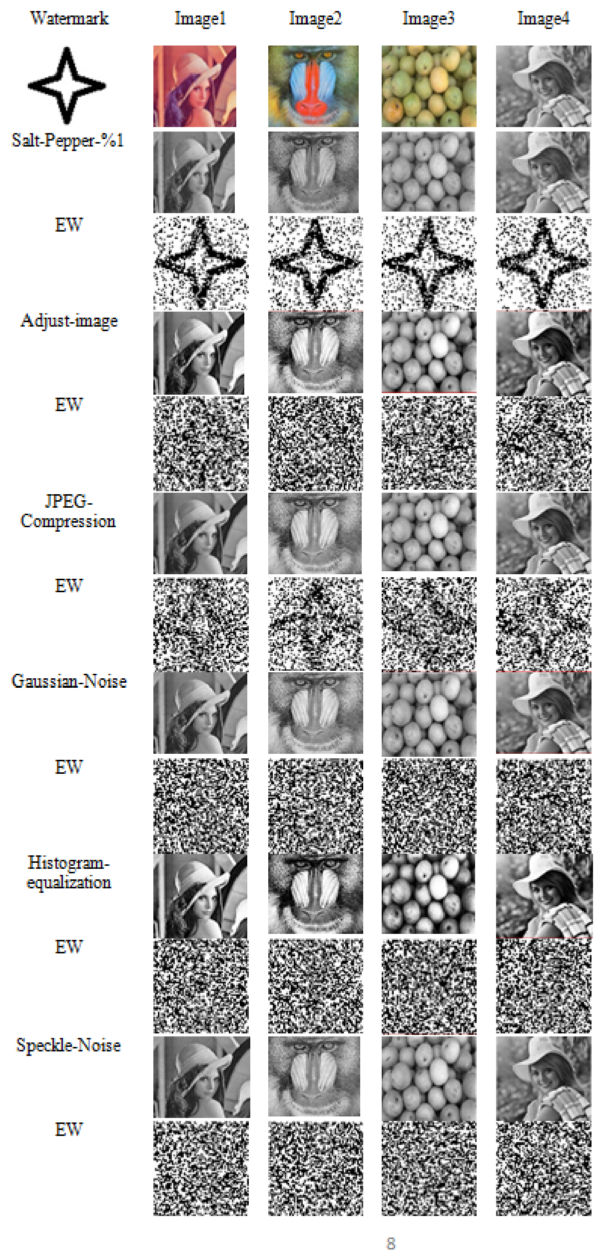

Figure 6: The Embedding and the Extraction of Images Using 2-Phases Hess Method After Attacks.

5.2. The Results of Section 4

Table 3 shows the PSNR-NC values using the 2-Phases Hess method using DWT before using attacks on images.

Image	Image1	Image2	Image3	Image4
PSNR	45.450	47.918	46.153	45.823
NC	1	1	1	1

Table 3: PSNR-NC Values for Images Using 2-Phases Hess Method and DWT.

The following figure shows the embedded and the extracted images:

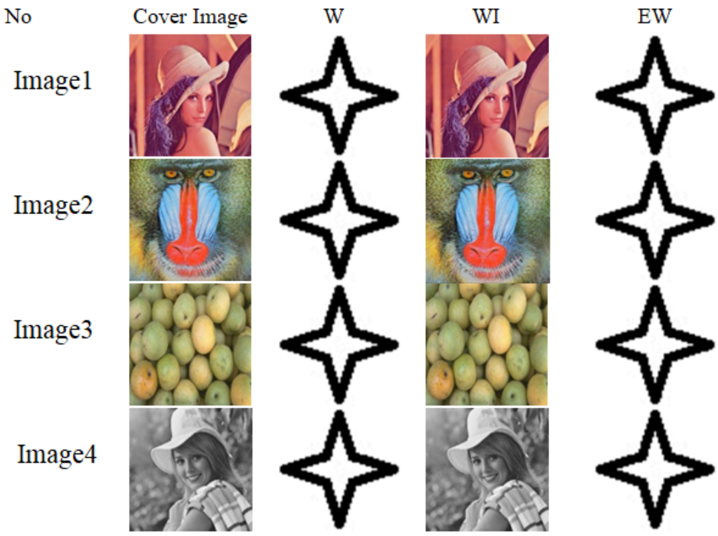

Figure 7: The Embedding and the Extraction of W before Attacks Using 2-Phases Hess Method and DWT.

The images used are attacked by 6 attacks to test the properties of the watermarking system (robustness and imperceptibility) as follows:

Attacks	Image1		Image2		Image3		Image4	
	PSNR	NC	PSNR	NC	PSNR	NC	PSNR	NC
Salt and Pepper %1	22.583	0.971	22.773	0.975	22.909	0.973	22.665	0.976
Adjust Image	22.141	0.531	19.389	0.620	23.483	0.571	21.393	0.661
JPEG Compression	58.619	0.999	58.487	1	58.618	1	58.553	1
Gaussian Noise	37.670	0.897	37.668	0.890	37.660	0.940	37.665	0.893
Speckle Noise	35.631	0.843	35.490	0.824	34.708	0.798	34.955	0.810
Histogram Equalization	19.048	0.745	16.475	0.567	15.935	0.529	18.149	0.694

Table 4: PSNR-NC Values for Images After Attacks Using 2-Phases Hess Method and DWT.

The following figure shows the embedded and the extracted images after attacks:

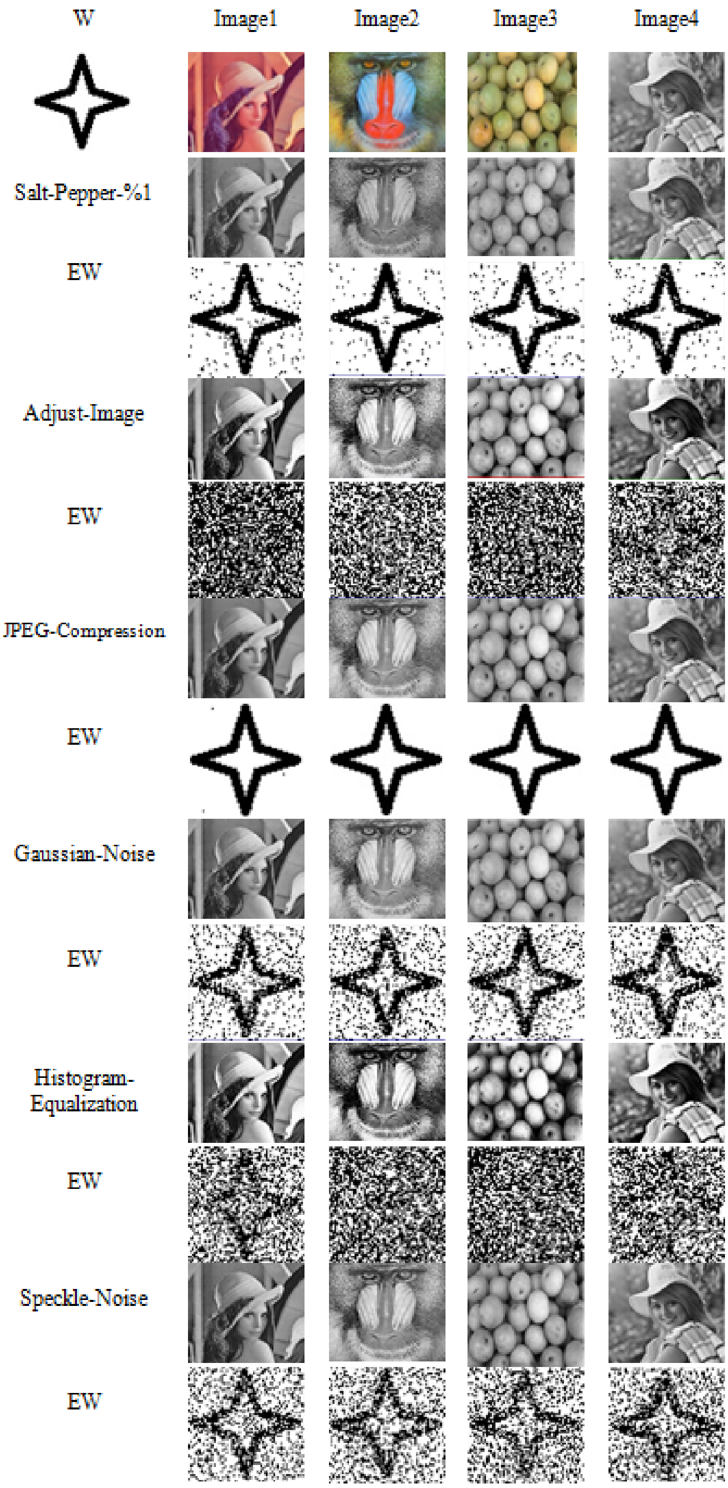

Figure 8: The Embedding and the Extraction of Images Using 2-Phases Hess Method and *DWT* After Attacks.

6. Comparison

Comparing the first watermarking proposed system with the second watermarking proposed system, it is obvious that the second system is better. The *DWT* improved the results obtained in the first one. The following is a comparison between methods used Hessenberg decomposition:

[Ref]	No Attacks		Salt and Pepper %1		JPEG Compression		Gaussian Noise	
	PSNR	NC	PSNR	NC	PSNR	NC	PSNR	NC
Hess – DWT	47.918	1	22.909	0.976	58.619	1	37.670	0.940
[9] HMD - FWHT-DWT	40.159	1	-------	1	-------	1	-------	0.990
[12] HMD – ArnoldT	35.452	1	-------	0.994	-------	0.805	-------	0.864

Table 5: PSNR and The NC value comparison with [9] and [12] method.

7. Conclusion

In this paper, two watermarking systems are proposed depending principally on the algebraic matrix decomposition method named the Hessenberg. The first watermarking system is done using the Hess decomposition method only, while the second system is built using DWT transformation in addition to Hess. Two phases are used in the embedding algorithms in the two systems. According to these systems, the results show that the watermarking system that used the Hess method and DWT is more robust and imperceptible.

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