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B-Algebras Which Generated by $\mathbb{Z}_n$ Group

Pramitha Shafika Wicaksono[1,*], Y. D. Sumanto[1] and Bambang Irawanto[1]

1 Department of Mathematics, Faculty of Science and Mathematics, Diponegoro University, Semarang, Indonesia.

Abstract: B-algebra is an algebraic structure formed from a non-empty set equipped with a binary operation with a 0 constant, B-algebra is a class of K-algebra that can be build from a group. This paper uses the literature study method from journals related to B-algebra, the set of all B-homomorphisms, and B-algebra generated from the group of the sets all integers modulo n. Based on the analysis carried out, it was concluded that the group of the sets all integers modulo n equipped with the addition operation modulo n can construct B-algebra, and the B-algebra is 0-commutative. If the function from the set of all integers modulo n to the sets of all integers modulo n is a group homomorphism then the function is also B-homomorphism.

Keywords: B-algebras, group, B-homomorphism, the sets all integers modulo n, homomorphism group.

1. Introduction

Algebraic structures are non-empty sets equipped with one or more binary operations that fulfill certain axioms or properties. Some examples of studies of algebraic structures are rings, semigroups, and others. As for the study of other algebraic structures such as K-algebras, B-algebra, and others. In 2006 K. H Dar and M. Akram introduced an algebraic structure, namely K-algebra [1]. It is known that K-algebras is divided into two classes based on its building group, namely $BCH/BCL/BCK$-algebra which is built on a commutative group, and B-algebra which is built on groups. This paper discussed the class of K-algebra that is B-algebra. In 2002 J. Neggers and Hee Sik Kim introduced B-algebra, B-algebra is a non-empty set of A with a constant 0 completed by a binary operation and satisfies certain axiom [2]. The algebraic structure of B-algebra is an algebraic structure that can be generated by a group with 0 identity elements [3].

Homomorphism is a mapping between two algebraic structures of the same type that preserves the binary operation of the algebraic structure. The concept of homomorphisms can be found in groups called group homomorphisms. Because B-algebra is an algebraic structure that can be built on groups, B-algebra also has the same concept as groups, namely homomorphism, in B-algebra the concept of a homomorphism is called B-homomorphism. The set of all modulo n integers that are completed by the addition operation modulo n is a group. This paper discusses one of the algebraic structures, namely B-algebra, and shows that B-algebra can be built from the set group of all modulo n integers which is equipped with a modulo n addition operation and shows the properties of B-algebra that is built from the set group of all a modulo n integer equipped with a modulo n addition operation.

* *E-mail: pramithawicaksono@yahoo.co.id*

2. B-Algebras and B-homomorphism

Definition 2.1 ([2]). *A non-empty set A completed by the binary operation "$*$" with constant 0 is B-algebra if it satisfies the axiom:*

(1). $x * x = 0$,

(2). $x * 0 = x$,

(3). $(x * y) * z = x * (z * (0 * y))$, *for every* $x, y, z \in A$.

Proposition 2.2 ([4]). *Let* $(A; *, 0)$ *be B-algebra, then*

(a). $(x * z) * (y * z) = x * y$,

(b). $0 * (x * y) = y * x$,

(c). $0 * (0 * x) = x$, *for every* $x, y, z \in A$.

Proof. Let $x, y, z \in A$ then

(a).
$$\begin{aligned}(x * y) * (y * z) &= x * ((y * z) * (0 * z)) \\ &= x * (y * ((0 * z) * (0 * z))) \\ &= x * (y * 0) \\ &= x * y\end{aligned}$$

(b).
$$\begin{aligned}y * x &= y * (x * 0) \\ &= y * (x * (0 * y) * (0 * y)) \\ &= y * ((x * y) * (0 * y)) \\ &= (y * y) * (x * y) \\ &= 0 * (x * y)\end{aligned}$$

(c). Let $x = 0 * y$, then

$$0 * y = (0 * y) * 0$$
$$0 * y = 0 * (0 * (0 * y))$$
$$x = (0 * (0 * x))$$

□

Theorem 2.3 ([4]). *Let* $(A; *, 0)$ *be B-algebra. If on A is defined operation "$\circ$" by* $x \circ y = x * (0 * y)$ *for every* $x, y \in A$, *then* $(A; \circ, 0)$ *is a group with identity element 0.*

Proof.

(1). Binary operation "$\circ$" is associative. Let $x, y, z \in A$ then

$$\begin{aligned}(x \circ y) \circ z &= (x \circ y) * (0 * z) \\ &= (x * (0 * y)) * (0 * z) \\ &= x * ((0 * z) * (0 * (0 * y)))\end{aligned}$$

$$
\begin{aligned}
&= x * ((0 * z) * y) \\
&= x * (0 * (y * (0 * z))) \\
&= x \circ (y * (0 * z)) \\
&= x \circ (y \circ z)
\end{aligned}
$$

(2). There exist $0 \in A$ as an identity element that is for every $x \in A$ implies

$$
\begin{aligned}
0 \circ x &= 0 * (0 * x) \\
&= x \\
\text{and} \quad x \circ 0 &= x * (0 * 0) \\
&= x * 0 \\
&= x
\end{aligned}
$$

(3). Each element in $(A; \circ, 0)$ has inverse. Let $x \in A$ there exists $x^{-1} = (0 * x) \in A$ then

$$
\begin{aligned}
x \circ x^{-1} &= x * \left(0 * x^{-1}\right) \\
&= x * (0 * (0 * x) \\
&= x * x \\
&= 0 \\
\text{and} \quad x^{-1} \circ x &= x^{-1} * (0 * x) \\
&= (0 * x) * (0 * x) \\
&= 0
\end{aligned}
$$

□

Proposition 2.4. *Let $(A; \circ, 0)$ be group with identity element 0. If defined $x * y = x \circ y^{-1}$ for every $x, y \in A$, then $(A; *, 0)$ is B-algebra.*

Proof. Let $x, y, z \in A$ then

(1). $x * x = x \circ x^{-1} = 0$

(2). $x * 0 = x \circ 0^{-1} = x \circ 0 = x$

(3). $(x * y) * z = \left(x \circ y^{-1}\right) \circ z^{-1}$

$$
\begin{aligned}
&= x \circ (y^{-1} \circ z^{-1}) \\
&= x \circ (z \circ y)^{-1} \\
&= x * (z \circ y) \\
&= x * (z * y^{-1}) \\
&= x * (z * (0 * y))
\end{aligned}
$$

□

Definition 2.5 ([5]). *Let $(A;*,0)$ be B-algebra, B-algebra said to be 0-commutative if $x*(0*y)=y*(0*x)$ for every $x,y\in A$.*

Proposition 2.6 ([4]). *If $(A;*,0)$ is 0-commutative B-algebra, then $(0*x)*(0*y)=y*x$, for every $x,y\in A$.*

Proof. Let $x,y\in A$ then $(0*x)*(0*y)=y*(0*(0*x))=y*x$. □

Theorem 2.7 ([6]). *If $(A;*,0)$ is 0-commutative B-algebra, then $(x*a)*(y*b)=(b*a)*(y*x)$, for every $x,y,a,b\in A$.*

Proof. Let $x,y,a,b\in A$, then

$$\begin{aligned}
(x*a)*(y*b) &= x*((y*b)*(0*a)) \\
&= x*(y*((0*a)*(0*b))) \\
&= x*(y*(b*a)) \\
&= x*(y*((b*a)*0)) \\
&= x*(y*(0*(0*(b*a)))) \\
&= (x*(0*(b*a)))*y \\
&= ((b*a)*(0*x))*y \\
&= (b*a)*(y*(0*(0*x))) \\
&= (b*a)*((y*x)*0) \\
&= (b*a)*(y*x)
\end{aligned}$$

□

Theorem 2.8. *Let $(A;\circ,0)$ be a commutative group. If defined $x*y=x\circ y^{-1}$, for every $x,y\in A$, then $(A;*,0)$ is 0-commutative B-algebra.*

Proof. Let $x,y\in A$ then

$$\begin{aligned}
x\circ y &= x\circ\left(y^{-1}\right)^{-1} \\
&= x\circ\left(0\circ y^{-1}\right)^{-1} \\
&= x*(0*y) \\
y\circ x &= y\circ\left(x^{-1}\right)^{-1} \\
&= y\circ\left(0\circ x^{-1}\right)^{-1} \\
&= y*(0*x)
\end{aligned}$$

So that if $x\circ y=y\circ x$ we obtained $x*(0*y)=y*(0*x)$, so based on Definition 1.5, $(A;*,0)$ is 0-commutative B-algebra. □

Definition 2.9 ([7]). *Let $(A;*,0)$ and $(B;\circ,0')$ be B-algebra. A function φ from A to B, donated by $\varphi:A\to B$ is called B-homomorphism if for every $x,y\in A$ applies $\varphi(x*y)=\varphi(x)\circ\varphi(y)$, where $\varphi(x),\varphi(y)\in B$.*

Example 2.10. *Let $(A;*,0)$ and $(B;\circ,0')$ be B-algebra. A trivial B-homomorphism $\theta:A\to B$ defined by $\theta(x)=0'$ is B-homomorphism for every $x\in A$.*

Definition 2.11. *Let $(A;*,0)$ and $(B;\circ,0')$ be B-algebra, then set all B-homomorphism from $(A;*,0)$ to $(B;\circ,0')$ donated by B-$Hom\,(A,B)$.*

Definition 2.12. *Let $(A;*,0)$ and $(B;\circ,0')$ be B-algebra. Let B-$Hom\,(A,B)$ is set all B-homomorphism from B-algebra $(A;*,0)$ to B-algebra $(B;\circ,0')$. In B-$Hom\,(A,B)$ defined operation " $\circledast$ " by $(\varphi\circledast\vartheta)(x)=\varphi(x)\circ\vartheta(x)$, for every $\varphi,\vartheta\in B-Hom\,(A,B)$, and $\theta(x)=0'$ for every $x\in A$.*

Example 2.13. *Let $(A;*,0)$ be not 0-commutative B-algebra. Suppose given a function $\theta, I: A\to A$ which are respectively defined with $\theta(x)=0$, $I(x)=x$ for every $x\in A$ then B-$Hom\,(A,A)$ with operation " $\circledast$ " and constant θ is not B-algebra.*

Theorem 2.14 ([8])**.** *Let $(A;*,0)$ be B-algebra and $(B;\circ,0')$ be 0-commutative B-algebra,then (B-$Hom\,(A,B)\,;\circledast,\theta)$ is 0-commutative B-algebra with $\circledast$ and θ as defined in Definition 2.12.*

Proof. Let $\varphi,\vartheta\in$ B-$Hom\,(A,B)$ and $x,y\in A$ then

$$\begin{aligned}(\varphi\circledast\vartheta)(x*y)&=\varphi(x*y)\circ\vartheta(x*y)\\&=(\varphi(x)\circ\varphi(y))\circ(\vartheta(x)\circ\vartheta(y))\\&=(\vartheta(y)\circ\varphi(y))\circ(\vartheta(x)\circ\varphi(x))\\&=\big(0'\circ(\varphi(y)\circ\vartheta(y))\big)\circ\big(0'\circ(\varphi(x)\circ\vartheta(x))\big)\\&=(\varphi(x)\circ\vartheta(x))\circ(\varphi(y)\circ\vartheta(y))\\&=(\varphi\circledast\vartheta)(x)\circ(\varphi\circledast\vartheta)(y)\end{aligned}$$

So that $\varphi\circledast\vartheta$ in B-$Hom\,(A,B)$ then operation " $\circledast$ " in B-$Hom\,(A,B)$ is a binary operation. Because $(B;\circ,0')$ is 0-commutative B-algebra then it is easy to show that (B-$Hom\,(A,B)\,;\circledast,\theta)$ is B-algebra. Let $\varphi,\vartheta,\in B-Hom\,(A,B)$ and $x\in A$ then

$$\begin{aligned}(\varphi\circledast(\theta\circledast\vartheta))(x)&=\varphi(x)\circ(\theta(x)\circ\vartheta(x))\\&=\varphi(x)\circ\big(0'\circ\vartheta(x)\big)\\&=\vartheta(x)\circ\big(0'\circ\varphi(x)\big)\\&=\vartheta(x)\circ(\theta(x)\circ\varphi(x))\\&=(\vartheta\circledast(\theta\circledast\varphi))(x)\end{aligned}$$

So it is proven if $(A;*,0)$ is B-algebra and $(B;\circ,0')$ is 0-commutative B-algebra, then (B-$Hom\,(A,B)\,;\circledast,\theta)$ is 0-commutative B-algebra. ☐

3. B-Algebra $(\mathbb{Z}_n;*,[0]_n)$ Defined from the Group $(\mathbb{Z}_n,+_n)$

Theorem 3.1. *Let $(\mathbb{Z}_n,+_n)$ be group, defined binary operations " $*$ " in $\mathbb{Z}_n$ with $[x]_n*[y]_n=[x]_n-_n[y]_n$, for every $[x]_n,[y]_n\in\mathbb{Z}_n$, then $\left(\mathbb{Z}_n;*,[0]_n\right)$ is B-algebra.*

Proof. It is known that $(\mathbb{Z}_n,+_n)$ is a group that has an identity element $[0]_n$ and defined with

$$[x]_n*[y]_n=[x]_n-_n[y]_n$$

$$= [x]_n +_n (-_n[y]_n)$$

$$= [x+(-y)]_n$$

Let $[x]_n, [y]_n, [z]_n \in \mathbb{Z}_n$ then for every $[x]_n, [y]_n \in \mathbb{Z}_n$.

1). $[x]_n * [x]_n = [x]_n -_n [x]_n$

$$= [x]_n +_n \left(-_n[x]_n\right)$$

$$= [x+(-x)]_n$$

$$= [0]_n$$

2). $[x]_n * [0]_n = [x]_n -_n [0]_n$

$$= [x]_n +_n (-_n[0]_n)$$

$$= [x+(-0)]_n$$

$$= [x]_n$$

3). $\left([x]_n * [y]_n\right) * [z]_n = \left([x]_n -_n [y]_n\right) -_n [z]_n$

$$= ([x]_n +_n \left(-_n[y]_n\right)) +_n (-_n[z]_n)$$

$$= [x]_n +_n ((-_n[y]_n) +_n (-_n[z]_n))$$

$$= [x]_n +_n (-_n([z]_n +_n [y]_n))$$

$$= [x]_n +_n \left(-_n[z+y]_n\right)$$

$$= [x]_n -_n [z+y]_n$$

$$= [x]_n * [z+y]_n$$

$$= [x]_n * ([z]_n +_n [y]_n)$$

$$= [x]_n * \left([z]_n +_n \left(-_n \left(-_n[y]_n\right)\right)\right)$$

$$= [x]_n * \left([z]_n +_n \left(-_n \left([0]_n +_n \left(-_n[y]_n\right)\right)\right)\right)$$

$$= [x]_n * \left([z]_n +_n \left(-_n \left([0]_n -_n [y]_n\right)\right)\right)$$

$$= [x]_n * ([z]_n -_n \left([0]_n -_n [y]_n\right))$$

$$= [x]_n * \left([z]_n * \left([0]_n * [y]_n\right)\right)$$

So based on Definition 1.1 it is proven that $(\mathbb{Z}_n; *, [0]_n)$ is B-algebra. □

Example 3.2. *Let $(\mathbb{Z}_4, +_4)$ with $\mathbb{Z}_4 = \{[0]_4, [1]_4, [2]_4, [3]_4\}$ be group against "$+_4$" operation that is the addition operation modulo 4 which is presented in the following table.*

$+_4$	$[0]_4$	$[1]_4$	$[2]_4$	$[3]_4$
$[0]_4$	$[0]_4$	$[1]_4$	$[2]_4$	$[3]_4$
$[1]_4$	$[1]_4$	$[2]_4$	$[3]_4$	$[0]_4$
$[2]_4$	$[2]_4$	$[3]_4$	$[0]_4$	$[1]_4$
$[3]_4$	$[3]_4$	$[0]_4$	$[1]_4$	$[2]_4$

Table 1. Operation Definition Table "$+_4$" in $\mathbb{Z}_4$

Defined

$$[x]_4 * [y]_4 = [x]_4 -_4 [y]_4,$$

for every $[x]_4, [y]_4 \in \mathbb{Z}_4$*, then based on Theorem 2.1,* $(\mathbb{Z}_4; *, [0]_4)$ *is B-algebra. So that it is obtained B-algebra* $(\mathbb{Z}_4; *, [0]_4)$ *which is presented in the following table.*

$*$	$[0]_4$	$[1]_4$	$[2]_4$	$[3]_4$
$[0]_4$	$[0]_4$	$[3]_4$	$[2]_4$	$[1]_4$
$[1]_4$	$[1]_4$	$[0]_4$	$[3]_4$	$[2]_4$
$[2]_4$	$[2]_4$	$[1]_4$	$[0]_4$	$[3]_4$
$[3]_4$	$[3]_4$	$[2]_4$	$[1]_4$	$[0]_4$

Table 2. **Operation Definition Table "$*$" in $\mathbb{Z}_4$**

Theorem 3.3. *Let* $(\mathbb{Z}_n, +_n)$ *be group and* $\left(\mathbb{Z}_n; *, [0]_n\right)$ *be B-algebra which is defined with* $[x]_n * [y]_n = [x]_n -_n [y]_n$*, for every* $[x]_n, [y]_n \in \mathbb{Z}_n$*, then* $\left(\mathbb{Z}_n; *, [0]_n\right)$ *is 0-commutative B-algebra.*

Proof. It is known that $(\mathbb{Z}_n, +_n)$ is commutative group that is, for every $[x]_n, [y]_n \in \mathbb{Z}_n$ implies

$$[x]_n +_n [y]_n = [x + y]_n = [y + x]_n = [y]_n +_n [x]_n.$$

From Theorem 2.1, $\left(\mathbb{Z}_n; *, [0]_n\right)$ is B-algebra. Let $[x]_n, [y]_n \in \mathbb{Z}_n$ then

$$\begin{aligned}
[x]_n * \left([0]_n * [y]_n\right) &= [x]_n * ([0]_n -_n [y]_n) \\
&= [x]_n * ([0]_n +_n \left(-_n[y]_n\right)) \\
&= [x]_n * (-_n \left([y]_n +_n \left(-_n[0]_n\right)\right)) \\
&= [x]_n -_n (-_n \left([y]_n +_n \left(-_n[0]_n\right)\right)) \\
&= [x]_n +_n \left(-_n \left(-_n \left([y]_n +_n \left(-_n[0]_n\right)\right)\right)\right) \\
&= [x]_n +_n ([y]_n +_n \left(-_n[0]_n\right)) \\
&= [x]_n +_n [y + (-0)]_n \\
&= [x]_n +_n [y]_n \\
&= [y]_n +_n [x]_n \\
&= [y]_n +_n (-_n \left(-_n[x]_n\right)) \\
&= [y]_n -_n \left(-_n[x]_n\right) \\
&= [y]_n -_n ([0]_n +_n \left(-_n[x]_n\right)) \\
&= [y]_n * [0 + (-x)]_n \\
&= [y]_n * ([0]_n -_n \left(-_n[x]_n\right)) \\
&= [y]_n * ([0]_n -_n [x]_n) \\
&= [y]_n * ([0]_n * [x]_n)
\end{aligned}$$

So based on Definition 1.5 it is proven that $\left(\mathbb{Z}_n; *, [0]_n\right)$ is 0-commutative B-algebra. □

Theorem 3.4. *Let* $(\mathbb{Z}_n, +_n)$ *be group and function* $\varphi : \mathbb{Z}_n \to \mathbb{Z}_n$*, function* φ *is a group homomorphism if and only if it exists* $[k]_n \in \mathbb{Z}_n$ *so that* $\varphi\left([m]_n\right) = [k]_n[m]_n$ *for every* $[m]_n \in \mathbb{Z}_n$*.*

Proof.

($\Leftarrow$) Suppose there is $[k]_n \in \mathbb{Z}_n$ so that $\varphi\left([m]_n\right) = [k]_n[m]_n$. Shown φ is the group homomorphism. Let $[x]_n, [y]_n \in \mathbb{Z}_n$ then

$$\begin{aligned}
\varphi\left([x]_n +_n [y]_n\right) &= [k]_n \left([x]_n +_n [y]_n\right) \\
&= [k]_n[x+y]_n \\
&= [k(x+y)]_n \\
&= [kx+ky]_n \\
&= [kx]_n +_n [ky]_n \\
&= [k]_n[x]_n +_n [k]_n[y]_n \\
&= \varphi\left([x]_n\right) +_n \varphi([y]_n)
\end{aligned}$$

So it is proven φ is the group homomorphism.

($\Rightarrow$) Suppose φ is the group homomorphism, then for every $[m]_n \in \mathbb{Z}_n$.

$$\varphi\left([m]_n\right) = \varphi\left(\underbrace{[1]_n +_n [1]_n +_n \dots +_n [1]_n}_{m}\right)$$

$$\varphi\left([m]_n\right) = \underbrace{\varphi([1]_n) +_n \varphi([1]_n) +_n \dots +_n \varphi([1]_n)}_{m}$$

$$\varphi\left([m]_n\right) = m\varphi\left([1]_n\right)$$

Suppose $\varphi\left([1]_n\right) = [k]_n \in \mathbb{Z}_n$ then

$$\begin{aligned}
\varphi\left([m]\right) &= m\varphi\left([1]_n\right) \\
&= m[k]_n \\
&= [mk]_n \\
&= [km]_n \\
&= [k]_n[m]_n
\end{aligned}$$

So if φ is a group homomorphism then exists $[k]_n \in \mathbb{Z}_n$ so that $\varphi\left([m]_n\right) = [k]_n[m]_n$ for every $[m]_n \in \mathbb{Z}_n$. $\square$

Theorem 3.5. *Let $(\mathbb{Z}_n, +_n)$ be group and group homomorphism $\varphi : \mathbb{Z}_n \to \mathbb{Z}_n$, if $(\mathbb{Z}_n; *, [0]_n)$ is B-algebra which is defined with $[x]_n * [y]_n = [x]_n -_n [y]_n$, for every $[x]_n, [y]_n \in \mathbb{Z}_n$, then $\varphi : \mathbb{Z}_n \to \mathbb{Z}_n$ is also B-homomorphism.*

Proof. It is known φ is group homomorphism from $\mathbb{Z}_n$ to $\mathbb{Z}_n$ then based on Theorem 2.3 there exists $[k]_n \in \mathbb{Z}_n$ so that $\varphi\left([m]_n\right) = [k]_n[m]_n$ for every $[m]_n \in \mathbb{Z}_n$. Let $[x]_n, [y]_n \in \mathbb{Z}_n$ then

$$\begin{aligned}
\varphi\left([x]_n * [y]_n\right) &= [k]_n \left([x]_n * [y]_n\right) \\
&= [k]_n \left([x]_n -_n [y]_n\right) \\
&= [k]_n \left([x]_n +_n \left(-_n[y]_n\right)\right) \\
&= [k]_n[x+(-y)]_n \\
&= [k\left(x+(-y)\right)]_n
\end{aligned}$$

$$
\begin{aligned}
&= [kx + (-ky)]_n \\
&= [kx]_n +_n \left(-_n [ky]_n\right) \\
&= [kx]_n -_n [ky]_n \\
&= [k]_n [x]_n -_n [k]_n [y]_n \\
&= [k]_n [x]_n * [k]_n [y]_n \\
&= \varphi\left([x]_n\right) * \varphi([y]_n)
\end{aligned}
$$

So it is proven that $\varphi : \mathbb{Z}_n \to \mathbb{Z}_n$ is also B-homomorphism. □

Example 3.6. *Let $(\mathbb{Z}_4, +_4)$ be group and $(\mathbb{Z}_4; *, [0]_4)$ be B-algebra. Function $\theta, I, \mu, \vartheta : \mathbb{Z}_4 \to \mathbb{Z}_4$ from $(\mathbb{Z}_4, +_4)$ to $(\mathbb{Z}_4, +_4)$ respectively are $\theta\left([x]_4\right) = [0]_4$, $I\left([x]_4\right) = [x]_4$, $\mu\left([x]_4\right) = [2]_4[x]_4$, and $\vartheta\left([x]_4\right) = [3]_4[x]_4$, for every $[x]_4 \in \mathbb{Z}_4$ so based on Theorem 2.3, $\theta, I, \mu, \vartheta$ are group homomorphism with Theorem 2.4 are B-homomorphism on $(\mathbb{Z}_4; *, [0]_4)$.*

Theorem 3.7. *Let $(\mathbb{Z}_n; *, [0]_n)$ be B-algebra and B-Hom$(\mathbb{Z}_n, \mathbb{Z}_n)$ is the set of all B-homomorphisms from B-algebra $(\mathbb{Z}_n; *, [0]_n)$ to B-algebra $(\mathbb{Z}_n; *, [0]_n)$. If in B-Hom$(\mathbb{Z}_n, \mathbb{Z}_n)$ defined binary operation "$\circledast$" with*

$$(\varphi \circledast \psi)\left([x]_n\right) = \varphi\left([x]_n\right) * \psi([x]_n)$$

for every $[x]_n \in \mathbb{Z}_n$ with $\theta\left([x]_n\right) = [0]_n$ for every $[x]_n \in \mathbb{Z}_n$, then (B-Hom$\left(\mathbb{Z}_n, \mathbb{Z}_n\right); \circledast, \theta$) is B-algebra.

Proof. Based on Theorem 2.2, $(\mathbb{Z}_n; *, [0])$ is 0-commutative B-algebra. Let $\varphi, \vartheta \in$ B-$Hom\left(\mathbb{Z}_n, \mathbb{Z}_n\right)$ and $[x]_n, [y]_n \in \mathbb{Z}_n$ then

$$
\begin{aligned}
(\varphi \circledast \vartheta)\left([x]_n * [y]_n\right) &= \varphi\left([x]_n * [y]_n\right) * \vartheta\left([x]_n * [y]_n\right) \\
&= \left(\varphi\left([x]_n\right) * \varphi([y]_n)\right) * \left(\vartheta\left([x]_n\right) * \vartheta([y]_n)\right) \\
&= \left(\vartheta\left([y]_n\right) * \varphi([y]_n)\right) * \left(\vartheta\left([x]_n\right) * \varphi([x]_n)\right) \\
&= \left([0]_n * \left(\varphi\left([y]_n\right) * \vartheta\left([y]_n\right)\right)\right) * \left([0]_n * \left(\varphi\left([x]_n\right) * \vartheta\left([x]_n\right)\right)\right) \\
&= \left(\varphi\left([x]_n\right) * \vartheta\left([x]_n\right)\right) * \left(\varphi\left([y]_n\right) * \vartheta\left([y]_n\right)\right) \\
&= (\varphi \circledast \vartheta)\left([x]_n\right) * (\varphi \circledast \vartheta)\left([y]_n\right)
\end{aligned}
$$

Because $(\mathbb{Z}_n; *, [0]_n)$ is 0-commutative B-algebra then for every $\varphi, \vartheta \in$ B-$Hom\left(\mathbb{Z}_n, \mathbb{Z}_n\right)$ obtained $\varphi \circledast \vartheta \in$ B-$Hom(\mathbb{Z}_n, \mathbb{Z}_n)$ so operation "$\circledast$" in B-$Hom(\mathbb{Z}_n, \mathbb{Z}_n)$ is a binary operation. Because $(\mathbb{Z}_n; *, [0]_n)$ is 0-commutative B-algebra then it is easy to show that (B-$Hom\left(\mathbb{Z}_n, \mathbb{Z}_n\right); \circledast, \theta$) is B-algebra. □

Example 3.8. *Let $(\mathbb{Z}_4; *, [0]_4)$ be B-algebra and B-Hom$\left(\mathbb{Z}_4, \mathbb{Z}_4\right) = \left\{\varphi_k\left([x]_4\right) \mid \varphi_k([x]_4) = [k]_4[x]_4, [x]_4 \in \mathbb{Z}_4\right\}$ is the set of all B-homomorphisms form B-algebra $(\mathbb{Z}_4; *, [0]_4)$ to B-algebra $(\mathbb{Z}_4; *, [0]_4)$. If in B-Hom$\left(\mathbb{Z}_4, \mathbb{Z}_4\right)$ defined binary operation "$\circledast$" with*

$$
\begin{aligned}
(\varphi_m \circledast \varphi_k)([x]_4) &= \varphi_m\left([x]_4\right) * \varphi_k\left([x]_4\right) \\
&= [m]_4[x]_4 * [k]_4[x]_4 \\
&= [mx]_4 * [kx]_4 \\
&= [mx]_4 -_4 [kx]_4
\end{aligned}
$$

$$= [mx]_4 +_4 \left(-_4[kx]_4\right)$$
$$= [mx + (-kx)]_4$$
$$= [(m + (-k))\, x]_4$$
$$= [m + (-k)]_4[x]_4$$
$$= ([m]_4 +_4 \left(-_4[k]_4\right))[x]_4$$
$$= \left([m]_4 -_4 [k]_4\right) [x]_4$$
$$= ([m]_4 * [k]_4)[x]_4$$
$$= \varphi_{[m]_4 * [k]_4}([x]_4)$$

for every $[x]_4 \in \mathbb{Z}_4$ *with* $\theta[x]_4 = [0]_4$ *for every* $[x]_4 \in \mathbb{Z}_4$*, then based on Theorem 3.7 (B-Hom* $(\mathbb{Z}_4, \mathbb{Z}_4)\,; \circledast, \theta)$ *is B-algebra. Operation "* $\circledast$ *" on B-Hom* $(\mathbb{Z}_8, \mathbb{Z}_8)$ *is presented in the following table.*

$\circledast$	φ_0	φ_1	φ_2	φ_3
φ_0	φ_0	φ_3	φ_2	φ_1
φ_1	φ_1	φ_0	φ_3	φ_2
φ_2	φ_2	φ_1	φ_0	φ_3
φ_3	φ_3	φ_2	φ_1	φ_0

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