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Fuzzy Parameterized Generalized Intuitionisitic Fuzzy Soft Expert Set

Research Article

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Abstract: In this paper, we introduced the theory of Fuzzy Parameterized Generalized Intuitionisitic Fuzzy Soft Expert Set (FPGIF-SES for short). The basic operations of Fuzzy parameterized Generalised Intuitionisitic Fuzzy Soft Expert Set namely complement, intersection, union, "AND"and "OR"are distinct along with illustrative examples.

Keywords: Fuzzy set, Soft set, Soft expert set, Fuzzy soft expert set, Fuzzy parameterized generalized Fuzzy soft expert set, Fuzzy parameterized Generalized Intuitionisitic Fuzzy Soft Expert.

1. Introduction

Intuitionisitic Fuzzy set (IFS) on a universe was introduced by Atanassov [1] in 1983, as a generalization of fuzzy set [2]. The theory of IFS is characterized by a membership function and a non-membership function. Molodtsov [4] initiated the theory of the soft set as a mathematical tool for dealing with uncertainties. After Molodtsov's work, Maji [6] have introduced the theory of fuzzy soft set, as a more universal theory, and as a grouping of fuzzy set and soft set where they studied its properties. Also Roy and Maji [5] used this theory to resolve some real time decision making problems.

Further, C.agman, [8] introduced the theory of fuzzy parameterized fuzzy soft set and their operations. As well as fuzzy parameterized fuzzy soft set aggregation operator to form fuzzy parameterized fuzzy soft set decision making method which allows constructing more efficient decision processes. Alkhazaleh and Salleh [7] introduced the theory of soft expert set and fuzzy soft expert set, where the user can know the opinion of all experts in one model without any operations. After Alkhazaleh's work, many researchers have worked with the theory of soft expert sets [3], [9], [12]. Majumdar & Samanta [11] has introduced the notion of generalised fuzzy soft sets and have applied this set in decision making. G.Geetharamani [10] has introduced the theory of Fuzzy Parameterized Generalized Fuzzy Soft Expert Set; it is a combination of generalized fuzzy soft expert set and fuzzy parameterized fuzzy soft expert.

In this paper, we shall introduce the concept of Fuzzy Parameterized Generalized Intuitionisitic fuzzy soft expert set, which is more effective and useful as we shall see. It is a combination of Intuitionisitic fuzzy soft expert set, generalised fuzzy soft expert and fuzzy parameterized fuzzy soft expert. Finally, we shall also define its basic operations, namely complement, union, intersection, AND, OR and study their properties.

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2. Preliminaries

In this section we recall some basic definitions related to this work. Let U be a universe of discourse and E a set of parameters. Let P(U) denotes the power set of U and $A \subseteq E$.

Definition 2.1. *A pair $(F,\ A)$ is called a soft set over U where F is a mapping given by* $F : A \to P\,(U)$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set $(F,\ A)$.

Definition 2.2. *An intuitionisitic fuzzy set A defined over a universe of discourse U is an object in the following form*

$$A = \{\langle x, \mu_A\,(x)\,, V_A\,(x)\rangle : x \in U\}$$

Where, the function $\mu_A : U \to [0,1]$ and $V_A : U \to [0,1]$ are the membership function and non-membership function respectively of every element $x \in U$ to set A and $0 \leq \mu_A\,(x) + V_A(x) \leq 1$ for every element $x \in U$. In the event that $0 \leq \mu_A\,(x) + V_A(x) \leq 1$, there is a degree of uncertainty that exists for element x with respect to set A. This degree of uncertainty, denoted as $\pi_A\,(X)$ and it is defined as $\pi_A\,(X) = 1 - \mu_A\,(x) - V_A(x)$. In general, a high degree of uncertainty implies that there are a lot things that are unknown about element x with respect to set A.

From now on, let A and B be intuitionistic fuzzy sets defined over a universal set U and are as defined

$$A = \{\langle x, \mu_A\,(x)\,, V_A\,(x)\rangle : x \in U\}$$

$$B = \{\langle x, \mu_B\,(x)\,, V_B\,(x)\rangle : x \in U\}$$

Definition 2.3. *The subset and equality of two intuitionistic fuzzy sets A and B are as defined*

- $A \subset B \leftrightarrow \mu_A\,(x) = \mu_B\,(x)$ *and* $V_A\,(x) = V_A\,(x)$ *for all* $x \in U$
- $A = B \leftrightarrow A \subset B$ *and* $B \subset A$

Definition 2.4. *The complement, union and intersection of two intuitionistic fuzzy sets A and B are as defined*

- $A^c = \{\langle x, V_A\,(x)\,, \mu_A\,(x)\rangle : x \in U\}$
- $A \cup B = \{\langle x,\ max(\mu_A\,(x)\,,\ \mu_B\,(x))\},\ min\,(V_A\,(x)\,, V_B\,(x))\rangle : x \in U\}$
- $A \cap B = \{\langle x,\ min(\mu_A\,(x)\,,\ \mu_B\,(x))\},\ max\,(V_A\,(x)\,, V_B\,(x))\rangle : x \in U\}$

Definition 2.5. *Let U be a universe, E be a set of parameters. Let $P(U)$ denote the set of all intuitionistic fuzzy sets of U. Let $A \subseteq E$. A pair (F, E) is an intuitionistic fuzzy soft set over U where F is a mapping given by $F : A \to P\,(U)$.*

From now on, Let U be a universe, E be a set of parameters and X be a set of experts. Let O be a set of opinions. Let $Z = \mathrm{E} \times X \times O$ and $A \subseteq Z$.

Definition 2.6. *A Pair (F, A) is called a soft expert set over U, where F is a mapping $F : A \to P\,(U)$, where $P(U)$ denotes the power set of U.*

Definition 2.7. *A Pair (F, A) is called a fuzzy soft expert set over U, where F is a mapping $F : A \to I^U$ and I^U denotes all fuzzy subsets of U.*

Definition 2.8. *The complement of a fuzzy soft expert set* (F, A) *is denoted by* $(F, A)^C$ *and is defined by* $(F, A) = (F^c, \urcorner A)$, *where* $F^c : \urcorner A \to I^U$ *is a mapping given by* $F^c(\alpha) = c(F(\urcorner\alpha))\ \forall\alpha \in \urcorner A$, *where c is a fuzzy complement.*

Definition 2.9. *The union of two fuzzy soft expert sets* (F, A) *and* (G, B) *over U, denoted by* $(F, A)\,\check{U}(G, B)$*, is the fuzzy soft expert set* (H, C) *such that* $C = A \cup B$ *and for all* $\varepsilon \in C$

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if } \varepsilon \in A - B \\ G(\varepsilon) & \text{if } \varepsilon \in B - A \\ s(F(\varepsilon),\ G(\varepsilon)) & \text{if } \varepsilon \in A \cap B \end{cases}$$

where s is an s-norm.

Definition 2.10. *The intersection of two fuzzy soft expert sets* (F, A) *and* (G, B) *over U denoted by* $(F, A)\,\check{n}(G, B)$ *is a fuzzy soft expert set* (H, C) *such that* $C = A \cup B$ *and* $\forall\varepsilon \in C$

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if } \varepsilon \in A - B \\ G(\varepsilon) & \text{if } \varepsilon \in B - A \\ t(F(\varepsilon),\ G(\varepsilon)) & \text{if } \varepsilon \in A \cap B \end{cases}$$

where t is a t-norm.

Definition 2.11. *If* (F, A) *and* (G, B) *are two fuzzy soft expert sets over U, then "*$(F, A)AND(G, B)$*" denoted by* $(F, A)\,\widetilde{\wedge}(G, B)$ *is defined by*

$$(F, A)\,\widetilde{\wedge}\,(G, B) = (H,\ A \times B)$$

such that $H(\alpha, \beta) = F(\alpha)\tilde{\cap}G(\beta)$*, for all* $(\alpha, \beta) \in A \times B$.

Definition 2.12. *If* (F, A) *and* (G, B) *are two fuzzy soft expert sets over U, then "*$(F, A)OR(G, B)$*" denoted by* $(F, A)\,\widetilde{\vee}(G, B)$ *is defined by*

$$(F, A)\,\widetilde{\vee}\,(G, B) = (H,\ A \times B)$$

such that $H(\alpha, \beta) = F(\alpha)\tilde{\cup}G(\beta)$*, for all* $(\alpha, \beta) \in A \times B$.

3. Fuzzy Parameterized Generalized Intuitionisitic Fuzzy Soft Expert Set (FPGIFSES in short)

Let U be a universal set of elements, E be a set of parameters, I^E denotes all intuitionistic fuzzy subsets of E and X be a set of experts. Let $O = \{0 = \text{disagree}, 1 = \text{agree}\}$ be a set of opinions, $Z = \psi \times X \times O$ and $A \subseteq Z$, where $\psi \subset I^E$, $\rho : Z \to I = [0, 1]$ and ρ be an intuitionistic fuzzy subsets of U.

Definition 3.1. *A Pair* $(F_\psi,\ A)$ *is called a Fuzzy Parameterized Intuitionistic Generalized Fuzzy Soft Expert Set (FPGIFSES in short) over U, where* F_ψ *is a mapping given by* $F_\psi : A \to I^U \times I$*, where* I^U *denotes the collection of all intuitionisitic fuzzy subset of U.*

Here for each parameter indicates not only the degree of belongingness of the elements but also the degree of possibility of such belongingness.

Example 3.2. *Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of universe and let $E = \{e_1, e_2, e_3\}$ a set of parameters. Let $\psi = \left\{\frac{e_1}{0.5}, \frac{e_2}{0.7}, \frac{e_3}{0.9}\right\}$ be a fuzzy subset of I^E and $X = \{m, n, r\}$ be a set of experts. Let $Z = \psi \times X \times O$ and $A \subseteq Z$, where $\Psi \subset I^E$, $\mu : Z \to I = [0, 1]$.*

Define a function, $F_\psi : A \to I^U \times I$ as follows

$$F_\psi\left(\frac{e_1}{0.5}, m, 1\right) = \left(\left\{\frac{u_1}{<0.1, 0.6>}, \frac{u_2}{<0.2, 0.7>}, \frac{u_3}{<0.6, 0.2>}, \frac{u_4}{<0.7, 0.3>}\right\}, 0.2\right)$$
$$F_\psi\left(\frac{e_1}{0.5}, n, 1\right) = \left(\left\{\frac{u_1}{<0.3, 0.5>}, \frac{u_2}{<0.2, 0.4>}, \frac{u_3}{<0.1, 0.2>}, \frac{u_4}{<0.1, 0.3>}\right\}, 0.1\right)$$
$$F_\psi\left(\frac{e_1}{0.5}, r, 1\right) = \left(\left\{\frac{u_1}{<0.4, 0.3>}, \frac{u_2}{<0.6, 0.3>}, \frac{u_3}{<0.2, 0.5>}, \frac{u_4}{<0.2, 0.6>}\right\}, 0.2\right)$$
$$F_\psi\left(\frac{e_2}{0.7}, m, 1\right) = \left(\left\{\frac{u_1}{<0.1, 0.6>}, \frac{u_2}{<0.3, 0.6>}, \frac{u_3}{<0.6, 0.4>}, \frac{u_4}{<0.4, 0.2>}\right\}, 0.4\right)$$
$$F_\psi\left(\frac{e_2}{0.7}, n, 1\right) = \left(\left\{\frac{u_1}{<0.2, 0.7>}, \frac{u_2}{<0.1, 0.8>}, \frac{u_3}{<0.3, 0.6>}, \frac{u_4}{<0.4, 0.5>}\right\}, 0.2\right)$$
$$F_\psi\left(\frac{e_2}{0.7}, r, 1\right) = \left(\left\{\frac{u_1}{<0.2, 0.7>}, \frac{u_2}{<0.3, 0.5>}, \frac{u_3}{<0.4, 0.6>}, \frac{u_4}{<0.4, 0.2>}\right\}, 0.3\right)$$
$$F_\psi\left(\frac{e_3}{0.9}, m, 1\right) = \left(\left\{\frac{u_1}{<0.3, 0.5>}, \frac{u_2}{<0.6, 0.3>}, \frac{u_3}{<0.8, 0.1>}, \frac{u_4}{<0.9, 0.1>}\right\}, 0.6\right)$$
$$F_\psi\left(\frac{e_3}{0.9}, n, 1\right) = \left(\left\{\frac{u_1}{<0.2, 0.6>}, \frac{u_2}{<0.3, 0.2>}, \frac{u_3}{<0.7, 0.5>}, \frac{u_4}{<0.6, 0.3>}\right\}, 0.5\right)$$
$$F_\psi\left(\frac{e_3}{0.9}, r, 1\right) = \left(\left\{\frac{u_1}{<0.2, 0.8>}, \frac{u_2}{<0.4, 0.2>}, \frac{u_3}{<0.6, 0.4>}, \frac{u_4}{<0.3, 0.2>}\right\}, 0.8\right)$$
$$F_\psi\left(\frac{e_1}{0.5}, m, 0\right) = \left(\left\{\frac{u_1}{<0.3, 0.4>}, \frac{u_2}{<0.6, 0.2>}, \frac{u_3}{<0.7, 0.2>}, \frac{u_4}{<0.6, 0.3>}\right\}, 0.7\right)$$
$$F_\psi\left(\frac{e_1}{0.5}, n, 0\right) = \left(\left\{\frac{u_1}{<0.7, 0.2>}, \frac{u_2}{<0.7, 0.2>}, \frac{u_3}{<0.5, 0.5>}, \frac{u_4}{<0.7, 0.1>}\right\}, 0.9\right)$$
$$F_\psi\left(\frac{e_1}{0.5}, r, 0\right) = \left(\left\{\frac{u_1}{<0.6, 0.2>}, \frac{u_2}{<0.6, 0.3>}, \frac{u_3}{<0.4, 0.6>}, \frac{u_4}{<0.5, 0.3>}\right\}, 0.6\right)$$
$$F_\psi\left(\frac{e_2}{0.7}, m, 0\right) = \left(\left\{\frac{u_1}{<0.2, 0.8>}, \frac{u_2}{<0.3, 0.7>}, \frac{u_3}{<0.8, 0.2>}, \frac{u_4}{<0.3, 0.2>}\right\}, 0.6\right)$$
$$F_\psi\left(\frac{e_2}{0.7}, n, 0\right) = \left(\left\{\frac{u_1}{<0.2, 0.5>}, \frac{u_2}{<0.4, 0.6>}, \frac{u_3}{<0.5, 0.4>}, \frac{u_4}{<0.7, 0.3>}\right\}, 0.7\right)$$
$$F_\psi\left(\frac{e_2}{0.7}, r, 0\right) = \left(\left\{\frac{u_1}{<0.4, 0.6>}, \frac{u_2}{<0.6, 0.2>}, \frac{u_3}{<0.1, 0.9>}, \frac{u_4}{<0.3, 0.2>}\right\}, 0.4\right)$$
$$F_\psi\left(\frac{e_3}{0.9}, m, 0\right) = \left(\left\{\frac{u_1}{<0.2, 0.7>}, \frac{u_2}{<0.4, 0.6>}, \frac{u_3}{<0.3, 0.6>}, \frac{u_4}{<0.1, 0.8>}\right\}, 0.3\right)$$
$$F_\psi\left(\frac{e_3}{0.9}, n, 0\right) = \left(\left\{\frac{u_1}{<0.9, 0.1>}, \frac{u_2}{<0.6, 0.3>}, \frac{u_3}{<0.8, 0.1>}, \frac{u_4}{<0.3, 0.4>}\right\}, 0.6\right)$$
$$F_\psi\left(\frac{e_3}{0.9}, r, 0\right) = \left(\left\{\frac{u_1}{<0.3, 0.5>}, \frac{u_2}{<0.5, 0.3>}, \frac{u_3}{<0.4, 0.2>}, \frac{u_4}{<0.2, 0.5>}\right\}, 0.5\right)$$

Then we can view the Fuzzy Parameterized Generalized Intuitionistic Fuzzy Soft Expert set (F_ψ,Z) as consisting of the following collections of approximations:

$$(F_\psi, Z) = \left\{\begin{array}{l}
\left(\left(\frac{e_1}{0.5}, m, 1\right), \left(\left\{\frac{u_1}{<0.1,0.6>}, \frac{u_2}{<0.2,0.7>}, \frac{u_3}{<0.6,0.2>}, \frac{u_4}{<0.7,0.3>}\right\}, 0.2\right)\right), \left(\left(\frac{e_1}{0.5}, n, 1\right), \left(\left\{\frac{u_1}{<0.3,0.5>}, \frac{u_2}{<0.2,0.4>}, \frac{u_3}{<0.1,0.2>}, \frac{u_4}{<0.1,0.3>}\right\}, 0.1\right)\right) \\
\left(\left(\frac{e_1}{0.5}, r, 1\right), \left(\left\{\frac{u_1}{<0.4,0.3>}, \frac{u_2}{<0.6,0.3>}, \frac{u_3}{<0.2,0.5>}, \frac{u_4}{<0.2,0.6>}\right\}, 0.2\right)\right), \left(\left(\frac{e_2}{0.7}, m, 1\right), \left(\left\{\frac{u_1}{<0.1,0.6>}, \frac{u_2}{<0.3,0.6>}, \frac{u_3}{<0.6,0.4>}, \frac{u_4}{<0.4,0.2>}\right\}, 0.4\right)\right) \\
\left(\left(\frac{e_2}{0.7}, n, 1\right), \left(\left\{\frac{u_1}{<0.2,0.7>}, \frac{u_2}{<0.1,0.8>}, \frac{u_3}{<0.3,0.6>}, \frac{u_4}{<0.4,0.5>}\right\}, 0.2\right)\right), \left(\left(\frac{e_2}{0.7}, r, 1\right), \left(\left\{\frac{u_1}{<0.2,0.7>}, \frac{u_2}{<0.3,0.5>}, \frac{u_3}{<0.4,0.6>}, \frac{u_4}{<0.4,0.2>}\right\}, 0.3\right)\right) \\
\left(\left(\frac{e_3}{0.9}, m, 1\right), \left(\left\{\frac{u_1}{<0.3,0.5>}, \frac{u_2}{<0.6,0.3>}, \frac{u_3}{<0.8,0.1>}, \frac{u_4}{<0.9,0.1>}\right\}, 0.6\right)\right), \left(\left(\frac{e_3}{0.9}, n, 1\right), \left(\left\{\frac{u_1}{<0.2,0.6>}, \frac{u_2}{<0.3,0.2>}, \frac{u_3}{<0.3,0.5>}, \frac{u_4}{<0.6,0.3>}\right\}, 0.5\right)\right) \\
\left(\left(\frac{e_3}{0.9}, r, 1\right), \left(\left\{\frac{u_1}{<0.2,0.8>}, \frac{u_2}{<0.4,0.2>}, \frac{u_3}{<0.6,0.4>}, \frac{u_4}{<0.3,0.2>}\right\}, 0.8\right)\right), \left(\left(\frac{e_1}{0.5}, m, 0\right), \left(\left\{\frac{u_1}{<0.3,0.4>}, \frac{u_2}{<0.6,0.2>}, \frac{u_3}{<0.7,0.2>}, \frac{u_4}{<0.6,0.3>}\right\}, 0.7\right)\right) \\
\left(\left(\frac{e_1}{0.5}, n, 0\right), \left(\left\{\frac{u_1}{<0.7,0.2>}, \frac{u_2}{<0.7,0.2>}, \frac{u_3}{<0.5,0.5>}, \frac{u_4}{<0.7,0.1>}\right\}, 0.9\right)\right), \left(\left(\frac{e_1}{0.5}, r, 0\right), \left(\left\{\frac{u_1}{<0.6,0.2>}, \frac{u_2}{<0.6,0.3>}, \frac{u_3}{<0.4,0.6>}, \frac{u_4}{<0.5,0.3>}\right\}, 0.6\right)\right) \\
\left(\left(\frac{e_2}{0.7}, m, 0\right), \left(\left\{\frac{u_1}{<0.2,0.8>}, \frac{u_2}{<0.3,0.7>}, \frac{u_3}{<0.8,0.2>}, \frac{u_4}{<0.3,0.2>}\right\}, 0.6\right)\right), \left(\left(\frac{e_2}{0.7}, n, 0\right), \left(\left\{\frac{u_1}{<0.2,0.5>}, \frac{u_2}{<0.4,0.6>}, \frac{u_3}{<0.5,0.4>}, \frac{u_4}{<0.7,0.3>}\right\}, 0.7\right)\right) \\
\left(\left(\frac{e_2}{0.7}, r, 0\right), \left(\left\{\frac{u_1}{<0.4,0.6>}, \frac{u_2}{<0.6,0.2>}, \frac{u_3}{<0.1,0.9>}, \frac{u_4}{<0.3,0.2>}\right\}, 0.4\right)\right), \left(\left(\frac{e_3}{0.9}, m, 0\right), \left(\left\{\frac{u_1}{<0.2,0.7>}, \frac{u_2}{<0.4,0.6>}, \frac{u_3}{<0.3,0.6>}, \frac{u_4}{<0.1,0.8>}\right\}, 0.3\right)\right) \\
\left(\left(\frac{e_3}{0.9}, n, 0\right), \left(\left\{\frac{u_1}{<0.9,0.1>}, \frac{u_2}{<0.6,0.3>}, \frac{u_3}{<0.8,0.1>}, \frac{u_4}{<0.3,0.4>}\right\}, 0.6\right)\right), \left(\left(\frac{e_3}{0.9}, r, 0\right), \left(\left\{\frac{u_1}{<0.3,0.5>}, \frac{u_2}{<0.5,0.3>}, \frac{u_3}{<0.4,0.2>}, \frac{u_4}{<0.2,0.5>}\right\}, 0.5\right)\right)
\end{array}\right\}$$

Definition 3.3. *Let (F_ψ, A) and (G_τ, B) be FPGIFSESs over the universe U. Then (F_ψ, A) is said to be a Fuzzy Parameterized Generalized Intuitionistic Fuzzy Soft Expert subset of (G_τ, B), if the following conditions are satisfied:*

(i) $A \subseteq B$

(ii) For all $\varepsilon \in A$, $F_\psi(\varepsilon)$ *is generalized intuitionistic fuzzy subset of* $G_\tau(\varepsilon)$.

This relationship is denoted as $(F_\psi, A) \subseteq (G_\tau, B)$. *In this case,* (G_τ, B) *is called a Fuzzy Parameterised Generalized Intuitionistic Fuzzy Soft Expert superset of* (F_ϕ, A).

Example 3.4. *Let* (F_ψ, A) *and* (G_τ, B) *be two FPGIFSESs over the universe U defined as follows*

$$(F_\psi, A) = \left\{ \begin{array}{l} \left(\left(\frac{e_1}{0.5}, m, 1\right), \left(\left\{\frac{u_1}{\langle 0.4,0.6\rangle}, \frac{u_2}{\langle 0.5,0.3\rangle}, \frac{u_3}{\langle 0.6,03\rangle}, \frac{u_4}{\langle 0.2,0.0\rangle}\right\}, 0.4\right)\right), \left(\left(\frac{e_1}{0.5}, n, 1\right), \left(\left\{\frac{u_1}{\langle 0.4,0.5\rangle}, \frac{u_2}{\langle 0.3,0.6\rangle}, \frac{u_3}{\langle 0.4,0.5\rangle}, \frac{u_4}{\langle 0.7,0.2\rangle}\right\}, 0.8\right)\right) \\ \left(\left(\frac{e_2}{0.3}, n, 1\right), \left(\left\{\frac{u_1}{\langle 0.6,0.4\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.4,0.4\rangle}, \frac{u_4}{\langle 0.2,0.8\rangle}\right\}, 0.2\right)\right), \left(\left(\frac{e_2}{0.3}, r, 0\right), \left(\left\{\frac{u_1}{\langle 0.5,04.\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.4,0.5\rangle}\right\}, 0.5\right)\right) \\ \left(\left(\frac{e_3}{0.6}, r, 0\right), \left(\left\{\frac{u_1}{\langle 0.5,0.3\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.3,0.5\rangle}\right\}, 0.5\right)\right), \left(\left(\frac{e_3}{0.6}, m, 0\right), \left(\left\{\frac{u_1}{\langle 0.5,0.2\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.7,0.3\rangle}, \frac{u_4}{\langle 0.5,0.4\rangle}\right\}, 0.7\right)\right) \end{array} \right\}$$

$$(G_\tau, B) = \left\{ \begin{array}{l} \left(\left(\frac{e_1}{0.6}, m, 1\right), \left(\left\{\frac{u_1}{\langle 0.5,0.4\rangle}, \frac{u_2}{\langle 0.6,0.4\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.4,0.6\rangle}\right\}, 0.5\right)\right), \left(\left(\frac{e_1}{0.6}, n, 1\right), \left(\left\{\frac{u_1}{\langle 0.6,0.1\rangle}, \frac{u_2}{\langle 0.3,0.4\rangle}, \frac{u_3}{\langle 0.6,0.2\rangle}, \frac{u_4}{\langle 0.2,0.8\rangle}\right\}, 0.5\right)\right) \\ \left(\left(\frac{e_2}{0.3}, n, 1\right), \left(\left\{\frac{u_1}{\langle 0.7,0.3\rangle}, \frac{u_2}{\langle 0.5,0.3\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.4,0.6\rangle}\right\}, 0.6\right)\right), \left(\left(\frac{e_3}{0.8}, m, 0\right), \left(\left\{\frac{u_1}{\langle 0.3,0.5\rangle}, \frac{u_2}{\langle 0.5,0.4\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.5,0.4\rangle}\right\}, 0.7\right)\right) \\ \left(\left(\frac{e_2}{0.3}, r, 0\right), \left(\left\{\frac{u_1}{\langle 0.6,0.2\rangle}, \frac{u_2}{\langle 0.5,0.4\rangle}, \frac{u_3}{\langle 0.6,0.4\rangle}, \frac{u_4}{\langle 0.6,0.3\rangle}\right\}, 0.4\right)\right), \left(\left(\frac{e_3}{0.8}, r, 1\right), \left(\left\{\frac{u_1}{\langle 0.7,0.2\rangle}, \frac{u_2}{\langle 0.4,0.3\rangle}, \frac{u_3}{\langle 0.7,0.2\rangle}, \frac{u_4}{\langle 0.6,0.4\rangle}\right\}, 0.5\right)\right) \\ \left(\left(\frac{e_3}{0.8}, r, 0\right), \left(\left\{\frac{u_1}{\langle 0.4,0.3\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.4,0.6\rangle}, \frac{u_4}{\langle 0.6,0.3\rangle}\right\}, 0.6\right)\right), \left(\left(\frac{e_3}{0.6}, n, 0\right), \left(\left\{\frac{u_1}{\langle 0.5,0.4\rangle}, \frac{u_2}{\langle 0.5,0.1\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.5,0.2\rangle}\right\}, 0.3\right)\right) \\ \left(\left(\frac{e_3}{0.6}, m, 1\right), \left(\left\{\frac{u_1}{\langle 0.5,0.4\rangle}, \frac{u_2}{\langle 0.3,0.5\rangle}, \frac{u_3}{\langle 0.4,0.5\rangle}, \frac{u_4}{\langle 0.5,0.5\rangle}\right\}, 0.6\right)\right) \end{array} \right\}$$

Clearly $(F_\psi, A) \subseteq (G_\tau, B)$.

Definition 3.5. *Let* (F_ψ, A) *and* (G_τ, B) *be FPGIFSESs over U. Then* (F_ψ, A) *and* (G_τ, B) *are said to be equal if* (F_ψ, A) *is a FPGIFSE subset of* (G_τ, B) *and* (G_τ, B) *is FPGIFSE subset of* (F_ψ, A)

Definition 3.6. *Let* (F_ψ, A) *be a FPGIFSES over a universe U. An agree- Fuzzy Parameterized Generalized Intuitionistic Fuzzy Soft Expert Set (agree-FPGIFSES) over U, denoted as* $(F_\psi, A)_1$ *is a Fuzzy Parameterized Generalized Intuitionistic Fuzzy Soft Expert subset of* (F_ψ, A) *which is defined as follows:*

$$(F_\psi, A)_1 = \{F_\psi(\alpha) : \alpha \in \psi \times X \times \{1\}\}$$

Example 3.7. *Consider the FPGIFSES given in Example 1. Then, the agree-FPGIFSES* $(F_\psi, A)_1$ *over U is:*

$$(F_\psi, A)_1 = \left\{ \begin{array}{l} \left(\left(\frac{e_1}{0.5}, m, 1\right), \left(\left\{\frac{u_1}{<0.1,0.6>}, \frac{u_2}{<0.2,0.7>}, \frac{u_3}{<0.6,0.2>}, \frac{u_4}{<0.7,0.3>}\right\}, 0.2\right)\right), \left(\left(\frac{e_1}{0.5}, n, 1\right), \left(\left\{\frac{u_1}{<0.3,0.5>}, \frac{u_2}{<0.2,0.4>}, \frac{u_3}{<0.1,0.2>}, \frac{u_4}{<0.1,0.3>}\right\}, 0.1\right)\right) \\ \left(\left(\frac{e_1}{0.5}, r, 1\right), \left(\left\{\frac{u_1}{<0.4,0.3>}, \frac{u_2}{<0.6,0.3>}, \frac{u_3}{<0.2,0.5>}, \frac{u_4}{<0.2,0.6>}\right\}, 0.2\right)\right), \left(\left(\frac{e_2}{0.7}, m, 1\right), \left(\left\{\frac{u_1}{<0.1,0.6>}, \frac{u_2}{<0.3,0.6>}, \frac{u_3}{<0.6,0.4>}, \frac{u_4}{<0.4,0.2>}\right\}, 0.4\right)\right) \\ \left(\left(\frac{e_2}{0.7}, n, 1\right), \left(\left\{\frac{u_1}{<0.2,0.7>}, \frac{u_2}{<0.1,0.8>}, \frac{u_3}{<0.3,0.6>}, \frac{u_4}{<0.4,0.5>}\right\}, 0.2\right)\right), \left(\left(\frac{e_2}{0.7}, r, 1\right), \left(\left\{\frac{u_1}{<0.2,0.7>}, \frac{u_2}{<0.3,0.5>}, \frac{u_3}{<0.4,0.6>}, \frac{u_4}{<0.4,0.2>}\right\}, 0.3\right)\right) \\ \left(\left(\frac{e_3}{0.9}, m, 1\right), \left(\left\{\frac{u_1}{<0.3,0.5>}, \frac{u_2}{<0.6,0.3>}, \frac{u_3}{<0.8,0.1>}, \frac{u_4}{<0.9,0.1>}\right\}, 0.6\right)\right), \left(\left(\frac{e_3}{0.9}, n, 1\right), \left(\left\{\frac{u_1}{<0.2,0.6>}, \frac{u_2}{<0.3,0.2>}, \frac{u_3}{<0.3,0.5>}, \frac{u_4}{<0.6,0.3>}\right\}, 0.5\right)\right) \\ \left(\left(\frac{e_3}{0.9}, r, 1\right), \left(\left\{\frac{u_1}{<0.2,0.8>}, \frac{u_2}{<0.4,0.2>}, \frac{u_3}{<0.6,0.4>}, \frac{u_4}{<0.3,0.2>}\right\}, 0.8\right)\right) \end{array} \right\}$$

Definition 3.8. *Let* (F_ψ, A) *be a FPGIFSES over a universe U. An disagree-Fuzzy Parameterized Generalized Intuitionistic Fuzzy Soft Expert Set (disagree-FPGIFSES) over U, denoted as* $(F_\psi, A)_0$ *is a Fuzzy Parameterized Generalized Intuitionistic Fuzzy Soft Expert subset of* (F_ψ, A) *which is defined as follows:*

$$(F_\psi, A)_0 = \{F_\psi(\alpha) : \alpha \in \psi \times X \times \{0\}\}$$

Example 3.9. *Consider the FPGIFSES given in example1. Then, the disagree-FPGIFSES* $(F_\psi, A)_0$ *over U is:*

$$(F_\psi, A)_0 = \left\{ \begin{array}{l} \left(\left(\frac{e_3}{0.9}, r, 0\right), \left(\left\{\frac{u_1}{<0.3,0.5>}, \frac{u_2}{<0.5,0.3>}, \frac{u_3}{<0.4,0.2>}, \frac{u_4}{<0.2,0.5>}\right\}, 0.5\right)\right), \left(\left(\frac{e_1}{0.5}, m, 0\right), \left(\left\{\frac{u_1}{<0.3,0.4>}, \frac{u_2}{<0.6,0.2>}, \frac{u_3}{<0.7,0.2>}, \frac{u_4}{<0.6,0.3>}\right\}, 0.7\right)\right) \\ \left(\left(\frac{e_1}{0.5}, n, 0\right), \left(\left\{\frac{u_1}{<0.7,0.2>}, \frac{u_2}{<0.7,0.2>}, \frac{u_3}{<0.5,0.5>}, \frac{u_4}{<0.7,0.1>}\right\}, 0.9\right)\right), \left(\left(\frac{e_1}{0.5}, r, 0\right), \left(\left\{\frac{u_1}{<0.6,0.2>}, \frac{u_2}{<0.6,0.3>}, \frac{u_3}{<0.4,0.6>}, \frac{u_4}{<0.5,0.3>}\right\}, 0.6\right)\right) \\ \left(\left(\frac{e_2}{0.7}, m, 0\right), \left(\left\{\frac{u_1}{<0.2,0.8>}, \frac{u_2}{<0.3,0.7>}, \frac{u_3}{<0.8,0.2>}, \frac{u_4}{<0.3,0.2>}\right\}, 0.6\right)\right), \left(\left(\frac{e_2}{0.7}, n, 0\right), \left(\left\{\frac{u_1}{<0.2,0.5>}, \frac{u_2}{<0.4,0.6>}, \frac{u_3}{<0.5,0.4>}, \frac{u_4}{<0.7,0.3>}\right\}, 0.7\right)\right) \\ \left(\left(\frac{e_2}{0.7}, r, 0\right), \left(\left\{\frac{u_1}{<0.4,0.6>}, \frac{u_2}{<0.6,0.2>}, \frac{u_3}{<0.1,0.9>}, \frac{u_4}{<0.3,0.2>}\right\}, 0.4\right)\right), \left(\left(\frac{e_3}{0.9}, m, 0\right), \left(\left\{\frac{u_1}{<0.2,0.7>}, \frac{u_2}{<0.4,0.6>}, \frac{u_3}{<0.3,0.6>}, \frac{u_4}{<0.1,0.8>}\right\}, 0.3\right)\right) \\ \left(\left(\frac{e_3}{0.9}, n, 0\right), \left(\left\{\frac{u_1}{<0.9,0.1>}, \frac{u_2}{<0.6,0.3>}, \frac{u_3}{<0.8,0.1>}, \frac{u_4}{<0.3,0.4>}\right\}, 0.6\right)\right) \end{array} \right\}$$

4. Basic Operations on Fuzzy Parameterized Generalized Intuitionisitic Fuzzy Soft Expert Set

In this section, we introduce some basic operations on FPGIFSES, namely the complement, union and intersection, "AND", "OR"of FPGIFSES, derive their properties and give some examples.

Definition 4.1. *Let (F_ψ, A) be a FPGIFSES over U. Then, the complement of (F_ψ, A) is denoted by $(F_\psi, A)^c$ and is defined by $(F_\psi, A)^c = \{(F_\psi{}^c, \urcorner A)$, where $F_\psi{}^c : \urcorner A \to I^U$ is a mapping given by*

$$F_\psi{}^c(\alpha) = c(F_\psi(\urcorner\alpha)), \ \ \forall \alpha \epsilon \urcorner A.$$

Where c is a generalized intuitionistic fuzzy complement and $\urcorner A \subset \{\psi^c \times X \times O\}$.

Example 4.2. *Consider the FPGIFSES (F_ψ, A) over a universe U aas given in Example 1. By using the generalized intuitionistic fuzzy complement, we obtain:*

$$(F_\psi, A)^c = \left\{ \begin{array}{l}
\left(\left(\frac{e_1}{0.5}, m, 1\right), \left(\left\{\frac{u_1}{<0.6,0.1>}, \frac{u_2}{<0.7,0.2>}, \frac{u_3}{<0.2,0.6>}, \frac{u_4}{<0.3,0.7>}\right\}, 0.8\right)\right), \left(\left(\frac{e_1}{0.5}, n, 1\right), \left(\left\{\frac{u_1}{<0.5,0.3>}, \frac{u_2}{<0.4,0.2>}, \frac{u_3}{<0.2,0.1>}, \frac{u_4}{<0.3,0.1>}\right\}, 0.9\right)\right) \\
\left(\left(\frac{e_1}{0.5}, r, 1\right), \left(\left\{\frac{u_1}{<0.3,0.4>}, \frac{u_2}{<0.3,0.6>}, \frac{u_3}{<0.5,0.2>}, \frac{u_4}{<0.6,0.2>}\right\}, 0.8\right)\right), \left(\left(\frac{e_2}{0.3}, m, 1\right), \left(\left\{\frac{u_1}{<0.6,0.1>}, \frac{u_2}{<0.6,0.3>}, \frac{u_3}{<0.4,0.6>}, \frac{u_4}{<0.2,0.4>}\right\}, 0.6\right)\right) \\
\left(\left(\frac{e_2}{0.3}, n, 1\right), \left(\left\{\frac{u_1}{<0.7,0.2>}, \frac{u_2}{<0.8,0.1>}, \frac{u_3}{<0.6,0.3>}, \frac{u_4}{<0.5,0.4>}\right\}, 0.8\right)\right), \left(\left(\frac{e_2}{0.3}, r, 1\right), \left(\left\{\frac{u_1}{<0.7,0.2>}, \frac{u_2}{<0.5,0.3>}, \frac{u_3}{<0.6,0.4>}, \frac{u_4}{<0.2,0.4>}\right\}, 0.7\right)\right) \\
\left(\left(\frac{e_3}{0.1}, m, 1\right), \left(\left\{\frac{u_1}{<0.5,0.3>}, \frac{u_2}{<0.3,0.6>}, \frac{u_3}{<0.1,0.8>}, \frac{u_4}{<0.1,0.9>}\right\}, 0.4\right)\right), \left(\left(\frac{e_3}{0.1}, n, 1\right), \left(\left\{\frac{u_1}{<0.6,0.2>}, \frac{u_2}{<0.2,0.3>}, \frac{u_3}{<0.5,0.3>}, \frac{u_4}{<0.3,0.6>}\right\}, 0.5\right)\right) \\
\left(\left(\frac{e_3}{0.1}, r, 1\right), \left(\left\{\frac{u_1}{<0.8,0.2>}, \frac{u_2}{<0.2,0.4>}, \frac{u_3}{<0.4,0.6>}, \frac{u_4}{<0.2,0.3>}\right\}, 0.2\right)\right), \left(\left(\frac{e_1}{0.5}, m, 0\right), \left(\left\{\frac{u_1}{<0.4,0.3>}, \frac{u_2}{<0.2,0.6>}, \frac{u_3}{<0.2,0.7>}, \frac{u_4}{<0.3,0.6>}\right\}, 0.3\right)\right) \\
\left(\left(\frac{e_1}{0.5}, n, 0\right), \left(\left\{\frac{u_1}{<0.2,0.7>}, \frac{u_2}{<0.2,0.7>}, \frac{u_3}{<0.5,0.5>}, \frac{u_4}{<0.1,0.7>}\right\}, 0.1\right)\right), \left(\left(\frac{e_1}{0.5}, r, 0\right), \left(\left\{\frac{u_1}{<0.2,0.6>}, \frac{u_2}{<0.3,0.6>}, \frac{u_3}{<0.6,0.4>}, \frac{u_4}{<0.3,0.5>}\right\}, 0.4\right)\right) \\
\left(\left(\frac{e_2}{0.3}, m, 0\right), \left(\left\{\frac{u_1}{<0.8,0.2>}, \frac{u_2}{<0.7,0.3>}, \frac{u_3}{<0.2,0.8>}, \frac{u_4}{<0.2,0.3>}\right\}, 0.4\right)\right), \left(\left(\frac{e_2}{0.3}, n, 0\right), \left(\left\{\frac{u_1}{<0.5,0.2>}, \frac{u_2}{<0.6,0.4>}, \frac{u_3}{<0.4,0.5>}, \frac{u_4}{<0.3,0.7>}\right\}, 0.3\right)\right) \\
\left(\left(\frac{e_2}{0.3}, r, 0\right), \left(\left\{\frac{u_1}{<0.6,0.4>}, \frac{u_2}{<0.2,0.6>}, \frac{u_3}{<0.9,0.1>}, \frac{u_4}{<0.2,0.3>}\right\}, 0.6\right)\right), \left(\left(\frac{e_3}{0.1}, m, 0\right), \left(\left\{\frac{u_1}{<0.7,0.2>}, \frac{u_2}{<0.6,0.4>}, \frac{u_3}{<0.6,0.3>}, \frac{u_4}{<0.8,0.1>}\right\}, 0.7\right)\right) \\
\left(\left(\frac{e_3}{0.1}, n, 0\right), \left(\left\{\frac{u_1}{<0.1,0.9>}, \frac{u_2}{<0.3,0.6>}, \frac{u_3}{<0.1,0.8>}, \frac{u_4}{<0.4,0.3>}\right\}, 0.4\right)\right), \left(\left(\frac{e_3}{0.1}, r, 0\right), \left(\left\{\frac{u_1}{<0.5,0.3>}, \frac{u_2}{<0.3,0.5>}, \frac{u_3}{<0.2,0.4>}, \frac{u_4}{<0.5,0.21>}\right\}, 0.5\right)\right)
\end{array} \right\}$$

Proposition 4.3. *If (F_ψ, A) is a fuzzy parameterized generalized intuitionisitic fuzzy soft expert set over U, then*

(1) $((F_\psi, A)^c)^c = (F_\psi, A)$

(2) $(F_\psi, A)^c_0 = (F_\psi, A)_1$

(3) $(F_\psi, A)^c_1 = (F_\psi, A)_0$

Definition 4.4. *Let (F_ψ, A) and (G_τ, B) be FPGIFSESs over a universe U. Then the union of (F_ψ, A) and (G_τ, B) is a FPGIFSES, (H_Ω, C) such that $C = \Omega \times X \times 0$ where $\Omega = \psi \cup \delta$ and for all $\varepsilon \in C$,*

$$H_\Omega(\varepsilon) = F_\phi(\varepsilon) \widetilde{\cup} G_\tau(\varepsilon)$$

where $\widetilde{\cup}$ is the generalized intuitionistic fuzzy union.

Example 4.5. *Let (F_ψ, A) and (G_τ, B) be two FPGIFSESs over the universe U defined as follows*

$$(F_\psi, A) = \left\{ \begin{array}{l}
\left(\left(\frac{e_1}{0.5}, m, 1\right), \left(\left\{\frac{u_1}{\langle 0.4,0.6\rangle}, \frac{u_2}{\langle 0.5,0.3\rangle}, \frac{u_3}{\langle 0.6,0.4\rangle}, \frac{u_4}{\langle 0.2,0.0\rangle}\right\}, 0.4\right)\right), \left(\left(\frac{e_1}{0.5}, n, 1\right), \left(\left\{\frac{u_1}{\langle 0.4,0.5\rangle}, \frac{u_2}{\langle 0.3,0.6\rangle}, \frac{u_3}{\langle 0.4,0.5\rangle}, \frac{u_4}{\langle 0.7,0.2\rangle}\right\}, 0.8\right)\right) \\
\left(\left(\frac{e_2}{0.3}, n, 1\right), \left(\left\{\frac{u_1}{\langle 0.6,0.4\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.4,0.4\rangle}, \frac{u_4}{\langle 0.2,0.8\rangle}\right\}, 0.2\right)\right), \left(\left(\frac{e_2}{0.3}, r, 0\right), \left(\left\{\frac{u_1}{\langle 0.5,04.\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.4,0.5\rangle}\right\}, 0.5\right)\right) \\
\left(\left(\frac{e_2}{0.3}, r, 1\right), \left(\left\{\frac{u_1}{\langle 0.5,0.5\rangle}, \frac{u_2}{\langle 0.6,0.3\rangle}, \frac{u_3}{\langle 0.3,0.6\rangle}, \frac{u_4}{\langle 0.2,0.3\rangle}\right\}, 0.0\right)\right), \left(\left(\frac{e_3}{0.6}, m, 1\right), \left(\left\{\frac{u_1}{\langle 0.5,0.4\rangle}, \frac{u_2}{\langle 0.3,0.5\rangle}, \frac{u_3}{\langle 0.4,0.5\rangle}, \frac{u_4}{\langle 0.5,0.5\rangle}\right\}, 0.6\right)\right) \\
\left(\left(\frac{e_3}{0.6}, m, 0\right), \left(\left\{\frac{u_1}{\langle 0.5,0.4\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.7,0.3\rangle}, \frac{u_4}{\langle 0.5,0.4\rangle}\right\}, 0.7\right)\right), \left(\left(\frac{e_3}{0.6}, n, 0\right), \left(\left\{\frac{u_1}{\langle 0.5,0.4\rangle}, \frac{u_2}{\langle 0.2,0.6\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.5,0.2\rangle}\right\}, 0.3\right)\right) \\
\left(\left(\frac{e_3}{0.6}, r, 1\right), \left(\left\{\frac{u_1}{\langle 0.5,0.3\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.4,0.5\rangle}, \frac{u_4}{\langle 0.3,0.7\rangle}\right\}, 0.6\right)\right), \left(\left(\frac{e_3}{0.6}, r, 0\right), \left(\left\{\frac{u_1}{\langle 0.5,0.2\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.4,0.3\rangle}, \frac{u_4}{\langle 0.3,0.5\rangle}\right\}, 0.5\right)\right)
\end{array} \right\}$$

$$(G_\tau, B) = \left\{ \begin{array}{l}
\left(\left(\frac{e_1}{0.6}, m, 1\right), \left(\left\{\frac{u_1}{\langle 0.5,0.4\rangle}, \frac{u_2}{\langle 0.6,0.4\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.4,0.6\rangle}\right\}, 0.5\right)\right), \left(\left(\frac{e_1}{0.6}, n, 1\right), \left(\left\{\frac{u_1}{\langle 0.3,0.5\rangle}, \frac{u_2}{\langle 0.3,0.4\rangle}, \frac{u_3}{\langle 0.2,0.6\rangle}, \frac{u_4}{\langle 0.2,0.7\rangle}\right\}, 0.5\right)\right) \\
\left(\left(\frac{e_2}{0.3}, n, 1\right), \left(\left\{\frac{u_1}{\langle 0.7,0.3\rangle}, \frac{u_2}{\langle 0.5,0.3\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.4,0.6\rangle}\right\}, 0.6\right)\right), \left(\left(\frac{e_3}{0.8}, m, 0\right), \left(\left\{\frac{u_1}{\langle 0.4,0.5\rangle}, \frac{u_2}{\langle 0.5,0.4\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.5,0.4\rangle}\right\}, 0.7\right)\right) \\
\left(\left(\frac{e_2}{0.3}, r, 0\right), \left(\left\{\frac{u_1}{\langle 0.6,0.5\rangle}, \frac{u_2}{\langle 0.5,0.4\rangle}, \frac{u_3}{\langle 0.6,0.4\rangle}, \frac{u_4}{\langle 0.6,0.3\rangle}\right\}, 0.4\right)\right), \left(\left(\frac{e_3}{0.8}, r, 1\right), \left(\left\{\frac{u_1}{\langle 0.7,0.2\rangle}, \frac{u_2}{\langle 0.4,0.3\rangle}, \frac{u_3}{\langle 0.7,0.2\rangle}, \frac{u_4}{\langle 0.6,0.4\rangle}\right\}, 0.5\right)\right) \\
\left(\left(\frac{e_3}{0.8}, r, 0\right), \left(\left\{\frac{u_1}{\langle 0.6,0.3\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.4,0.6\rangle}, \frac{u_4}{\langle 0.6,0.3\rangle}\right\}, 0.6\right)\right)
\end{array} \right\}$$

Then by using the generalized intuitionistic fuzzy union, we have $(H_\Omega, C) = (F_\phi, A)\widetilde{\cup}(G_\tau, B)$ *is a FPGIFSES defined as*

$$(H_\Omega, C) = \left\{ \begin{array}{l} \left(\left(\frac{e_1}{0.6}, m, 1\right), \left(\left\{\frac{u_1}{<0.5,0.4>}, \frac{u_2}{<0.6,0.3>}, \frac{u_3}{<0.6,0.3>}, \frac{u_4}{<0.4,0.0>}\right\}, 0.5\right)\right), \left(\left(\frac{e_1}{0.6}, n, 1\right), \left(\left\{\frac{u_1}{<0.4,0.5>}, \frac{u_2}{<0.3,0.4>}, \frac{u_3}{<0.6,0.5>}, \frac{u_4}{<0.7,0.2>}\right\}, 0.8\right)\right) \\ \left(\left(\frac{e_3}{0.8}, m, 0\right), \left(\left\{\frac{u_1}{<0.5,0.4>}, \frac{u_2}{<0.5,0.4>}, \frac{u_3}{<0.7,0.3>}, \frac{u_4}{<0.5,0.4>}\right\}, 0.7\right)\right), \left(\left(\frac{e_3}{0.8}, r, 1\right), \left(\left\{\frac{u_1}{<0.7,0.2>}, \frac{u_2}{<0.4,0.3>}, \frac{u_3}{<0.7,0.2>}, \frac{u_4}{<0.6,0.4>}\right\}, 0.5\right)\right) \\ \left(\left(\frac{e_3}{0.8}, r, 0\right), \left(\left\{\frac{u_1}{<0.6,0.2>}, \frac{u_2}{<0.4,0.5>}, \frac{u_3}{<0.4,0.3>}, \frac{u_4}{<0.6,0.3>}\right\}, 0.5\right)\right), \left(\left(\frac{e_2}{0.3}, r, 0\right), \left(\left\{\frac{u_1}{<0.6,0.4>}, \frac{u_2}{<0.5,0.4>}, \frac{u_3}{<0.6,0.3>}, \frac{u_4}{<0.6,0.3>}\right\}, 0.6\right)\right) \\ \left(\left(\frac{e_2}{0.3}, n, 1\right), \left(\left\{\frac{u_1}{<0.7,0.3>}, \frac{u_2}{<0.5,0.3>}, \frac{u_3}{<0.6,0.3>}, \frac{u_4}{<0.4,0.6>}\right\}, 0.6\right)\right), \left(\left(\frac{e_2}{0.3}, r, 1\right), \left(\left\{\frac{u_1}{\langle 0.5,0.5\rangle}, \frac{u_2}{\langle 0.6,0.3\rangle}, \frac{u_3}{\langle 0.3,0.6\rangle}, \frac{u_4}{\langle 0.2,0.3\rangle}\right\}, 0.0\right)\right) \\ \left(\left(\frac{e_3}{0.6}, m, 1\right), \left(\left\{\frac{u_1}{<0.5,0.4>}, \frac{u_2}{<0.3,0.5>}, \frac{u_3}{<0.4,0.5>}, \frac{u_4}{<0.5,0.5>}\right\}, 0.6\right)\right), \left(\left(\frac{e_3}{0.6}, n, 0\right), \left(\left\{\frac{u_1}{\langle 0.5,0.4\rangle}, \frac{u_2}{\langle 0.2,0.6\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.5,0.2\rangle}\right\}, 0.3\right)\right) \end{array} \right\}$$

Proposition 4.6. *Let* (F_ψ, A), (G_τ, B) *and* (H_Ω, C) *are three FPGIFSESs over U. Then the following results hold true.*

(1) $(F_\psi, A)\widetilde{\cup}(G_\tau, B) = (G_\tau, B)\widetilde{\cup}(F_\psi, A)$ *(commutative law)*

(2) $(F_\psi, A)\widetilde{\cup}((G_\tau, B)\widetilde{\cup}(H_\Omega, C)) = ((F_\psi, A)\widetilde{\cup}(G_\tau, B))\check{\widetilde{\cup}}(H_\Omega, C)$ *(associative law)*

(3) $(F_\psi, A)\widetilde{\cup}(F_\psi, A) = (F_\psi, A)$

Definition 4.7. *Let* (F_ψ, A) *and* (G_τ, B) *be FPGIFSESs over a universe U. Then the intersection of* (F_ψ, A) *and* (G_τ, B) *is a FPGIFSES such that* (H_Ω, C) *such that* $C = \Omega \times X \times 0$ *where* $\Omega = \psi \cap \delta$ *and for all* $\varepsilon \in C$ $H_\Omega(\varepsilon) = F_\psi(\varepsilon)\widetilde{\cap}G_\tau(\varepsilon)$, *where* $\widetilde{\cap}$ *is the generalized intuitionistic fuzzy union.*

Example 4.8. *Let* (F_ψ, A) *and* (G_τ, B) *be two FPGIFSESs over the universe U defined in Example 3. Then by using intuitionistic fuzzy intersection, we obtain* $(H_\Omega, C) = (F_\psi, A)\widetilde{\cap}(G_\tau, B)$ *is a FPGIFSES defined as*

$$(F_\psi, A) = \left\{ \begin{array}{l} \left(\left(\frac{e_1}{0.5}, m, 1\right), \left(\left\{\frac{u_1}{\langle 0.4,0.6\rangle}, \frac{u_2}{\langle 0.5,0.3\rangle}, \frac{u_3}{\langle 0.6,0.4\rangle}, \frac{u_4}{\langle 0.2,0.0\rangle}\right\}, 0.4\right)\right), \left(\left(\frac{e_1}{0.5}, n, 1\right), \left(\left\{\frac{u_1}{\langle 0.4,0.5\rangle}, \frac{u_2}{\langle 0.3,0.6\rangle}, \frac{u_3}{\langle 0.4,0.5\rangle}, \frac{u_4}{\langle 0.7,0.2\rangle}\right\}, 0.8\right)\right) \\ \left(\left(\frac{e_2}{0.3}, n, 1\right), \left(\left\{\frac{u_1}{\langle 0.6,0.4\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.4,0.4\rangle}, \frac{u_4}{\langle 0.2,0.8\rangle}\right\}, 0.2\right)\right), \left(\left(\frac{e_2}{0.3}, r, 0\right), \left(\left\{\frac{u_1}{\langle 0.5,04.\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.4,0.5\rangle}\right\}, 0.5\right)\right) \\ \left(\left(\frac{e_2}{0.3}, r, 1\right), \left(\left\{\frac{u_1}{\langle 0.5,0.5\rangle}, \frac{u_2}{\langle 0.6,0.3\rangle}, \frac{u_3}{\langle 0.3,0.6\rangle}, \frac{u_4}{\langle 0.2,0.3\rangle}\right\}, 0.0\right)\right), \left(\left(\frac{e_3}{0.6}, m, 1\right), \left(\left\{\frac{u_1}{\langle 0.5,0.4\rangle}, \frac{u_2}{\langle 0.3,0.5\rangle}, \frac{u_3}{\langle 0.4,0.5\rangle}, \frac{u_4}{\langle 0.5,0.5\rangle}\right\}, 0.6\right)\right) \\ \left(\left(\frac{e_3}{0.6}, m, 0\right), \left(\left\{\frac{u_1}{\langle 0.5,0.4\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.7,0.3\rangle}, \frac{u_4}{\langle 0.5,0.4\rangle}\right\}, 0.7\right)\right), \left(\left(\frac{e_3}{0.6}, n, 0\right), \left(\left\{\frac{u_1}{\langle 0.5,0.4\rangle}, \frac{u_2}{\langle 0.2,0.6\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.5,0.2\rangle}\right\}, 0.3\right)\right) \\ \left(\left(\frac{e_3}{0.6}, r, 1\right), \left(\left\{\frac{u_1}{\langle 0.5,0.3\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.4,0.5\rangle}, \frac{u_4}{\langle 0.3,0.7\rangle}\right\}, 0.6\right)\right), \left(\left(\frac{e_3}{0.6}, r, 0\right), \left(\left\{\frac{u_1}{\langle 0.5,0.2\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.4,0.3\rangle}, \frac{u_4}{\langle 0.3,0.5\rangle}\right\}, 0.5\right)\right) \end{array} \right\}$$

$$(G_\tau,\ B) = \left\{ \begin{array}{l} \left(\left(\frac{e_1}{0.6}, m, 1\right), \left(\left\{\frac{u_1}{\langle 0.5,0.4\rangle}, \frac{u_2}{\langle 0.6,0.4\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.4,0.6\rangle}\right\}, 0.5\right)\right), \left(\left(\frac{e_1}{0.6}, n, 1\right), \left(\left\{\frac{u_1}{\langle 0.3,0.5\rangle}, \frac{u_2}{\langle 0.3,0.4\rangle}, \frac{u_3}{\langle 0.2,0.6\rangle}, \frac{u_4}{\langle 0.2,0.7\rangle}\right\}, 0.5\right)\right) \\ \left(\left(\frac{e_2}{0.3}, n, 1\right), \left(\left\{\frac{u_1}{\langle 0.7,0.3\rangle}, \frac{u_2}{\langle 0.5,0.3\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.4,0.6\rangle}\right\}, 0.6\right)\right), \left(\left(\frac{e_3}{0.8}, m, 0\right), \left(\left\{\frac{u_1}{\langle 0.4,0.5\rangle}, \frac{u_2}{\langle 0.5,0.4\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.5,0.4\rangle}\right\}, 0.7\right)\right) \\ \left(\left(\frac{e_2}{0.3}, r, 0\right), \left(\left\{\frac{u_1}{\langle 0.6,0.5\rangle}, \frac{u_2}{\langle 0.5,0.4\rangle}, \frac{u_3}{\langle 0.6,0.4\rangle}, \frac{u_4}{\langle 0.6,0.3\rangle}\right\}, 0.4\right)\right), \left(\left(\frac{e_3}{0.8}, r, 1\right), \left(\left\{\frac{u_1}{\langle 0.7,0.2\rangle}, \frac{u_2}{\langle 0.4,0.3\rangle}, \frac{u_3}{\langle 0.7,0.2\rangle}, \frac{u_4}{\langle 0.6,0.4\rangle}\right\}, 0.5\right)\right) \\ \left(\left(\frac{e_3}{0.8}, r, 0\right), \left(\left\{\frac{u_1}{\langle 0.6,0.3\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.4,0.6\rangle}, \frac{u_4}{\langle 0.6,0.3\rangle}\right\}, 0.6\right)\right) \end{array} \right\}$$

Then by using the generalized intuitionistic fuzzy intersection, we have $(H_\Omega, C) = (F_\psi, A)\widetilde{\cap}(G_\tau, B)$ *is a FPGIFSES defined as*

$$(H_\Omega, C) = \left\{ \begin{array}{l} \left(\left(\frac{e_1}{0.5}, m, 1\right), \left(\left\{\frac{u_1}{<0.4,0.6>}, \frac{u_2}{<0.5,0.4>}, \frac{u_3}{<0.6,0.4>}, \frac{u_4}{<0.2,0.6>}\right\}, 0.4\right)\right), \left(\left(\frac{e_1}{0.5}, n, 1\right), \left(\left\{\frac{u_1}{<0.3,0.5>}, \frac{u_2}{<0.3,0.6>}, \frac{u_3}{<0.2,0.6>}, \frac{u_4}{<0.2,0.7>}\right\}, 0.5\right)\right) \\ \left(\left(\frac{e_3}{0.6}, m, 0\right), \left(\left\{\frac{u_1}{<0.4,0.5>}, \frac{u_2}{<0.4,0.5>}, \frac{u_3}{<0.6,0.3>}, \frac{u_4}{<0.5,0.4>}\right\}, 0.7\right)\right), \left(\left(\frac{e_3}{0.6}, r, 1\right), \left(\left\{\frac{u_1}{<0.5,0.3>}, \frac{u_2}{<0.4,0.5>}, \frac{u_3}{<0.4,0.5>}, \frac{u_4}{<0.3,0.7>}\right\}, 0.5\right)\right) \\ \left(\left(\frac{e_3}{0.6}, r, 0\right), \left(\left\{\frac{u_1}{<0.5,0.3>}, \frac{u_2}{<0.4,0.5>}, \frac{u_3}{<0.4,0.6>}, \frac{u_4}{<0.3,0.5>}\right\}, 0.5\right)\right), \left(\left(\frac{e_2}{0.3}, r, 0\right), \left(\left\{\frac{u_1}{<0.5,0.5>}, \frac{u_2}{<0.4,0.5>}, \frac{u_3}{<0.6,0.4>}, \frac{u_4}{<0.4,0.5>}\right\}, 0.4\right)\right) \\ \left(\left(\frac{e_2}{0.3}, n, 1\right), \left(\left\{\frac{u_1}{<0.6,0.4>}, \frac{u_2}{<0.4,0.8>}, \frac{u_3}{<0.4,0.4>}, \frac{u_4}{<0.2,0.8>}\right\}, 0.2\right)\right), \left(\left(\frac{e_3}{0.6}, n, 0\right), \left(\left\{\frac{u_1}{<0.5,0.4>}, \frac{u_2}{<0.2,0.6>}, \frac{u_3}{<0.6,0.3>}, \frac{u_4}{<0.5,0.2>}\right\}, 0.3\right)\right) \\ \left(\left(\frac{e_2}{0.3}, r, 1\right), \left(\left\{\frac{u_1}{<0.5,0.5>}, \frac{u_2}{<0.6,0.3>}, \frac{u_3}{<0.3,0.6>}, \frac{u_4}{<0.2,0.3>}\right\}, 0.0\right)\right), \left(\left(\frac{e_3}{0.6}, m, 1\right), \left(\left\{\frac{u_1}{<0.5,0.4>}, \frac{u_2}{<0.3,0.5>}, \frac{u_3}{<0.4,0.5>}, \frac{u_4}{<0.5,0.5>}\right\}, 0.6\right)\right) \end{array} \right\}$$

Proposition 4.9. *Let* (F_ψ, A), (G_τ, B) *and* (H_Ω, C) *be any three FPGIFSESs over U. Then the following results hold true.*

(1) $(F_\psi, A)\widetilde{\cap}(G_\tau, B) = (G_\tau, B)\widetilde{\cap}(F_\psi, A)$ *(commutative law)*

(2) $(F_\psi, A)\widetilde{\cap}((G_\tau, B)\widetilde{\cap}(H_\Omega, C)) = ((F_\psi, A)\widetilde{\cap}(G_\tau, B))\widetilde{\cap}(H_\Omega, C)$ *(associative law)*

(3) $(F_\psi, A)\widetilde{\cap}(F_\psi, A) = (F_\psi, A)$.

Proposition 4.10. *Let* (F_ψ, A), (G_τ, B) *and* (H_Ω, C) *are three FPGIFSESs over U. Then the following results hold true.*

(1) $(F_\psi, A)\widetilde{\cup}((G_\tau, B)\widetilde{\cap}(H_\Omega, C)) = ((F_\psi, A)\widetilde{\cup}(G_\tau, B))\widetilde{\cap}((F_\psi, A)\widetilde{\cup}(H_\Omega, C))$

(2) $(F_\psi, A)\widetilde{\cap}((G_\tau, B)\widetilde{\cup}(H_\Omega, C)) = ((F_\psi, A)\widetilde{\cap}(G_\tau, B))\widetilde{\cup}((F_\psi, A)\widetilde{\cap}(H_\Omega, C))$.

Proposition 4.11. *Let (F_ψ, A) and (G_τ, B) are two FPGIFSESs over U. Then the following De Morgan's Laws hold true:*

(1) $((F_\psi, A)\widetilde{\cup}(G_\tau, B))^c = (F_\psi, A)^c\widetilde{\cap}(G_\tau, B)^c$

(2) $((F_\psi, A)\widetilde{\cap}(G_\tau, B))^c = (F_\psi, A)^c\widetilde{\cup}(G_\tau, B)^c$

Definition 4.12. *Let (F_ψ, A) and (G_τ, B) be two FPGIFSES over U. Then "$(F_\psi, A)AND(G_\tau, B)$"denoted by "$(F_\psi, A)\widetilde{\wedge}(G_\tau, B)$"is defined as $(F_\psi, A)\widetilde{\wedge}(G_\tau, B) = (H_\Omega, A \times B)$ such that $H(\alpha, \beta) = F(\alpha)\widetilde{\cap}G(\beta)$, for all $(\alpha, \beta) \in A \times B$.*

Example 4.13. *Let (F_ψ, A) and (G_τ, B) be FPGIFSESs over a universe U.*

$$\left(F_\psi, A\right) = \left\{\begin{array}{l} \left(\left(\frac{e_1}{0.5}, m, 1\right), \left(\left\{\frac{u_1}{\langle 0.4,0.6\rangle}, \frac{u_2}{\langle 0.5,0.3\rangle}, \frac{u_3}{\langle 0.2,0.6\rangle}, \frac{u_4}{\langle 0.2,0.0\rangle}\right\}, 0.4\right)\right), \left(\left(\frac{e_3}{0.6}, m, 0\right), \left(\left\{\frac{u_1}{\langle 0.5,0.3\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.7,0.3\rangle}, \frac{u_4}{\langle 0.5,0.4\rangle}\right\}, 0.7\right)\right) \\ \left(\left(\frac{e_2}{0.3}, n, 1\right), \left(\left\{\frac{u_1}{\langle 0.6,0.4\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.4,0.4\rangle}, \frac{u_4}{\langle 0.2,0.8\rangle}\right\}, 0.8\right)\right), \left(\left(\frac{e_2}{0.3}, r, 1\right), \left(\left\{\frac{u_1}{\langle 0.5,0.5\rangle}, \frac{u_2}{\langle 0.6,0.3\rangle}, \frac{u_3}{\langle 0.3,0.2\rangle}, \frac{u_4}{\langle 0.2,0.3\rangle}\right\}, 0.5\right)\right) \\ \left(\left(\frac{e_3}{0.6}, n, 0\right), \left(\left\{\frac{u_1}{\langle 0.5,0.4\rangle}, \frac{u_2}{\langle 0.5,0.4\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.5,0.2\rangle}\right\}, 0.3\right)\right) \end{array}\right\}$$

$$(G_\tau, B) = \left\{\begin{array}{l} \left(\left(\frac{e_1}{0.6}, n, 1\right), \left(\left\{\frac{u_1}{\langle 0.4,0.5\rangle}, \frac{u_2}{\langle 0.3,0.4\rangle}, \frac{u_3}{\langle 0.6,0.4\rangle}, \frac{?_4}{\langle 0.2,0.8\rangle}\right\}, 0.5\right)\right), \left(\left(\frac{e_2}{0.3}, r, 0\right), \left(\left\{\frac{u_1}{\langle 0.3,0.2\rangle}, \frac{u_2}{\langle 0.5,0.4\rangle}, \frac{u_3}{\langle 0.5,0.3\rangle}, \frac{u_4}{\langle 0.6,0.3\rangle}\right\}, 0.7\right)\right) \\ \left(\left(\frac{e_3}{0.8}, r, 1\right), \left(\left\{\frac{u_1}{\langle 0.7,0.2\rangle}, \frac{u_2}{\langle 0.4,0.3\rangle}, \frac{u_3}{\langle 0.7,0.2\rangle}, \frac{u_4}{\langle 0.6,0.4\rangle}\right\}, 0.6\right)\right) \end{array}\right\}$$

Then by using the generalized intuitionistic fuzzy intersection, we obtain $(F_\psi, A)\,\check{\wedge}\,(G_\tau, B) = (H_\Omega, C)$. Where $C = A \times B$, (H_Ω, C) is a FPGIFSES defined as

$$(H_\Omega, C) = \left\{\begin{array}{l} \left(\left(\frac{e_1}{0.5}, m, 1\right), \left(\frac{e_1}{0.6}, n, 1\right), \left\{\frac{u_1}{\langle 0.4,0.6\rangle}, \frac{u_2}{\langle 0.3,0.4\rangle}, \frac{u_3}{\langle 0.2,0.6\rangle}, \frac{u_4}{\langle 0.2,0.8\rangle}\right\}, 0.4\right) \\ \left(\left(\frac{e_1}{0.5}, m, 1\right), \left(\frac{e_2}{0.3}, r, 0\right), \left\{\frac{u_1}{\langle 0.3,0.6\rangle}, \frac{u_2}{\langle 0.5,0.4\rangle}, \frac{u_3}{\langle 0.2,0.6\rangle}, \frac{u_4}{\langle 0.2,0.3\rangle}\right\}, 0.4\right) \\ \left(\left(\frac{e_1}{0.5}, m, 1\right), \left(\frac{e_3}{0.8}, r, 1\right), \left\{\frac{u_1}{\langle 0.4,0.6\rangle}, \frac{u_2}{\langle 0.4,0.3\rangle}, \frac{u_3}{\langle 0.2,0.6\rangle}, \frac{u_4}{\langle 0.2,0.4\rangle}\right\}, 0.4\right) \\ \left(\left(\frac{e_3}{0.6}, m, 0\right), \left(\frac{e_1}{0.6}, n, 1\right), \left\{\frac{u_1}{\langle 0.4,0.5\rangle}, \frac{u_2}{\langle 0.3,0.5\rangle}, \frac{u_3}{\langle 0.6,0.4\rangle}, \frac{u_4}{\langle 0.2,0.8\rangle}\right\}, 0.5\right) \\ \left(\left(\frac{e_3}{0.6}, m, 0\right), \left(\frac{e_2}{0.3}, r, 0\right), \left\{\frac{u_1}{\langle 0.5,0.3\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.5,0.3\rangle}, \frac{u_4}{\langle 0.5,0.4\rangle}\right\}, 0.7\right) \\ \left(\left(\frac{e_3}{0.6}, m, 0\right), \left(\frac{e_3}{0.8}, r, 1\right), \left\{\frac{u_1}{\langle 0.5,0.3\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.7,0.3\rangle}, \frac{u_4}{\langle 0.5,0.4\rangle}\right\}, 0.6\right) \\ \left(\left(\frac{e_2}{0.3}, n, 1\right), \left(\frac{e_1}{0.6}, n, 1\right), \left\{\frac{u_1}{\langle 0.4,0.5\rangle}, \frac{u_2}{\langle 0.3,0.5\rangle}, \frac{u_3}{\langle 0.4,0.4\rangle}, \frac{u_4}{\langle 0.2,0.8\rangle}\right\}, 0.5\right) \\ \left(\left(\frac{e_2}{0.3}, n, 1\right), \left(\frac{e_2}{0.3}, r, 0\right), \left\{\frac{u_1}{\langle 0.3,0.4\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.4,0.4\rangle}, \frac{u_4}{\langle 0.2,0.8\rangle}\right\}, 0.7\right) \\ \left(\left(\frac{e_2}{0.3}, n, 1\right), \left(\frac{e_3}{0.8}, r, 1\right), \left\{\frac{u_1}{\langle 0.6,0.4\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.4,0.4\rangle}, \frac{u_4}{\langle 0.2,0.8\rangle}\right\}, 0.6\right) \\ \left(\left(\frac{e_2}{0.3}, r, 1\right), \left(\frac{e_1}{0.6}, n, 1\right), \left\{\frac{u_1}{\langle 0.4,0.5\rangle}, \frac{u_2}{\langle 0.3,0.4\rangle}, \frac{u_3}{\langle 0.3,0.4\rangle}, \frac{u_4}{\langle 0.2,0.8\rangle}\right\}, 0.5\right) \\ \left(\left(\frac{e_2}{0.3}, r, 1\right), \left(\frac{e_2}{0.3}, r, 0\right), \left\{\frac{u_1}{\langle 0.3,0.5\rangle}, \frac{u_2}{\langle 0.5,0.4\rangle}, \frac{u_3}{\langle 0.3,0.3\rangle}, \frac{u_4}{\langle 0.2,0.3\rangle}\right\}, 0.5\right) \\ \left(\left(\frac{e_2}{0.3}, r, 1\right), \left(\frac{e_3}{0.8}, r, 1\right), \left\{\frac{u_1}{\langle 0.5,0.5\rangle}, \frac{u_2}{\langle 0.4,0.3\rangle}, \frac{u_3}{\langle 0.3,0.2\rangle}, \frac{u_4}{\langle 0.2,0.4\rangle}\right\}, 0.5\right) \\ \left(\left(\frac{e_3}{0.6}, n, 0\right), \left(\frac{e_1}{0.6}, n, 1\right), \left\{\frac{u_1}{\langle 0.4,0.5\rangle}, \frac{u_2}{\langle 0.3,0.4\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.2,0.8\rangle}\right\}, 0.3\right) \\ \left(\left(\frac{e_3}{0.6}, n, 0\right), \left(\frac{e_2}{0.3}, r, 0\right), \left\{\frac{u_1}{\langle 0.3,0.4\rangle}, \frac{u_2}{\langle 0.5,0.4\rangle}, \frac{u_3}{\langle 0.5,0.3\rangle}, \frac{u_4}{\langle 0.5,0.3\rangle}\right\}, 0.3\right) \\ \left(\left(\frac{e_3}{0.6}, n, 0\right), \left(\frac{e_3}{0.8}, r, 1\right), \left\{\frac{u_1}{\langle 0.5,0.4\rangle}, \frac{u_2}{\langle 0.4,0.4\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.5,0.4\rangle}\right\}, 0.3\right) \end{array}\right\}$$

Definition 4.14. *Let (F_ψ, A) and (G_τ, B) be two FPGIFSES over U. Then "$(F_\psi, A)OR(G_\tau, B)$"denoted by "$(F_\psi, A)\widetilde{\vee}(G_\tau, B)$"is defined as $(F_\psi, A)\widetilde{\vee}(G_\tau, B) = (H_\Omega, A \times B)$ such that $H(\alpha, \beta) = F(\alpha)\widetilde{\cup}G(\beta)$, for all $(\alpha, \beta) \in A \times B$.*

Example 4.15. *Let (F_ψ, A) and (G_τ, B) be two FPGIFSES over U defined as given below.*

$$\left(F_\psi, A\right) = \left\{\begin{array}{l} \left(\left(\frac{e_1}{0.5}, m, 1\right), \left(\left\{\frac{u_1}{\langle 0.4,0.6\rangle}, \frac{u_2}{\langle 0.5,0.3\rangle}, \frac{u_3}{\langle 0.2,0.6\rangle}, \frac{u_4}{\langle 0.2,0.0\rangle}\right\}, 0.4\right)\right), \left(\left(\frac{e_3}{0.6}, m, 0\right), \left(\left\{\frac{u_1}{\langle 0.5,0.3\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.7,0.3\rangle}, \frac{u_4}{\langle 0.5,0.4\rangle}\right\}, 0.7\right)\right) \\ \left(\left(\frac{e_2}{0.3}, n, 1\right), \left(\left\{\frac{u_1}{\langle 0.6,0.4\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.4,0.4\rangle}, \frac{u_4}{\langle 0.2,0.8\rangle}\right\}, 0.8\right)\right), \left(\left(\frac{e_2}{0.3}, r, 1\right), \left(\left\{\frac{u_1}{\langle 0.5,0.5\rangle}, \frac{u_2}{\langle 0.6,0.3\rangle}, \frac{u_3}{\langle 0.3,0.2\rangle}, \frac{u_4}{\langle 0.2,0.3\rangle}\right\}, 0.5\right)\right) \\ \left(\left(\frac{e_3}{0.6}, n, 0\right), \left(\left\{\frac{u_1}{\langle 0.5,0.4\rangle}, \frac{u_2}{\langle 0.5,0.4\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.5,0.2\rangle}\right\}, 0.3\right)\right) \end{array}\right\}$$

$$(G_\tau, B) = \left\{ \begin{array}{l} \left(\left(\frac{e_1}{0.6}, n, 1\right), \left(\left\{\frac{u_1}{\langle 0.4,0.5\rangle}, \frac{u_2}{\langle 0.3,0.4\rangle}, \frac{u_3}{\langle 0.6,0.4\rangle}, \frac{u_4}{\langle 0.2,0.8\rangle}\right\}, 0.5\right)\right), \left(\left(\frac{e_2}{0.3}, r, 0\right), \left(\left\{\frac{u_1}{\langle 0.3,0.2\rangle}, \frac{u_2}{\langle 0.5,0.4\rangle}, \frac{u_3}{\langle 0.5,0.3\rangle}, \frac{u_4}{\langle 0.6,0.3\rangle}\right\}, 0.7\right)\right) \\ \left(\left(\frac{e_3}{0.8}, r, 1\right), \left(\left\{\frac{u_1}{\langle 0.7,0.2\rangle}, \frac{u_2}{\langle 0.4,0.3\rangle}, \frac{u_3}{\langle 0.7,0.2\rangle}, \frac{u_4}{\langle 0.6,0.4\rangle}\right\}, 0.6\right)\right) \end{array} \right\}$$

Then by using the generalized intuitionistic fuzzy union, we obtain $(F_\psi, A) \,\widetilde{\vee}\, (G_\tau, B) = (H_\Omega, C)$ *where* $C = A \times B$, (H_Ω, C) *is a FPGIFSES defined as*

$$(H_\Omega, C) = \left\{ \begin{array}{l}
\left(\left(\frac{e_1}{0.5}, m, 1\right), \left(\frac{e_1}{0.6}, n, 1\right), \left\{\frac{u_1}{\langle 0.4,0.5\rangle}, \frac{u_2}{\langle 0.5,0.3\rangle}, \frac{u_3}{\langle 0.6,0.4\rangle}, \frac{u_4}{\langle 0.2,0.0\rangle}\right\}, 0.5\right) \\
\left(\left(\frac{e_1}{0.5}, m, 1\right), \left(\frac{e_2}{0.3}, r, 0\right), \left\{\frac{u_1}{\langle 0.4,0.2\rangle}, \frac{u_2}{\langle 0.5,0.3\rangle}, \frac{u_3}{\langle 0.5,0.3\rangle}, \frac{u_4}{\langle 0.6,0.0\rangle}\right\}, 0.7\right) \\
\left(\left(\frac{e_1}{0.5}, m, 1\right), \left(\frac{e_3}{0.8}, r, 1\right), \left\{\frac{u_1}{\langle 0.7,0.2\rangle}, \frac{u_2}{\langle 0.5,0.3\rangle}, \frac{u_3}{\langle 0.7,0.2\rangle}, \frac{u_4}{\langle 0.6,0.4\rangle}\right\}, 0.6\right) \\
\left(\left(\frac{e_3}{0.6}, m, 0\right), \left(\frac{e_1}{0.6}, n, 1\right), \left\{\frac{u_1}{\langle 0.5,0.3\rangle}, \frac{u_2}{\langle 0.4,0.4\rangle}, \frac{u_3}{\langle 0.7,0.3\rangle}, \frac{u_4}{\langle 0.5,0.6\rangle}\right\}, 0.7\right) \\
\left(\left(\frac{e_3}{0.6}, m, 0\right), \left(\frac{e_2}{0.3}, r, 0\right), \left\{\frac{u_1}{\langle 0.5,0.2\rangle}, \frac{u_2}{\langle 0.5,0.4\rangle}, \frac{u_3}{\langle 0.7,0.3\rangle}, \frac{u_4}{\langle 0.6,0.3\rangle}\right\}, 0.7\right) \\
\left(\left(\frac{e_3}{0.6}, m, 0\right), \left(\frac{e_3}{0.8}, r, 1\right), \left\{\frac{u_1}{\langle 0.7,0.2\rangle}, \frac{u_2}{\langle 0.4,0.3\rangle}, \frac{u_3}{\langle 0.7,0.2\rangle}, \frac{u_4}{\langle 0.6,0.4\rangle}\right\}, 0.7\right) \\
\left(\left(\frac{e_2}{0.3}, n, 1\right), \left(\frac{e_1}{0.6}, n, 1\right), \left\{\frac{u_1}{\langle 0.6,0.4\rangle}, \frac{u_2}{\langle 0.4,0.4\rangle}, \frac{u_3}{\langle 0.6,0.4\rangle}, \frac{u_4}{\langle 0.2,0.8\rangle}\right\}, 0.8\right) \\
\left(\left(\frac{e_2}{0.3}, n, 1\right), \left(\frac{e_2}{0.3}, r, 0\right), \left\{\frac{u_1}{\langle 0.6,0.2\rangle}, \frac{u_2}{\langle 0.5,0.4\rangle}, \frac{u_3}{\langle 0.5,0.3\rangle}, \frac{u_4}{\langle 0.6,0.3\rangle}\right\}, 0.8\right) \\
\left(\left(\frac{e_2}{0.3}, n, 1\right), \left(\frac{e_3}{0.8}, r, 1\right), \left\{\frac{u_1}{\langle 0.7,0.2\rangle}, \frac{u_2}{\langle 0.4,0.3\rangle}, \frac{u_3}{\langle 0.7,0.2\rangle}, \frac{u_4}{\langle 0.6,0.4\rangle}\right\}, 0.8\right) \\
\left(\left(\frac{e_2}{0.3}, r, 1\right), \left(\frac{e_1}{0.6}, n, 1\right), \left\{\frac{u_1}{\langle 0.5,0.5\rangle}, \frac{u_2}{\langle 0.6,0.3\rangle}, \frac{u_3}{\langle 0.6,0.2\rangle}, \frac{u_4}{\langle 0.2,0.3\rangle}\right\}, 0.5\right) \\
\left(\left(\frac{e_2}{0.3}, r, 1\right), \left(\frac{e_2}{0.3}, r, 0\right), \left\{\frac{u_1}{\langle 0.5,0.2\rangle}, \frac{u_2}{\langle 0.6,0.3\rangle}, \frac{u_3}{\langle 0.5,0.2\rangle}, \frac{u_4}{\langle 0.6,0.3\rangle}\right\}, 0.7\right) \\
\left(\left(\frac{e_2}{0.3}, r, 1\right), \left(\frac{e_3}{0.8}, r, 1\right), \left\{\frac{u_1}{\langle 0.7,0.2\rangle}, \frac{u_2}{\langle 0.6,0.3\rangle}, \frac{u_3}{\langle 0.7,0.2\rangle}, \frac{u_4}{\langle 0.6,0.3\rangle}\right\}, 0.6\right) \\
\left(\left(\frac{e_3}{0.6}, n, 0\right), \left(\frac{e_1}{0.6}, n, 1\right), \left\{\frac{u_1}{\langle 0.5,0.4\rangle}, \frac{u_2}{\langle 0.5,0.4\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.5,0.2\rangle}\right\}, 0.5\right) \\
\left(\left(\frac{e_3}{0.6}, n, 0\right), \left(\frac{e_2}{0.3}, r, 0\right), \left\{\frac{u_1}{\langle 0.5,0.2\rangle}, \frac{u_2}{\langle 0.5,0.4\rangle}, \frac{u_3}{\langle 0.6,0.3\rangle}, \frac{u_4}{\langle 0.6,0.2\rangle}\right\}, 0.7\right) \\
\left(\left(\frac{e_3}{0.6}, n, 0\right), \left(\frac{e_3}{0.8}, r, 1\right), \left\{\frac{u_1}{\langle 0.7,0.2\rangle}, \frac{u_2}{\langle 0.5,0.3\rangle}, \frac{u_3}{\langle 0.7,0.2\rangle}, \frac{u_4}{\langle 0.6,0.2\rangle}\right\}, 0.6\right)
\end{array} \right\}$$

5. Conclusion

In this work we have introduced the concept of fuzzy parameterized generalized intuitionistic fuzzy soft expert set and studied some of its properties. Finally, the complement, intersection and union operations have been defined on the fuzzy parameterized generalised fuzzy soft expert set.

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