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Common Fixed Point Theorems For Compatible Maps in Generalized Intuitionistic Fuzzy Metric Space

Research Article

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Abstract: The purpose of this paper is to obtain common fixed point theorems for compatible maps in generalized intuitionistic fuzzy metric spaces. Our results extend, generalize and fuzzify several fixed point theorems in Q -fuzzy metric spaces.

MSC: 54H25, 47H10.

Keywords: Fixed point, Generalized intuitionistic fuzzy metric spaces.

1. Introduction

The concept of fuzzy sets, introduced by Zadeh [13] plays an important role in topology and analysis. Since then, there are many authors to study the fuzzy sets with applications. Especially Kramosil and Michlek [5] put forward a new concept of fuzzy metric space. George and Veeramani [3] revised the notion of fuzzy metric space with the help of continuous t-norm. As a result, many fixed point theorems for various forms of mappings are obtained in fuzzy metric spaces. Dhage [2] introduced the definition of D-metric space and proved many new fixed point theorems in D-metric spaces. Recently, Mustafa and Sims [7] presented a new definition of G-metric space and made great contribution to the development of Dhage theory. Fixed point theorems using the notion of compatibility of maps or by using its generalized or weaker forms are obtained by many authors in varied spaces. In [12] Guangpeng Sun and Kai yang introduced the notion of Q- fuzzy metric space. In this study we introduce the notion of generalized intuitionistic fuzzy metric space, which can be considered as a generalization of fuzzy metric space. We show some new fixed point theorems in such generalized intuitionistic fuzzy metric spaces. The results presented in this paper improve and extend some known results.

2. Generalized Intuitionistic Fuzzy Metric Spaces

Definition 2.1. *A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if it satisfies the following conditions:*

(1). $*$ is associative and commutative

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(2). $*$ is continuous

(3). $a * 1 = a$ for all $a \in [0,1]$

(4). $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0,1]$.

Definition 2.2. *A binary operation $\diamondsuit : [0,1] \times [0,1] \to [0,1]$ is said to be a continuous t- conorm if it satisfies the following conditions:*

(1). $\diamondsuit$ is associative and commutative.

(2). $\diamondsuit$ is continuous.

(3). $a \diamondsuit 0 = a$ for all $a \in [0,1]$

(4). $a \diamondsuit b \leq c \diamondsuit d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0,1]$.

Definition 2.3. *A 3-tuple $(X, Q, *)$ is called a Q-fuzzy metric space if X is an arbitray (non-empty) set, $*$ is a continuous t-norm, and Q is a fuzzy set on $X^3 \times (0, \infty)$, satisfying the following conditions for each $x, y, z, a \in X$ and $t, s > 0$:*

(1). $Q(x, x, y, t) > 0$ and $Q(x, x, y, t) \leq Q(x, y, z, t)$ for all $x, y, z \in X$ with $z \neq y$

(2). $Q(x, y, z, t) = 1$ if and only if $x = y = z$

(3). $Q(x, y, z, t) = Q\{p(x, y, z), t\}$, (symmetry) where p is a permutation function,

(4). $Q(x, a, a, t) * Q(a, y, z, s) \leq Q(x, y, z, t + s)$

(5). $Q(x, y, z, \cdot) : (0, \infty) \to [0,1]$ is continuous

A Q-fuzzy metric space is said to be symmetric if $Q(x, y, y, t) = Q(x, x, y, t)$ for all $x, y \in X$.

Definition 2.4. *A 5-tuple $(X, Q, H, *, \diamondsuit)$ is said to be an generalized intuitionistic fuzzy metric space (for short GIFMS) if X is an arbitrary set, $*$ is a continuous t-norm, $\diamondsuit$ is a continuous t-conorm and Q, H are fuzzy set on $X^3 \to (0, \infty)$ satisfying the following conditions. For every $x, y, z, a \in X$ and $t, s > 0$*

(1). $Q(x, y, z, t) + H(x, y, z, t) \leq 1$

(2). $Q(x, x, y, t) > 0$, for all $x \neq y$

(3). $Q(x, x, y, t) = Q(x, y, z, t)$ for $y \neq z$

(4). $Q(x, y, z, t) = 1$ iff $x = y = z$

(5). $Q(x, y, z, t) = Q\{p(x, y, z), t\}$, where p is a permutation function.

(6). $Q(x, a, at) * Q(a, y, z, s) \leq Q(x, y, z, t + s)$

(7). $Q(x, y, z, \cdot) : (0, \infty) \to [0,1]$ is continuous

(8). Q is non decreasing function on R^+ $\lim_{t \to \infty} Q(x, y, z, t) = 1$ and $\lim_{t \to 0} Q(x, y, z, t) = 0$, for all $x, y, z \in X$, $t > 0$

(9). $H(x, x, y, t) < 1$, for all $x \neq y$

(10). $H(x, x, y, t) \geq H(x, y, z, t)$ for $y \neq z$

(11). $H(x, y, z, t) = 0$ *iff* $x = y = z$

(12). $H(x, y, z, t) = H\{p(x, y, z), t\}$ *where p is a permutation function.*

(13). $H(x, a, at)\Diamond H(a, y, z, s) \geq H(x, y, z, t + s)$

(14). $H(x, y, z, \cdot) : (0, \infty) \to [0, 1]$ *is continuous*

(15). H is a non- increasing function on R^+ $\lim_{t\to\infty} H(x, y, z, t) = 0$ *and* $\lim_{t\to 0} H(x, y, z, t) = 1$ *for all* $x, y, z \in X,\ t > 0$

In this case, the pair (Q, H) is called an generalized intuitionistic fuzzy metric on X.

Example 2.5. *Let* (X, Q) *be a Q-metric space, for all* $x, y, z \in X$ *and every* $t > 0$*, Consider Q, H to be fuzzy sets on* $X^3 \times (0, \infty)$ *defined by* $Q(x, y, z, t) = \frac{t}{t+Q(x,y,z,t)}$ *and* $H(x, y, z, t) = \frac{Q(x,y,z,t)}{t+Q(x,y,z,t)}$ *and denote* $a*b = ab$ *and* $a\Diamond b = \min\{a+b, 1\}$*. Then* $(X, Q, H, *, \Diamond)$ *is an generalized intuitionistic fuzzy metric space. Notice that the above example holds even with the t-norm* $a * b = \min\{a, b\}$ *and t-conorm* $a\Diamond b = \max\{a, b\}$*.*

Remark 2.6. *In an generalized intuitionistic fuzzy metric space* $Q(x, y, z, \cdot)$ *is non-decreasing and* $H(x, y, z, \cdot)$ *is non-increasing for all* $x, y, z \in X$*.*

Definition 2.7. *Let* $x \in X$*, where* $(X, Q, H, *, \Diamond)$ *is an generalized intuitionistic fuzzy metric space. Then, for* $r \in (0, 1)$ *and* $t > 0$*, the set* $B_{Q,H}(x, r, t) = \{y \in X : Q(x, y, y, t) > 1 - r \text{ and } H(x, y, y, t) < r\}$ *is said to be an open ball with centre x and radius r with respect to t. Note that every open ball* $B_{Q,H}(x, r, t)$ *is an open set.*

Definition 2.8. *Let* $(X, Q, H, *, \Diamond)$ *be an generalized intuitionistic fuzzy metric space, then*

(1). A sequence $\{x_n\}$ *in X is said to be convergent to x if* $\lim_{n\to\infty} Q(x_n, x_n, x, t) = 1$ *and* $\lim_{n\to\infty} H(x_n, x_n, x, t) = 0$*.*

(2). A sequence $\{x_n\}$ *in X is said to be Cauchy sequence if* $\lim_{n,m\to\infty} Q(x_n, x_n, x_m, t) = 1$ *and* $\lim_{n,m\to\infty} H(x_n, x_n, x_m, t) = 0$ *that is, for any* $\varepsilon > 0$ *and for each* $t > 0$*, there exists* $n_0 \in N$ *such that* $Q(x_n, x_n, x_m, t) > 1 - \varepsilon$ *and* $H(x_n, x_n, x_m, t) < \varepsilon$ *for* $n, m \geq n_0$*.*

(3). A generalized intuitionistic fuzzy metric space $(X, Q, H, *, \Diamond)$ *is said to be complete if every Cauchy sequence in X is convergent.*

Definition 2.9. *Let f and g be two self mappings of a generalized intuitionistic fuzzy metric space* $(X, Q, H, *, \Diamond)$ *If f and g satisfy the following conditions: There exists a sequence* $\{x_n\}$ *such that* $\lim_{n\to\infty} Q(fx_n, u, u, t) = \lim_{n\to\infty} Q(gx_n, u, u, t) = 1$ *and* $\lim_{n\to\infty} H(fx_n, u, u, t) = \lim_{n\to\infty} H(gx_n, u, u, t) = 0$ *for some* $u \in X$ *and* $t > 0$*, we say that f and g have the property (E.A).*

Definition 2.10. *Let* $(X, Q, H, *, \Diamond)$ *be a generalized intuitionistic fuzzy metric space. The following conditions are satisfied :* $\lim_{n\to\infty} Q(x_n, y_n, z_n, t_n) = Q(x, y, z, t)$ *and* $\lim_{n\to\infty} H(x_n, y_n, z_n, t_n) = H(x, y, z, t)$*. Whenever* $\lim_{n\to\infty} x_n = x$ *;* $\lim_{n\to\infty} y_n = y$ *;* $\lim_{n\to\infty} z_n = z$ *and* $\lim_{n\to\infty} Q(x, y, z, t_n) = Q(x, y, z, t)$ *,* $\lim_{n\to\infty} H(x, y, z, t_n) = H(x, y, z, t)$ *then Q, H are called convergent function on* $X^3 \times (0, \infty)$*.*

Lemma 2.11. *Let* $(X, Q, H, *, \Diamond)$ *be a generalized intuitionistic fuzzy metric space. Then Q, H are continuous function on* $X^3 \times (0, \infty)$*.*

Proof. Since $\lim_{n\to\infty} x_n = x$; $\lim_{n\to\infty} y_n = y$; $\lim_{n\to\infty} z_n = z$. $\lim_{n\to\infty} Q(x, y, z, t_n) = Q(x, y, z, t)$ and $\lim_{n\to\infty} H(x, y, z, t_n) = H(x, y, z, t)$. There is $n_0 \in N$ such that $|t - t_n| < \varepsilon$ and $|t - t_n| > \delta$ for $n \geq n_0$ and $\varepsilon < t/2$ and $\delta > t/2$. We know that Q(x, y, z, t) is non -decreasing and H(x, y, z, t) is non-increasing with respect to t, So, we have

$$\begin{aligned} Q(x_n, y_n, z_n, t) &\geq Q(x_n, y_n, z_n, t - \varepsilon) \\ &\geq Q(x_n, x, x, \frac{\varepsilon}{3}) * Q(x, y_n, z_n, t - \frac{4\varepsilon}{3}) \\ &\geq Q(x_n, x, x, \frac{\varepsilon}{3}) * Q(y_n, y, y, \frac{\varepsilon}{3}) * Q(y, x, z_n, t - \frac{5\varepsilon}{3}) \\ &\geq Q(x_n, x, x, \frac{\varepsilon}{3}) * Q(y_n, y, y, \frac{\varepsilon}{3}) * Q(z_n, z, z, \frac{\varepsilon}{3}) * Q(z, y, z, t - 2\varepsilon) \quad \text{and} \end{aligned}$$

$$\begin{aligned} H(x_n, y_n, z_n, t) &\leq H(x_n, y_n, z_n, t - \delta) \\ &\leq H(x_n, x, x, \frac{\delta}{3}) \diamondsuit H(x, y_n, z_n, t - \frac{4\delta}{3}) \\ &\leq H(x_n, x, x, \frac{\delta}{3}) \diamondsuit H(y_n, y, y, \frac{\delta}{3}) \diamondsuit H(y, x, z_n, t - \frac{5\delta}{3}) \\ &\leq H(x_n, x, x, \frac{\delta}{3}) \diamondsuit H(y_n, y, y, \frac{\delta}{3}) \diamondsuit H(z_n, z, z, \frac{\delta}{3}) \diamondsuit H(z, y, z, t - 2\delta) \end{aligned}$$

$$\begin{aligned} Q(x, y, z, t + 2\,\varepsilon) &\geq Q(x, y, z, t_n + \varepsilon) \\ &\geq Q(x, x_n, x_n, \frac{\varepsilon}{3}) * Q(x_n, y, z, t_n + \frac{2\varepsilon}{3}) \\ &\geq Q(x, x_n, x_n, \frac{\varepsilon}{3}) * Q(y, y_n, y_n, \frac{\varepsilon}{3}) * Q(y_n, x_n, z, t_n + \frac{\varepsilon}{3}) \\ &\geq Q(x, x_n, x_n, \frac{\varepsilon}{3}) * Q(y, y_n, y_n, \frac{\varepsilon}{3}) * Q(z, z_n, z_n, \frac{\varepsilon}{3}) * Q(z, y, x, t_n) \quad \text{and} \end{aligned}$$

$$\begin{aligned} H(x, y, z, t + 2\,\delta) &\leq H(x, y, z, t_n + \delta) \\ &\leq H(x, x_n, x_n, \frac{\delta}{3}) \diamondsuit H(x_n, y, z, t_n + \frac{2\delta}{3}) \\ &\leq H(x, x_n, x_n, \frac{\delta}{3}) \diamondsuit H(y, y_n, y_n, \frac{\delta}{3}) \diamondsuit H(y_n, x_n, z, t_n + \frac{\delta}{3}) \\ &\leq H(x, x_n, x_n, \frac{\delta}{3}) \diamondsuit H(y, y_n, y_n, \frac{\delta}{3}) \diamondsuit H(z, z_n, z_n, \frac{\delta}{3}) \diamondsuit H(z, y, x, t_n) \end{aligned}$$

Let n $\to \infty$, by continuity of the function Q, H with respect to t, we can get $Q(x, y, z, t+2\varepsilon) \geq Q(z, y, x, t) \geq Q(z, y, x, t-2\varepsilon)$ and $H(x, y, z, t + 2\delta) \leq H(z, y, x, t) \leq H(z, y, x, t - 2\delta)$. Therefore Q, H are continuous function on $X^3 \times (0, \infty)$. □

Definition 2.12. *Let f and g be self maps on generalized intuitionsitic fuzzy metric space (X, Q, H, ∗, ♢). Then the mappings are said to be weakly compatible if they commute at their coincidence point, that is, $fx = gx$ implies that $fgx = gfx$.*

Definition 2.13. *Let f and g be self maps on generalized intuitionistic fuzzy metric space (X, Q, H, ∗,♢). The pair (f,g) is said to be compatible if $\lim_{n\to\infty} Q(fgx_n, gfx_n, gfx_n, t) = 1$ and $\lim_{n\to\infty} H(fgx_n, gfx_n, gfx_n, t) = 0$ Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some $z \in X$.*

3. Main Theorem

The following theorem deals with four self maps.

Theorem 3.1. *Let A, B, S and T be four self maps on a complete generalized intuitionistic fuzzy metric space (X, Q, H, ∗, ♢) such that Q, H are symmetric with continuous t-norm, $t * t \geq t$ and continuous t-conorm $(1-t)\diamondsuit(1-t) \leq 1-t$ satisfying the following conditions:*

(1). $AX \subseteq TX$, $BX \subseteq SX$

(2). S and T are continuous

(3). $\{A, S\}$ and $\{B, T\}$ are compatible pairs of maps

(4). For all $x, y, z \in X$, $k \in (0,1)$, $t > 0$ and $\beta \in (0,2)$ with

$$Q(Ax, By, Bz, kt) \geq \left\{ \begin{array}{l} Q(Sx, Ty, Tz, t) * Q(Ax, Sx, Sz, t) * Q(By, Ty, Tz, t) \\ *Q(By, Sx, Sz, (2-\beta)t) * Q(Ax, Ty, Ty, \beta t) \end{array} \right\}$$

$$H(Ax, By, Bz, kt) \leq \left\{ \begin{array}{l} H(Sx, Ty, Tz, t) \diamondsuit H(Ax, Sx, Sz, t) \diamondsuit H(By, Ty, Tz, t) \\ \diamondsuit H(By, Sx, Sz, (2-\beta)t) \diamondsuit H(Ax, Ty, Ty, \beta t) \end{array} \right\}$$

Then A, B, S and T have unique common fixed point in X.

Proof. Let x_0 be any arbitrary point in X. Then by condition (1) we can construct a sequence $\{y_n\}$ in X such that $y_{2n-1} = Tx_{2n-1} = Ax_{2n-2}$ and $y_{2n} = Sx_{2n} = Bx_{2n-1}$, $n = 1, 2, \ldots$. From the condition (4), we have

$$\begin{aligned} Q(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt) &= Q(Ax_{2n}, Bx_{2n+1}, Bx_{2n+1}, kt) \\ &\geq \left\{ \begin{array}{l} Q(Sx_{2n}, Tx_{2n+1}, Tx_{2n+1}, t) * Q(Ax_{2n}, Sx_{2n}, Sx_{2n+1}, t) * \\ Q(Bx_{2n+1}, Tx_{2n+1}, Tx_{2n+1}, t) * Q(Ax_{2n}, Tx_{2n+1}, Tx_{2n+1}, \beta t) * \\ Q(Bx_{2n+1}, Sx_{2n}, Sx_{2n+1}, (2-\beta)t) \end{array} \right\} \\ &= \left\{ \begin{array}{l} Q(y_{2n}, y_{2n+1}, y_{2n+1}, t) * Q(y_{2n+1}, y_{2n}, y_{2n+1}, t) * \\ Q(y_{2n+2}, y_{2n+1}, y_{2n+1}, t) * Q(y_{2n+1}, y_{2n+1}, y_{2n+1}, \beta t) * \\ Q(y_{2n+2}, y_{2n}, y_{2n+1}, (2-\beta)t) \end{array} \right\} \end{aligned}$$

$$\begin{aligned} Q(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt) &\geq \{Q(y_{2n}, y_{2n+1}, y_{2n+1}, t) * Q(y_{2n+2}, y_{2n+1}, y_{2n+1}, t) * Q(y_{2n+2}, y_{2n}, y_{2n+1}, (2-\beta)t\} \\ &= \left\{ \begin{array}{l} Q(y_{2n}, y_{2n+1}, y_{2n+1}, t) * Q(y_{2n+2}, y_{2n+1}, y_{2n+1}, t) \\ *Q(y_{2n+2}, y_{2n}, y_{2n+1}, (1+q)t) \end{array} \right\} \end{aligned}$$

$$\begin{aligned} H(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt) &= H(Ax_{2n}, Bx_{2n+1}, Bx_{2n+1}, kt) \\ &\leq \left\{ \begin{array}{l} H(Sx_{2n}, Tx_{2n+1}, Tx_{2n+1}, t) \diamondsuit H(Ax_{2n}, Sx_{2n}, Sx_{2n+1}, t) \diamondsuit \\ H(Bx_{2n+1}, Tx_{2n+1}, Tx_{2n+1}, t) \diamondsuit H(Ax_{2n}, Tx_{2n+1}, Tx_{2n+1}, \beta t) \diamondsuit \\ H(Bx_{2n+1}, Sx_{2n}, Sx_{2n+1}, (2-\beta)t) \end{array} \right\} \\ &= \left\{ \begin{array}{l} H(y_{2n}, y_{2n+1}, y_{2n+1}, t) \diamondsuit H(y_{2n+1}, y_{2n}, y_{2n+1}, t) \diamondsuit \\ H(y_{2n+2}, y_{2n+1}, y_{2n+1}, t) \diamondsuit H(y_{2n+1}, y_{2n+1}, y_{2n+1}, \beta t) \diamondsuit \\ H(y_{2n+2}, y_{2n}, y_{2n+1}, (2-\beta)t) \end{array} \right\} \end{aligned}$$

$$\begin{aligned} H(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt) &\leq \{H(y_{2n}, y_{2n+1}, y_{2n+1}, t) \diamondsuit H(y_{2n+2}, y_{2n+1}, y_{2n+1}, t) \diamondsuit H(y_{2n+2}, y_{2n}, y_{2n+1}, (2-\beta)t)\} \\ &= \left\{ \begin{array}{l} H(y_{2n}, y_{2n+1}, y_{2n+1}, t) \diamondsuit H(y_{2n+2}, y_{2n+1}, y_{2n+1}, t) \\ \diamondsuit H(y_{2n+2}, y_{2n}, y_{2n+1}, (1+q)t) \end{array} \right\} \end{aligned}$$

Here $\beta = 1 - q$, $q \in (0, 1)$

$$Q(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt) \geq \left\{ \begin{array}{l} Q(y_{2n}, y_{2n+1}, y_{2n+1}, t) * Q(y_{2n+2}, y_{2n+1}, y_{2n+1}, t) \\ *Q(y_{2n+2}, y_{2n+1}, y_{2n+1}, t) * Q(y_{2n+2}, y_{2n}, y_{2n+1}, qt) \end{array} \right\}$$

$$\geq \left\{ \begin{array}{l} Q(y_{2n}, y_{2n+1}, y_{2n+1}, t) * Q(y_{2n+2}, y_{2n+1}, y_{2n+1}, t) \\ *Q(y_{2n+1}, y_{2n}, y_{2n+1}, qt) \end{array} \right\}$$

$$H(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt) \leq \left\{ \begin{array}{l} H(y_{2n}, y_{2n+1}, y_{2n+1}, t) \diamondsuit H(y_{2n+2}, y_{2n+1}, y_{2n+1}, t) \\ H(y_{2n+2}, y_{2n+1}, y_{2n+1}, t) \diamondsuit H(y_{2n+2}, y_{2n}, y_{2n+1}, qt) \end{array} \right\}$$

$$\leq \left\{ \begin{array}{l} H(y_{2n}, y_{2n+1}, y_{2n+1}, t) \diamondsuit H(y_{2n+2}, y_{2n+1}, y_{2n+1}, t) \\ \diamondsuit H(y_{2n+1}, y_{2n}, y_{2n+1}, qt) \end{array} \right\}$$

Since the t-norm and t-conorm are continuous and $Q(x, y, \cdot)$ and $H(x, y, \cdot)$ are continuous. Letting $q \to 1$ we get

$$Q(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt) \geq \{Q(y_{2n}, y_{2n+1}, y_{2n+1}, t) * Q(y_{2n+2}, y_{2n+1}, y_{2n+1}, t) * Q(y_{2n+1}, y_{2n}, y_{2n+1}, t)\}$$

$$H(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt) \leq \{H(y_{2n}, y_{2n+1}, y_{2n+1}, t) \diamondsuit H(y_{2n+2}, y_{2n+1}, y_{2n+1}, t) \diamondsuit H(y_{2n+1}, y_{2n}, y_{2n+1}, t)\}$$

Since Q, H are symmetric and, $t * t \geq t$ and $(1-t) \diamondsuit (1-t) \leq 1-t$ we have

$$Q(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt) \geq \{Q(y_{2n}, y_{2n+1}, y_{2n+1}, t) * Q(y_{2n+1}, y_{2n+2}, y_{2n+2}, t)\}.$$

In general $Q(y_{n+1}, y_{n+2}, y_{n+2}, kt) \geq \{Q(y_n, y_{n+1}, y_{n+1}, t) * Q(y_{n+1}, y_{n+2}, y_{n+2}, t)\}$. Then

$$Q(y_{n+1}, y_{n+2}, y_{n+2}, t) \geq \left\{ Q\left(y_n, y_{n+1}, y_{n+1}, \frac{t}{k}\right) * Q\left(y_{n+1}, y_{n+2}, y_{n+2}, \frac{t}{k}\right) \right\}.$$

Thus $Q(y_{n+1}, y_{n+2}, y_{n+2}, kt) \geq \left\{Q\left(y_n, y_{n+1}, y_{n+1}, t\right) * Q\left(y_n, y_{n+1}, y_{n+1}, \frac{t}{k}\right) * Q\left(y_{n+1}, y_{n+2}, y_{n+2}, \frac{t}{k}\right)\right\}$. By repeatedly applying this,

$$Q(y_{n+1}, y_{n+2}, y_{n+2}, kt) \geq \left\{ \begin{array}{l} Q\left(y_n, y_{n+1}, y_{n+1}, t\right) * Q\left(y_n, y_{n+1}, y_{n+1}, \frac{t}{k}\right) \\ * \cdots * Q\left(y_n, y_{n+1}, y_{n+1}, \frac{t}{k^p}\right) * Q\left(y_{n+1}, y_{n+2}, y_{n+2}, \frac{t}{k^p}\right) \end{array} \right\}$$

and $H(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt) \leq \{H(y_{2n}, y_{2n+1}, y_{2n+1}, t) \diamondsuit\ H(y_{2n+1}, y_{2n+2}, y_{2n+2}, t)\}$. In general

$$H(y_{n+1}, y_{n+2}, y_{n+2}, kt) \leq \{H(y_n, y_{n+1}, y_{n+1}, t) \diamondsuit H(y_{n+1}, y_{n+2}, y_{n+2}, t)\}.$$

Then

$$H(y_{n+1}, y_{n+2}, y_{n+2}, t) \leq \left\{ H\left(y_n, y_{n+1}, y_{n+1}, \frac{t}{k}\right) \diamondsuit H\left(y_{n+1}, y_{n+2}, y_{n+2}, \frac{t}{k}\right) \right\}.$$

Thus $H(y_{n+1}, y_{n+2}, y_{n+2}, kt) \leq \left\{H\left(y_n, y_{n+1}, y_{n+1}, t\right) \diamondsuit H\left(y_n, y_{n+1}, y_{n+1}, \frac{t}{k}\right) \diamondsuit\ H\left(y_{n+1}, y_{n+2}, y_{n+2}, \frac{t}{k}\right)\right\}$. By repeatedly applying this,

$$H(y_{n+1}, y_{n+2}, y_{n+2}, kt) \leq \left\{ \begin{array}{l} H\left(y_n, y_{n+1}, y_{n+1}, t\right) \diamondsuit H\left(y_n, y_{n+1}, y_{n+1}, \frac{t}{k}\right) \\ \diamondsuit \cdots \diamondsuit H\left(y_n, y_{n+1}, y_{n+1}, \frac{t}{k^p}\right) \diamondsuit H\left(y_{n+1}, y_{n+2}, y_{n+2}, \frac{t}{k^p}\right) \end{array} \right\}$$

Since $Q(x, y, z, \cdot)$ is non decreasing and $H(x, y, z, \cdot)$ is non increasing, by lemma , we have

$$Q(y_{n+1}, y_{n+2}, y_{n+2}, kt) \geq \left\{ \begin{array}{l} Q\left(y_n, y_{n+1}, y_{n+1}, t\right) * Q\left(y_n, y_{n+1}, y_{n+1}, t\right) \\ * \cdots * Q\left(y_n, y_{n+1}, y_{n+1}, t\right) * Q\left(y_{n+1}, y_{n+2}, y_{n+2}, \frac{t}{k^p}\right) \end{array} \right\}$$

$$H(y_{n+1}, y_{n+2}, y_{n+2}, kt) \leq \left\{ \begin{array}{l} H\left(y_n, y_{n+1}, y_{n+1}, t\right) \diamondsuit H\left(y_n, y_{n+1}, y_{n+1}, t\right) \\ \diamondsuit \cdots \diamondsuit H\left(y_n, y_{n+1}, y_{n+1}, t\right) \diamondsuit H\left(y_{n+1}, y_{n+2}, y_{n+2}, \frac{t}{k^p}\right) \end{array} \right\}$$

Since t * t $\geq$ t, we have

$$Q(y_{n+1}, y_{n+2}, y_{n+2}, kt) \geq \left\{ Q\left(y_n, y_{n+1}, y_{n+1}, t\right) * Q\left(y_{n+1}, y_{n+2}, y_{n+2}, \frac{t}{k^p}\right) \right\}$$

$Q\left(y_{n+1}, y_{n+2}, y_{n+2}, \frac{t}{k^p}\right) \to 1$ as $p \to \infty$. Therefore, $Q(y_{n+1}, y_{n+2}, y_{n+2}, kt) \geq Q\left(y_n, y_{n+1}, y_{n+1}, t\right)$.

Since (1-t)$\diamondsuit$(1-t) $\leq$ (1- t), we have

$$H(y_{n+1}, y_{n+2}, y_{n+2}, kt) \leq \left\{ H\left(y_n, y_{n+1}, y_{n+1}, t\right) \diamondsuit H\left(y_{n+1}, y_{n+2}, y_{n+2}, \frac{t}{k^p}\right) \right\}$$

$H\left(y_{n+1}, y_{n+2}, y_{n+2}, \frac{t}{k^p}\right) \to 0$ as $p \to \infty$. Therefore, $H(y_{n+1}, y_{n+2}, y_{n+2}, kt) \leq H\left(y_n, y_{n+1}, y_{n+1}, t\right)$. By lemma $\{y_n\}$ is a Cauchy sequence in X.

Since X is complete $\{y_n\}$ converges to a point u $\in$ X. Since $\{Ax_{2n-2}\}$, $\{Tx_{2n-1}\}$, $\{Sx_{2n}\}$ and $\{Bx_{2n-1}\}$ are subsequence of $\{y_n\}$ we have,

$$Ax_{2n-2} \to u \text{ and } Tx_{2n-1} \to u$$
$$Sx_{2n} \to u \text{ and } Bx_{2n-1} \to u$$

and $S$and T are continuous, therefore we have,

$$SAx_{2n-2} \to Su \text{ and } TTx_{2n-1} \to Tu$$
$$SSx_{2n} \to Su \text{ and } TBx_{2n-1} \to Tu$$

Since the pair $\{A, S\}$ is compatible, $Q(ASx_{2n}, SAx_{2n}, SAx_{2n}, t) \to 1$ and $H(ASx_{2n}, SAx_{2n}, SAx_{2n}, t) \to 0$ as $n \to \infty$. Consider,

$$Q(ASx_{2n}, Su, Su, t) \geq \left\{ Q\left(ASx_{2n}, SAx_{2n}, SAx_{2n}, \frac{t}{2}\right) * Q\left(Su, Su, SAx_{2n}, \frac{t}{2}\right) \right\} \text{ and}$$
$$H(ASx_{2n}, Su, Su, t) \leq \left\{ H\left(ASx_{2n}, SAx_{2n}, SAx_{2n}, \frac{t}{2}\right) \diamondsuit H\left(Su, Su, , SAx_{2n}, \frac{t}{2}\right) \right\}$$

Taking limit $n \to \infty$ we get $ASx_{2n} \to Su$. $\{B, T\}$ is also compatible, then $Q(BTx_{2n-1}, TBx_{2n-1}, TBx_{2n-1}, t) \to 1$ and $H(BTx_{2n-1}, TBx_{2n-1}, TBx_{2n-1}, t) \to 0$ as $n \to \infty$. Then,

$$Q(BTx_{2n-1}, Tu, Tu, t) \geq Q\left(BTx_{2n-1}, TBx_{2n-1}, TBx_{2n-1}, \frac{t}{2}\right) * Q\left(TBx_{2n-1}, Tu, Tu, \frac{t}{2}\right) \text{ and}$$
$$H(BTx_{2n-1}, Tu, Tu, t) \leq H\left(BTx_{2n-1}, TBx_{2n-1}, TBx_{2n-1}, \frac{t}{2}\right) \diamondsuit H\left(TBx_{2n-1}, Tu, Tu, \frac{t}{2}\right).$$

This implies $BTx_{2n-1} \to Tu$ as $n \to \infty$. Substituting $x = Sx_{2n-1}$ and $y = x_{2n+2} = z$ with $\beta = 1$ in condition (4), we obtain

$$Q(ASx_{2n+1}, Bx_{2n+1}, Bx_{2n+2}, kt) \geq \left\{ \begin{array}{l} Q(SSx_{2n+1}, Tx_{2n+2} + Tx_{2n+2}, t) * Q(ASx_{2n+1}, SSx_{2n+1}, Sx_{2n+2}, t) \\ Q(Bx_{2n+2}, Tx_{2n+2}, Tx_{2n+2}, t) * Q(Bx_{2n+2}, SSx_{2n+2}, Sx_{2n+2}, t) * \\ Q(ASx_{2n+1}, Tx_{2n+2}, Tx_{2n+2}, t) \end{array} \right\}$$

as $n \to \infty$, we get

$$Q(Su,u,u,kt) \geq \{Q(Su,u,u,t) * Q(Su,Su,u,t) * Q(u,u,u,t) * Q(u,Su,u,t) * Q(Su,u,u,t)\}$$

$$H(ASx_{2n+1},Bx_{2n+1},Bx_{2n+2},kt) \leq \left\{ \begin{array}{l} H(SSx_{2n+1},Tx_{2n+2},Tx_{2n+2},t)\diamondsuit H(ASx_{2n+1},SSx_{2n+1},Sx_{2n+2},t)\diamondsuit \\ H(Bx_{2n+2},Tx_{2n+2},Tx_{2n+2},t)\diamondsuit H(Bx_{2n+2},SSx_{2n+2},Sx_{2n+2},t)\diamondsuit \\ H(ASx_{2n+1},Tx_{2n+2},Tx_{2n+2},t) \end{array} \right\}$$

as $n \to \infty$, we get

$$H(Su,u,u,kt) \leq \{H(Su,u,u,t)\diamondsuit H(Su,Su,u,t)\diamondsuit\ H(u,u,u,t)\diamondsuit H(u,Su,u,t)\diamondsuit\ H(Su,u,u,t)\}$$

Thus, $Q(Su,u,u,kt) \geq Q(Su,u,u,t)$ and $H(Su,u,u,kt) \leq H(Su,u,u,t)$since Q, H are symmetric. Hence by lemma, we obtain $Su = u$. Put $x = u$ and $y = x_{2n+1} = z$ with $\beta = 1$ in condition (4), we have

$$Q(Au,Bx_{2n+1},Bx_{2n+1},kt) \geq \left\{ \begin{array}{l} Q(Su,Tx_{2n+1},Tx_{2n+1},t) * Q(Au,Sx_{2n+1},Sx_{2n+1},t)* \\ Q(Bx_{2n+1},Tx_{2n+1},Tx_{2n+1},t) * Q(Bx_{2n+1},Su,Sx_{2n+1},t)* \\ Q(Au,Tx_{2n+1},Tx_{2n+1},t) \end{array} \right\}$$

Taking limit $n \to \infty$, we obtain

$$Q(Au,u,u,kt) \geq \{Q(Su,u,u,t) * Q(Au,Su,u,t) * Q(u,u,u,t) * Q(u,Su,u,t) * Q(Au,u,u,t)\} \quad \text{and}$$

$$H(Au,Bx_{2n+1},Bx_{2n+1},kt) \leq \left\{ \begin{array}{l} H(Su,Tx_{2n+1},Tx_{2n+1},t)\diamondsuit H(Au,Sx_{2n+1},Sx_{2n+1},t)\diamondsuit \\ H(Bx_{2n+1},Tx_{2n+1},Tx_{2n+1},t)\diamondsuit H(Bx_{2n+1},Su,Sx_{2n+1},t)\diamondsuit \\ H(Au,Tx_{2n+1},Tx_{2n+1},t) \end{array} \right\}$$

Taking limit $n \to \infty$, we obtain

$$H(Au,u,u,kt) \leq \{H(Su,u,u,t)\diamondsuit H(Au,Su,u,t)\diamondsuit\ H(u,u,u,t)\diamondsuit H(u,Su,u,t)\diamondsuit\ H(Au,u,u,t)\}$$

Thus $Q(Au,u,u,kt) \geq Q(Au,u,u,t)$ and $H(Au,u,u,kt) \leq H(Au,u,u,t)$. Therefore by lemma, we obtain $Au = u$. Now setting, $x = x_{2n+1}$, $y = Tx_{2n+2} = z$ and $\beta = 1$

$$Q(Ax_{2n+1},BTx_{2n+2},BTx_{2n+2},kt) \geq \left\{ \begin{array}{l} Q(Sx_{2n+1},TTx_{2n+2},TTx_{2n+2},t) * Q(Ax_{2n+1},Sx_{2n+1},STx_{2n+2},t)* \\ Q(BTx_{2n+2},TTx_{2n+2},TTx_{2n+2},t) * Q(BTx_{2n+2},Sx_{2n+2},STx_{2n+2},t)* \\ Q(Ax_{2n+1},TTx_{2n+2},TTx_{2n+2},t) \end{array} \right\}$$

Taking limit $n \to \infty$ we get,

$$Q(u,Tu,Tu,kt) \geq \{Q(u,Tu,Tu,t) * Q(u,u,Su,t) * Q(Tu,Tu,Tu,t) * Q(Tu,u,Su,t) * Q(u,Tu,Tu,t)\}$$

and

$$H(Ax_{2n+1},BTx_{2n+2},BTx_{2n+2},kt) \leq \left\{ \begin{array}{l} H(Sx_{2n+1},TTx_{2n+2},TTx_{2n+2},t)\diamondsuit H(Ax_{2n+1},Sx_{2n+1},STx_{2n+2},t)\diamondsuit \\ H(BTx_{2n+2},TTx_{2n+2},TTx_{2n+2},t)\diamondsuit H(BTx_{2n+2},Sx_{2n+2},STx_{2n+2},t)\diamondsuit \\ H(Ax_{2n+1},TTx_{2n+2},TTx_{2n+2},t) \end{array} \right\}$$

Taking limit $n \to \infty$ we get,

$$H(u, Tu, Tu, kt) \leq \{H(u, Tu, Tu, t) \diamondsuit H(u, u, Su, t) \diamondsuit\ H(Tu, Tu, Tu, t) \diamondsuit H(Tu, u, Su, t) \diamondsuit\ H(u, Tu, Tu, t)\}$$

Thus $Q(u, Tu, Tu, kt) \geq Q(u, Tu, Tu, t)$ and $H(u, Tu, Tu, kt) \leq H(u, Tu, Tu, t)$. Since Q, H are symmetric. Therefore, $Tu = u$. Similarly on substituting $x = x_{2n+1}$, $y = u = z$ and $\beta = 1$ in contractive condition

$$Q(Ax_{2n+1}, Bu, Bu, t) \geq \left\{ \begin{array}{l} Q(Sx_{2n+1}, Tu, Tu, t) * Q(Ax_{2n+1}, Sx_{2n+1}, Su, t)* \\ Q(Bu, Tu, Tu, t) * Q(Bu, Sx_{2n+1}, Su, t) * Q(Ax_{2n+1}, Tu, Tu, t) \end{array} \right\}$$

Taking limit $n \to \infty$, we get

$$Q(u, Bu, Bu, kt) \geq \{Q(u, u, u, t) * Q(u, u, u, t)*\ Q(Bu, u, u, t) * Q(Bu, u, u, t) *\ Q(u, u, u, t)\}$$

And

$$H(Ax_{2n+1}, Bu, Bu, t) \leq \left\{ \begin{array}{l} H(Sx_{2n+1}, Tu, Tu, t) \diamondsuit H(Ax_{2n+1}, Sx_{2n+1}, Su, t) \diamondsuit \\ H(Bu, Tu, Tu, t) \diamondsuit H(Bu, Sx_{2n+1}, Su, t) \diamondsuit H(Ax_{2n+1}, Tu, Tu, t) \end{array} \right\}$$

Taking limit $n \to \infty$, we get

$$H(u, Bu, Bu, kt) \leq \{H(u, u, u, t) \diamondsuit H(u, u, u, t) \diamondsuit\ H(Bu, u, u, t) \diamondsuit H(Bu, u, u, t) \diamondsuit\ H(u, u, u, t)\}$$

Thus $Bu = u$. Hence $Au = Bu = Su = Tu = u$. Thus u is a common fixed point of A, B, S and T.

Uniqueness: Let us assume that w be another common fixed point of A, B, S and T. Put $x = u$ and $y = z = w$ in condition (4),

$$Q(Au, Bw, Bw, t) \geq \left\{ \begin{array}{l} Q(Su, Tw, Tw, t) * Q(Au, Su, Sw, t)* \\ Q(Bw, Tw, Tw, t) * Q(Bw, Su, Sw, t) * Q(Au, Tw, Tw, t) \end{array} \right\}$$

$$H(Au, Bw, Bw, t) \leq \left\{ \begin{array}{l} H(Su, Tw, Tw, t) \diamondsuit H(Au, Su, Sw, t) \diamondsuit \\ H(Bw, Tw, Tw, t) \diamondsuit H(Bw, Su, Sw, t) \diamondsuit H(Au, Tw, Tw, t) \end{array} \right\}$$

Again by lemma, we obtain $z = w$. ☐

Corollary 3.2. *Let A, S and T be self maps on a complete generalized intuitionistic fuzzy metric space $(X, Q, H, *, \diamondsuit)$ such that Q is symmetric with continuous t-norm, $t * t \geq t$ and H is symmetric with continuous t-conorm $(1-t) \diamondsuit (1-t) \leq (1-t)$, satisfying the following conditions:*

(1). $AX \subseteq TX$, $AX \subseteq SX$

(2). S and T are continuous

(3). $\{A, S\}$ and $\{A, T\}$ are compatible pairs of maps

(4). for all $x, y, z \in X$, $k \in (0, 1)$, $t > 0$ and $\beta \in (0, 2)$ with

$$Q(Ax, Ay, Az, kt) \geq \left\{ \begin{array}{l} Q(Sx, Ty, Tz, t) * Q(Ax, Sx, Sz, t) * Q(Ay, Ty, Tz, t)* \\ Q(Ay, Sx, Sz, (2-\beta)t) * Q(Ax, Ty, Ty, \beta t) \end{array} \right\} \text{ and}$$

$$H(Ax, Ay, Az, kt) \leq \left\{ \begin{array}{l} H(Sx, Ty, Tz, t) \diamondsuit H(Ax, Sx, Sz, t) \diamondsuit H(Ay, Ty, Tz, t) \diamondsuit \\ H(Ay, Sx, Sz, (2-\beta)t) \diamondsuit H(Ax, Ty, Ty, \beta t) \end{array} \right\}$$

Then A, S and T have unique common fixed point in X.

Proof. Proof is obtained by substituting $A = B$ in Theorem 3.1. $\square$

Corollary 3.3. *Let A, B and T be self maps on a complete generalized intuitionistic fuzzy metric space $(X, Q, H, *, \diamondsuit)$ such that Q is symmetric with continuous t-norm, $t * t \geq t$ and H is symmetric with continuous t-conorm $(1-t)\diamondsuit(1-t) \leq (1-t)$, satisfying the following conditions:*

(1). $AX \subseteq TX$, $BX \subseteq TX$

(2). T is continuous

(3). $\{A, T\}$ and $\{B, T\}$ are compatible pairs of maps

(4). for all $x, y, z \in X$, $k \in (0, 1)$, $t > 0$ and $\beta \in (0, 2)$ with

$$Q(Ax, By, Bz, kt) \geq \left\{ \begin{array}{l} Q(Tx, Ty, Tz, t) * Q(Ax, Tx, Tz, t) * Q(By, Ty, Tz, t)* \\ Q(By, Ty, Tz, (2-\beta)t) * Q(Ax, Ty, Ty, \beta t) \end{array} \right\} \textit{ and}$$

$$H(Ax, By, Bz, kt) \leq \left\{ \begin{array}{l} H(Tx, Ty, Tz, t)\diamondsuit H(Ax, Tx, Tz, t)\diamondsuit H(By, Ty, Tz, t)\diamondsuit \\ H(By, Ty, Tz, (2-\beta)t)\diamondsuit H(Ax, Ty, Ty, \beta t) \end{array} \right\}$$

Then A, B and T have unique common fixed point in X.

Proof. Proof is obtained by substituting $S = T$ in Theorem 3.1. $\square$

Corollary 3.4. *Let A and T be self maps on a complete generalized intuitionistic fuzzy metric space $(X, Q, H, *, \diamondsuit)$ with continuous t-norm, and continuous t-conorm satisfying the following conditions:*

(1). $AX \subseteq TX$

(2). T is continuous

(3). the pair $\{A, T\}$ is compatible

(4). for all $x, y, z \in X$, $k \in (0, 1)$, $t > 0$ and $\beta \in (0, 2)$ with

$$M(Ax, Ay, Az, kt) \geq \left\{ \begin{array}{l} M(Tx, Ty, Tz, t) * M(Ax, Tx, Tz, t) * M(Ay, Ty, Tz, t)* \\ M(Ay, Tx, Tz, (2-\beta)t) * M(Ax, Ty, Ty, \beta t) \end{array} \right\} \textit{ and}$$

$$N(Ax, Ay, Az, kt) \leq \left\{ \begin{array}{l} N(Tx, Ty, Tz, t)\diamondsuit N(Ax, Tx, Tz, t)\diamondsuit N(Ay, Ty, Tz, t)\diamondsuit \\ N(Ay, Tx, Tz, (2-\beta)t)\diamondsuit N(Ax, Ty, Ty, \beta t) \end{array} \right\}$$

Then A and T have unique common fixed point in X.

Corollary 3.5. *Let A and B be self maps on a complete generalized intuitionistic fuzzy metric space $(X, Q, H, *, \diamondsuit)$ with continuous t-norm and continuous t-conorm if there exist a constant $k \in (0, 1)$, $t > 0$ and $\beta \in (0, 2)$ with*

$$M(Ax, By, Bz, kt) \geq \left\{ \begin{array}{l} M(x, y, z, t) * M(Ax, x, z, t) * M(By, y, z, t)* \\ M(By, x, z, (2-\beta)t) * M(Ax, y, y, \beta t) \end{array} \right\} \textit{ and}$$

$$N(Ax, By, Bz, kt) \leq \left\{ \begin{array}{l} N(x, y, z, t)\diamondsuit N(Ax, x, z, t)\diamondsuit N(By, y, z, t)\diamondsuit \\ N(By, x, z, (2-\beta)t)\diamondsuit N(Ax, y, y, \beta t) \end{array} \right\}$$

Then A and B have unique common fixed point in X.

Proof. Proof is obtained by substituting $SX = TX = IX$ in theorem 3.1. □

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