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Weakly (1,2)*-fg-Closed Sets in Fuzzy Bitopological Spaces.

Research Article

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Abstract: In this paper, a new class of closed set called weakly (1,2)*-fg-closed set and new class of maps namely Weakly (1,2)*-fg continuous map, Weakly (1,2)*-fg open and Weakly (1,2)*-fg closed maps are introduced and their properties are studied.

Keywords: (1,2)*-fwg closed , (1,2)*-fwg continuous, (1,2)*-fwg open maps,(1,2)*-fwg closed maps.

1. Introduction

Levine [2] introduced the concept of Generalized closed sets in topological spaces. Malghan [3] introduced the concept of generalized closed maps in topological spaces. Devi [1] introduced and studied sg-closed maps and gs-closed maps. Recently, Sheik John [6] defined ω-closed maps and studied some of their properties. Ravi and Ganesan [4] have introduced the concept of $\ddot{g}$-closed sets and studied their most fundamental properties in topological spaces. In this paper, we introduce a new class of generalized closed sets called weakly (1,2)*-fg-closed sets which contains the above mentioned class. Also, we investigate the relationships among the related generalized closed sets. In the last section, we introduce Weakly (1,2)*-fg continuous map, Weakly (1,2)*-fg open and Weakly (1,2)*-fg closed maps are introduced and their properties are studied.

2. Preliminaries

Definition 2.1 ([5]). *A subset A of a bitopological space X is called*

(1). (1,2)-π-open set if A is the finite union of regular (1,2)*-open sets. The complement of (1,2)*-π-open sets are called (1,2)*-π-closed set.*

(2). (1,2)-πg-closed set if $\tau_{1,2}$-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is (1,2)*-π-open in X.*

The complement of (1,2)-πg-closed set is called (1,2)*-πg-open set.*

Definition 2.2. *Let (X, τ_1, τ_2) and (Y, σ_1,σ_2) be two bitopological spaces. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1,\sigma_2)$ is called*

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(1). completely (1,2)-continuous [5] (resp. (1,2)*-R-map [2]) if $f^{-1}(V)$ is regular (1,2)*-open in X for each $\sigma_{1,2}$-open (resp. regular (1,2)*-open) set V of Y.*

(2). perfectly (1,2)-continuous [5] if $f^{-1}(V)$ is both $\tau_{1,2}$-open and $\tau_{1,2}$-closed in X for each $\sigma_{1,2}$-open set V of Y.*

We introduce the following definitions,

Definition 2.3. *A subset A of a fuzzy bitopological space X is called*

(1). (1,2)-fπ-open set if A is the finite union of regular (1,2)*-fuzzy open sets. The complement of (1,2)*-fπ-open sets are called (1,2)*-fπ-closed set.*

(2). (1,2)-fπg-closed set if $\tau_{1,2}$-cl(A) $\leq$ U whenever A $\leq$ U and U is (1,2)*-fπ-open in X. The complement of (1,2)*-fπg-closed set is called (1,2)*-fπg-open set.*

Definition 2.4. *Let (X, τ_1, τ_2) and (Y, σ_1,σ_2) be two fuzzy bitopological spaces. A function f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1,σ_2) is called*

(1). completely (1,2)-fuzzy continuous (resp. (1,2)*-fR-map) if $f^{-1}(V)$ is regular (1,2)*-fuzzy open in X for each $\sigma_{1,2}$-open (resp. regular (1,2)*-fuzzy open) set V of Y.*

(2). perfectly (1,2)-fuzzy continuous if $f^{-1}(V)$ is both $\tau_{1,2}$-open and $\tau_{1,2}$-closed in X for each $\sigma_{1,2}$-open set V of Y.*

Definition 2.5. *A subset A of a fuzzy bitopological space X is called*

(1). weakly (1,2)-fuzzy generalized closed (briefly, (1,2)*-fwg-closed) set if $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\leq$ U whenever A $\leq$ U and U is $\tau_{1,2}$-open in X.*

(2). weakly (1,2)-fuzzy πg-closed (briefly, (1,2)*-fwπg-closed) set if $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\leq$ U whenever A $\leq$ U and U is (1,2)*-fπ-open in X.*

(3). regular weakly (1,2)-fuzzy generalized closed (briefly, (1,2)*-frwg-closed) set if $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\leq$ U whenever A $\leq$ U and U is regular (1,2)*-fuzzy open in X.*

Remark 2.6. *Every $\tau_{1,2}$-open set is (1,2)*-fsg-open but not conversely.*

Remark 2.7. *For a subset of a fuzzy bitopological space, we have following implications: regular (1,2)*-fuzzy open $\rightarrow$ (1,2)*-fuzzy π-open $\rightarrow$ $\tau_{1,2}$-open.*

Definition 2.8. *A subset A of a fuzzy bitopological space X is said to be nowhere dense if $\tau_{1,2}$-int($\tau_{1,2}$-cl(A)) = ϕ.*

Definition 2.9. *Let f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1,σ_2) be a function. Then f is said to be contra- (1,2)*-fg-continuous if $f^{-1}(V)$ is (1,2)*-fg-closed in X for every $\sigma_{1,2}$-open set of Y.*

3. Weakly (1,2)*-fg-Closed Sets

We introduce the definition of weakly (1,2)*-fg-closed sets in fuzzy bitopological spaces and study the relationships of such sets.

Definition 3.1. *A subset A of a fuzzy bitopological space X is called a weakly (1,2)*-fg-closed (briefly, (1,2)*-fwg-closed) set if $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\leq$ U whenever A $\leq$ U and U is (1,2)*-τ_{12}-open in X.*

Theorem 3.2. *Every (1,2)*-fg-closed set is (1,2)*-fwg-closed but not conversely.*

Example 3.3. *Consider the fbts* (X,τ_1,τ_2) *where* $X = \{a,b\}$, $\tau_1 = \{0,1,\mu = \frac{0.4}{a} + \frac{0.6}{b}\}$; $\tau_2\{0,1\}$, $\tau_{1,2}$*-open set*=$\{0,1,\mu = \frac{0.4}{a} + \frac{0.6}{b}\}$ *and* $\tau_{1,2}$ *closed set*=$\{0,1,\mu' = \frac{0.6}{a} + \frac{0.4}{b}\}$, $\tau_{1,2}$*-closed set containing* $\lambda = \frac{0.4}{a} + \frac{0.5}{b}$ *is 1. Therefore* $\tau_{1,2} - cl(\lambda) = 1$. $\tau_{1,2}$*-open set containing* λ *are 1 and* μ. *Therefore* $\tau_{1,2} - cl(\lambda) = \chi \nleq \mu$. *Therefore* λ *is not (1,2)*-fg-closed. Also* $\tau_{1,2} - cl(\tau_{1,2} - int(\lambda)) = 0$. *Therefore* λ *is (1,2)*- fwg-closed.*

Theorem 3.4. *Every (1,2)*-fwg-closed set is (1,2)*-fwπg-closed but not conversely.*

Proof. Let A be any (1,2)*-fwg-closed set and U be any (1,2)*-fπ-open set containing A. Then U is a $\tau_{1,2}$-open set containing A. We have $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\leq$ U. Thus, A is (1,2)*-fwπg-closed □

Example 3.5. *Consider the fbts* (X,τ_1,τ_2) *where* $X = \{a,b,c\}$, $\tau_1 = \{0,1,\mu = \frac{1}{a} + \frac{0}{b} + \frac{0}{c}, \lambda = \frac{0}{a} + \frac{0}{b} + \frac{1}{c}, \eta = \frac{1}{a} + \frac{0}{b} + \frac{1}{c}\}$, $\tau_2 = \{0,1\}$, $\tau_{1,2}$*-open sets are* $0, 1$, $\mu = \frac{1}{a} + \frac{0}{b} + \frac{0}{c}$, $\lambda = \frac{0}{a} + \frac{0}{b} + \frac{1}{c}$, $\eta = \frac{1}{a} + \frac{0}{b} + \frac{1}{c}$ *and* $\tau_{1,2}$ *closed sets are* $0, 1$, $\mu' = \frac{0}{a} + \frac{1}{b} + \frac{1}{c}$, $\lambda' = \frac{1}{a} + \frac{1}{b} + \frac{0}{c}$, $\eta' = \frac{0}{a} + \frac{1}{b} + \frac{0}{c}$. $\tau_{1,2}$*-open set containing* $\upsilon = \frac{0.5}{a} + \frac{0}{b} + \frac{1}{c}$ *are* η *and 1. Therefore* $\tau_{12} - cl\left(\tau_{12} - int\left(\upsilon\right)\right) = \mu' \nleq \eta$ *and hence* υ *is not (1,2)*-fwg-closed but it is (1,2)*-fwπg-closed.*

Theorem 3.6. *Every (1,2)*-fwg-closed set is (1,2)*-frwg-closed but not conversely.*

Proof. Let A be any (1,2)*-fwg-closed set and U be any regular (1,2)*-fuzzy open set containing A. Then U is a $\tau_{1,2}$ -open set containing A. We have $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\leq$ U. Thus, A is (1,2)*-frwg-closed. □

Example 3.7. *Consider the fbts* (X,τ_1,τ_2) *where X=* $\{a,b,c\}$, $\tau_1 = \{0,1,\mu = \frac{1}{a} + \frac{0}{b} + \frac{0}{c}, \lambda = \frac{0}{a} + \frac{0}{b} + \frac{1}{c}, \eta = \frac{1}{a} + \frac{0}{b} + \frac{1}{c}\}$, $\tau_2 = \{0,1\}$, $\tau_{1,2}$*-open sets are* $0, 1$, $\mu = \frac{1}{a} + \frac{0}{b} + \frac{0}{c}$, $\lambda = \frac{0}{a} + \frac{0}{b} + \frac{1}{c}$, $\eta = \frac{1}{a} + \frac{0}{b} + \frac{1}{c}$ *and closed sets are* $0, 1$, $\mu' = \frac{0}{a} + \frac{1}{b} + \frac{1}{c}$, $\lambda' = \frac{1}{a} + \frac{1}{b} + \frac{0}{c}$, $\eta' = \frac{0}{a} + \frac{1}{b} + \frac{0}{c}$. $\tau_{1,2}$*-open set containing* $\upsilon = \frac{0.5}{a} + \frac{0}{b} + \frac{1}{c}$ *are* η *and 1. Therefore* $\tau_{12} - cl\left(\tau_{12} - int\left(\upsilon\right)\right) = \mu' \nleq \eta$ *and hence* υ *is not (1,2)*-fwg-closed but it is (1,2)*-frwg-closed.*

Theorem 3.8. *If a subset A of a fuzzy bitopological space X is both* $\tau_{1,2}$*-closed and (1,2)*-fαg-closed, then it is (1,2)*-fwg-closed in X.*

Proof. Let A be an (1,2)*-fαg-closed set in X and U be any $\tau_{1,2}$-open set containing A. Then U $\geq$ (1,2)*-αcl(A) = A $\cup$ $\tau_{1,2}$-cl($\tau_{1,2}$-int($\tau_{1,2}$-cl(A))). Since A is $\tau_{1,2}$-closed, U $\geq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) and hence A is (1,2)*-fwg-closed in X. □

Theorem 3.9. *If a subset A of a fuzzy bitopological space X is both* $\tau_{1,2}$*-open and (1,2)*-fwg-closed, then it is* $\tau_{1,2}$*-closed.*

Proof. Since A is both $\tau_{1,2}$-open and (1,2)*-fwg-closed, A $\geq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) = $\tau_{1,2}$-cl(A) and hence A is $\tau_{1,2}$-closed in X. □

Corollary 3.10. *If a subset A of a fuzzy bitopological space X is both* $\tau_{1,2}$*-open and (1,2)*-fwg-closed, then it is both regular (1,2)*-fuzzy open and regular (1,2)*-fuzzy closed in X.*

Theorem 3.11. *Let X be a fuzzy bitopological space and* $A \leq X$ *be* $\tau_{1,2}$*-open. Then, A is (1,2)*-fwg-closed if and only if A is (1,2)*-fg-closed.*

Proof. Let A be (1,2)*-fg-closed. By Proposition 3.2, it is (1,2)*-fwg-closed.

Conversely, let A be (1,2)*-fwg-closed. Since A is $\tau_{1,2}$-open, by Theorem 3.9, A is $\tau_{1,2}$-closed. Hence A is (1,2)*-fg-closed. □

Theorem 3.12. *If a set A is (1,2)*-fwg-closed then* $\tau_{1,2}$*-cl(*$\tau_{1,2}$*-int(A))* $-$ *A contains no non-empty (1,2)*-fsg-closed set.*

Proof. Let F be a (1,2)*-fsg-closed set such that F $\leq \tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $-$ A. Since F^{c}is (1,2)*-fsg-open and A $\leq$ F^c, from the definition of (1,2)*-fwg-closedness, it follows that $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\leq$ F^c. That is F $\leq$ ($\tau_{1,2}$-cl($\tau_{1,2}$-int(A)))^c. This implies that F $\leq$ ($\tau_{1,2}$-cl($\tau_{1,2}$-int(A))) $\cap$ ($\tau_{1,2}$-cl($\tau_{1,2}$-int(A)))^c $= \phi$. □

Theorem 3.13. *If a subset A of a fuzzy bitopological space X is nowhere dense, then it is (1,2)*-fwg-closed.*

Proof. Since $\tau_{1,2}$-int(A) $\leq \tau_{1,2}$-int($\tau_{1,2}$-cl(A)) and A is nowhere dense, $\tau_{1,2}$-int(A) $= \phi$. Therefore $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $=$ ϕ and hence A is (1,2)*-fwg-closed in X. □

The converse of Theorem 3.13 need not be true as seen in the following example.

Example 3.14. *Consider the fbts* (X, τ_1, τ_2) *where* $X = \{a, b\}$, $\tau_1 = \{0, 1, \mu = \frac{0.4}{a} + \frac{0.6}{b}\}$; $\tau_2\,\{0, 1\}$, $\tau_{1,2}$*-open set*$=\{0, 1, \mu = \frac{0.4}{a} + \frac{0.6}{b}\}$ *and* $\tau_{1,2}$ *closed set* $=\{0, 1, \mu' = \frac{0.6}{a} + \frac{0.4}{b}\}$. *Let* $\lambda = \frac{0.4}{a} + \frac{0.6}{b}$ *be any fuzzy subset of X.* $\tau_{1,2} - cl(\tau_{1,2} - int(\lambda)) = 0$, *hence* λ *is (1,2)*-fwg-closed but it is not nowhere dense.*

Remark 3.15. *From the above discussions and known results . We obtain the following diagram, where A → B represents A implies B but not conversely.*

Diagram

$\tau_{1,2}$*-closed → (1,2)*-fwg-closed →(1,2)*-fw*π*g-closed → (1,2)*-frwg-closed.*

None of the above implications is reversible as shown in the above examples.

Definition 3.16. *A subset A of a fuzzy bitopological space X is called (1,2)*-fwg-open set if* A^c *is (1,2)*-fwg-closed in X.*

Proposition 3.17. *Every (1,2)*-fg-open set is (1,2)*-fwg-open but not conversely.*

Example 3.18. *Consider the fbts* (X, τ_1, τ_2) *where* $X = \{a, b\}$, $\tau_1 = \{0, 1, \mu = \frac{0.4}{a} + \frac{0.6}{b}\}$; $\tau_2\,\{0, 1\}$, $\tau_{1,2}$*-open set*$=\{0, 1, \mu = \frac{0.4}{a} + \frac{0.6}{b}\}$ *and* $\tau_{1,2}$ *closed set*$=\{0, 1, \mu' = \frac{0.6}{a} + \frac{0.4}{b}\}$*-closed set containing* $\lambda = \frac{0.4}{a} + \frac{0.5}{b}$ *is 1. Therefore* $\tau_{1,2} - cl(\lambda) = 1$. $\tau_{1,2}$*-open set containing* λ *are 1 and* μ. *Therefore* $\tau_{1,2} - cl(\lambda) = \chi \nleq \mu$. *Therefore* λ *is not (1,2)*-fg-closed and* λ^c *is not (1,2)*-fg-open but it is (1,2)*-fwg open.*

Theorem 3.19. *A subset A of a fuzzy bitopological space X is (1,2)*-fwg-open if G* $\leq \tau_{1,2}$*-int(*$\tau_{1,2}$*-cl(A)) whenever G* $\leq$ *A and G is (1,2)*-fsg-closed.*

Proof. Let A be any (1,2)*-fwg-open. Then A^c is (1,2)*-fwg-closed. Let G be a (1,2)*-fsg-closed set contained in A. Then G^c is a (1,2)*-fsg-open set containing A^c. Since A^c is (1,2)*-fwg-closed, we have $\tau_{1,2}$-cl($\tau_{1,2}$-int(A^c)) $\leq$ G^c. Therefore G $\leq$ $\tau_{1,2}$-int($\tau_{1,2}$-cl(A)).

Conversely, we suppose that G $\leq \tau_{1,2}$-int($\tau_{1,2}$-cl(A)) whenever G $\leq$ A and G is (1,2)*-fsg-closed. Then G^c is a (1,2)*-fsg-open set containing A^c and G$^c \geq$ ($\tau_{1,2}$-int($\tau_{1,2}$-cl(A)))^c. It follows that G^c $\geq \tau_{1,2}$-cl($\tau_{1,2}$-int(A^c)). Hence A^c is (1,2)*-fwg-closed and so A is (1,2)*-fwg-open. □

Definition 3.20. *Let A* $\leq$ *X. The (1,2)*-kernel of A is defined as the intersection of all* $\tau_{1,2}$*-open supersets of the set A and is denoted by (1,2)*-ker(A).*

Lemma 3.21. *The following properties hold for subsets U, V of a space X:*

(1). $x \in$ *(1,2)*-ker(U) if and only if* $U \cap F \neq \phi$ *for any* $\tau_{1,2}$*-closed set F containing x,*

(2). $U \leq$ *(1,2)*-ker(U) and U = (1,2)*-ker(U) if U is* $\tau_{1,2}$*-open in X,*

(3). if $U \leq V$, then (1,2)-ker(U) $\leq$ (1,2)*-ker(V).*

Theorem 3.22. *The following are equivalent for a function f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1,\sigma_2)$.*

(1). f is contra (1,2)-fg-continuous,*

(2). the inverse image of every $\sigma_{1,2}$-closed set of Y is (1,2)-fg-open.*

Proof. Let U be any $\sigma_{1,2}$-closed set of Y. Since $Y \setminus U$ is $\sigma_{1,2}$-open, then by (i), it follows that $f^{-1}(Y \setminus U) = X \setminus f^{-1}(U)$is (1,2)*-fg-closed. This shows that $f^{-1}(U)$ is (1,2)*-fg-open in X. Converse is similar. □

Theorem 3.23. *Suppose that (1,2)*-FGC(X) is closed under arbitrary intersections. Then the following are equivalent for a function f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1,\sigma_2)$.*

(1). f is contra (1,2)-fg-continuous,*

(2). the inverse image of every $\sigma_{1,2}$-closed set of Y is (1,2)-fg-open in X,*

(3). for each $x \in X$ and each $\sigma_{1,2}$-closed set B in Y with $f(x) \in B$, there exists a (1,2)-fg-open set A in X such that $x \in A$ and $f(A) \leq B$,*

(4). f((1,2)-g-cl(A)) $\leq$ (1,2)*-ker(f(A)) for every subset A of X,*

(5). (1,2)-g-cl(f^{-1}(B)) $\leq$ f^{-1}((1,2)*-ker(B)) for every subset B of Y.*

Proof.
(1) $\Rightarrow$ (3). Let $x \in X$ and B be a $\sigma_{1,2}$-closed set in Y with $f(x) \in B$. By (i), it follows that $f^{-1}(Y \setminus B) = X \setminus f^{-1}(B)$ is (1,2)*-fg-closed and so $f^{-1}(B)$ is (1,2)*-g-open. Take $A = f^{-1}(B)$. We obtain that $x \in A$ and $f(A) \leq B$.
(3) $\Rightarrow$ (2). Let B be $\sigma_{1,2}$-closed set in Y with $x \in f^{-1}(B)$. Since $f(x) \in B$, by (iii) there exists a (1,2)*-fg-open set A in X containing x such that $f(A) \leq B$. It follows that $x \in A \leq f^{-1}(B)$. Hence $f^{-1}(B)$ is (1,2)*-fg-open.
(2) $\Rightarrow$ (1). Follows from the previous Theorem.
(2) $\Rightarrow$ (4). Let A be any subset of X. Let $y \notin$ (1,2)*-ker(f(A)). Then there exists a $\sigma_{1,2}$-closed set F containing y such that $f(A) \cap F = \phi$. Hence, we have $A \cap f^{-1}(F) = \phi$ and (1,2)*-g-cl(A) $\cap\, f^{-1}(F) = \phi$. Hence, we obtain f((1,2)*-g-cl(A)) $\cap\, F = \phi$ and $y \notin$ f((1,2)*-g-cl(A)). Thus, f((1,2)*-g-cl(A)) $\leq$ (1,2)*-ker(f(A)).
(4) $\Rightarrow$ (5). Let B be any subset of Y. By (iv), f((1,2)*-g-cl(f^{-1}(B))) $\leq$ (1,2)*-ker(B) and (1,2)*-g-cl(f^{-1}(B)) $\leq$ f^{-1}((1,2)*-ker(B)).
(5) $\Rightarrow$ (1). Let B be any $\sigma_{1,2}$-open set of Y. By (v), (1,2)*-g-cl(f^{-1}(B)) $\leq$ f^{-1}((1,2)*-ker(B)) $=$ f^{-1}(B) and (1,2)*-g-cl(f^{-1}(B)) $=$ f^{-1}(B). We obtain that f^{-1}(B) is (1,2)*-fg-closed in X. □

4. Weakly (1,2)*-fg-Continuous Maps

Definition 4.1. *A map f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1,\sigma_2)$ is called ((1,2)*-fg, s)-continuous if the inverse image of each regular (1,2)*-fuzzy open set of Y is (1,2)*-fg-closed in X.*

Definition 4.2. *A space X is called (1,2)*-fg-connected if X is not the union of two disjoint nonempty (1,2)*-fg-open sets.*

Definition 4.3. *A subset of a fuzzy bitopological space X is said to be*

(1). almost (1,2)-fuzzy connected if X cannot be written as a disjoint union of two non-empty regular (1,2)*-fuzzy open sets.*

(2). (1,2)-fuzzy connected if X cannot be written as a disjoint union of two non-empty $\tau_{1,2}$-open sets.*

Definition 4.4. *Let X and Y be two bitopological spaces. A map f : (X, τ_1, τ_2) $\to$ (Y, σ_1,σ_2) is called weakly (1,2)*-fg-continuous (briefly (1,2)*-fwg-continuous) if f^{-1}(U)is a (1,2)*-fwg-open set in X for each $\sigma_{1,2}$-open set U of Y.*

Theorem 4.5. *Every (1,2)*-fg-continuous map is (1,2)*-fwg-continuous.*

Proof. It follows from Proposition 3.17 □

The converse of Theorem 4.5 need not be true as seen from the following example.

Example 4.6. *Let (X, τ_1, τ_2) be a fuzzy bitopological space where $X = \{a, b, c\}$. $\tau_2 = \{\phi, X\}$, $\tau_1 = 0, 1$, $\lambda = \frac{0.7}{a} + \frac{0.3}{b} + \frac{0}{c}$, $\mu = \frac{0.7}{a} + \frac{0}{b} + \frac{0}{c}$. τ_{12}-open sets are $0, 1, \lambda = \frac{0.7}{a} + \frac{0.3}{b} + \frac{0}{c}$, $\mu = \frac{0.7}{a} + \frac{0}{b} + \frac{0}{c}$. Let (Y, σ_1, σ_2) be a fuzzy bitopological space where $Y = \{a, b, c\}$. $\sigma_1 = 0, 1$, $\lambda = \frac{1}{a} + \frac{0}{b} + \frac{0}{c}$ and $\sigma_2 = \{0,1\}$. σ_{12}-open sets are $\sigma_1 = 0, 1$, $\lambda = \frac{1}{a} + \frac{0}{b} + \frac{0}{c}$. Therefore (1,2)*-fg open sets are 0,1, λ, $\frac{\alpha_1}{a} + \frac{\alpha_2}{b} + \frac{\alpha_3}{c}$ where $\alpha_1 < 1$ (or) $\alpha_2 < 1$. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is an (1,2)*-fg-open map but it is not (1,2)*-fuzzy open map.*

Theorem 4.7. *A map f : (X, τ_1, τ_2) $\to$ (Y, σ_1,σ_2) is (1,2)*-fwg-continuous if and only if f^{-1}(U)is a (1,2)*-fwg-closed set in X for each $\sigma_{1,2}$-closed set U of Y.*

Proof. Let U be any $\sigma_{1,2}$-closed set of Y. According to the assumption $f^{-1}(U^c) = X \setminus f^{-1}(U)$is (1,2)*-fwg-open in X, so f^{-1}(U)is (1,2)*-fwg-closed in X.

The converse can be proved in a similar manner. □

Theorem 4.8. *Suppose that X and Y are fuzzy bitopological spaces and (1,2)*-FGC(X) is closed under arbitrary intersections. If a map f : (X, τ_1, τ_2) $\to$ (Y, σ_1,σ_2) is contra (1,2)*-fg-continuous and Y is (1,2)*-fuzzy regular, then f is (1,2)*-fg-continuous.*

Proof. Let x be an arbitrary point of X and V be an $\sigma_{1,2}$-open set of Y containing f(x). Since Y is (1,2)*-fuzzy regular, there exists an $\sigma_{1,2}$-open set G in Y containing f(x) such that $\sigma_{1,2}$-cl(G) $\leq$ V. Since f is contra (1,2)*-fg-continuous, there exists U $\in$ (1,2)*-FGO(X) containing x such that f(U) $\leq$ $\sigma_{1,2}$-cl(G). Then f(U) $\leq$ $\sigma_{1,2}$-cl(G) $\leq$ V. Hence, f is (1,2)*-fg-continuous. □

Theorem 4.9. *Suppose that X and Y are fuzzy bitopological spaces and the family of (1,2)*-fg-closed sets in X is closed under arbitrary intersections. If a map f : (X, τ_1, τ_2) $\to$ (Y, σ_1,σ_2) is contra (1,2)*-fg-continuous and Y is (1,2)*-fuzzy regular, then f is (1,2)*-fwg-continuous.*

Proof. The proof is obvious from Theorem 4.5. □

Definition 4.10. *A fuzzy bitopological space X is said to be locally (1,2)*-fg-indiscrete if every (1,2)*-fg-open set of X is $\tau_{1,2}$-closed in X.*

Theorem 4.11. *Let f : (X, τ_1, τ_2) $\to$ (Y, σ_1,σ_2) be a map. If f is (1,2)*-fg-continuous and X is locally (1,2)*-fg-indiscrete, then f is (1,2)*-fuzzy continuous.*

Proof. Let V be an $\sigma_{1,2}$-open in Y. Since f is (1,2)*-fg-continuous, f^{-1}(V)is (1,2)*-fg-open in X. Since X is locally (1,2)*-fg-indiscrete, f^{-1}(V)is $\tau_{1,2}$-closed in X. Hence f is (1,2)*-fuzzy continuous. □

Theorem 4.12. *Let f : (X, τ_1, τ_2) $\to$ (Y, σ_1,σ_2) be a map. If f is contra (1,2)*-fg-continuous and X is locally (1,2)*-fg-indiscrete, then f is (1,2)*-fwg-continuous.*

Proof. Let f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1,σ_2) be contra (1,2)*-fg-continuous and X is locally (1,2)*-fg-indiscrete. By Theorem 4.11, f is (1,2)*-fuzzy continuous, then f is (1,2)*-fwg-continuous by Theorem 4.5._ □

Corollary 4.13. *Let Y be a (1,2)*-fuzzy regular space and f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1,σ_2) be a map. Suppose that the collection of (1,2)*-fg-closed sets in X is closed under arbitrary intersections. Then if f is ((1,2)*-fg, s)-continuous, f is (1,2)*-fwg-continuous.*

Proof. Let f be ((1,2)*-fg, s)-continuous. Then f is (1,2)*-fg-continuous. Thus, f is (1,2)*-fwg-continuous by Theorem 4.5. □

Proposition 4.14. *If f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1,σ_2) is perfectly (1,2)*-fuzzy continuous and (1,2)*-fwg-continuous, then it is (1,2)*-fR-map.*

Proof. Let V be any regular (1,2)*-fuzzy open subset of Y. According to the assumption, f^{-1}(V)is both $\tau_{1,2}$-open and $\tau_{1,2}$-closed in X. Since f^{-1}(V)is $\tau_{1,2}$-closed, it is (1,2)*-fwg-closed. We have f^{-1}(V)is both $\tau_{1,2}$-open and (1,2)*-fwg-closed. Hence, by Corollary 3.10, it is regular (1,2)*-fuzzy open in X, so f is (1,2)*-fR-map. □

Definition 4.15. *A bitopological space X is called (1,2)*-fg-compact (resp. (1,2)*-fuzzy compact) if every cover of X by (1,2)*-fg-open (resp. $\tau_{1,2}$-open) sets has finite subcover.*

Definition 4.16. *A bitopological space X is weakly (1,2)*-fg-compact (briefly, (1,2)*-fwg-compact) if every (1,2)*-fwg-open cover of X has a finite subcover.*

Remark 4.17. *Every (1,2)*-fwg-compact space is (1,2)*-fg-compact.*

Theorem 4.18. *Let f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1,σ_2) be surjective (1,2)*-fwg-continuous map. If X is (1,2)*-fwg-compact, then Y is (1,2)*-fuzzy compact.*

Proof. Let {A_i : i $\in$I} be an $\sigma_{1,2}$-open cover of Y. Then { $f^{-1}(A_i)$: i $\in$I} is a (1,2)*-fwg-open cover in X. Since X is (1,2)*-fwg-compact, it has a finite subcover, say {$f^{-1}(A_1)$, $f^{-1}(A_2)$,. . ., $f^{-1}(A_n)$}. Since f is surjective {A_1, A_2, . . ., A_n} is a finite subcover of Y and hence Y is (1,2)*-fuzzy compact. □

Definition 4.19. *A fuzzy bitopological space X is weakly (1,2)*-fg-connected (briefly, (1,2)*-fwg-connected) if X cannot be written as the disjoint union of two non-empty (1,2)*-fwg-open sets.*

Theorem 4.20. *If a fuzzy bitopological space X is (1,2)*-fwg-connected, then X is almost (1,2)*-fuzzy connected (resp. (1,2)*-fg-connected).*

Proof. It follows from the fact that each regular (1,2)*-fuzzy open set (resp. (1,2)*-fg-open set) is (1,2)*-fwg-open. □

Theorem 4.21. *For a fuzzy bitopological space X, the following statements are equivalent:*

(1). X is (1,2)-fwg-connected.*

(2). The empty set ϕ and X are only subsets which are both (1,2)-fwg-open and (1,2)*-fwg-closed.*

(3). Each (1,2)-fwg-continuous map from X into a discrete fuzzy bitopological space Y which has at least two points is a constant map.*

Proof.

(1) $\Rightarrow$ (2). Let S $\leq$ X be any proper subset, which is both (1,2)*-fwg-open and (1,2)*-fwg-closed. Its complement X $\setminus$ S is also (1,2)*-fwg-open and (1,2)*-fwg-closed. Then X = S $\cup$ (X $\setminus$ S) is a disjoint union of two non-empty (1,2)*-fwg-open sets which is a contradiction with the fact that X is (1,2)*-fwg-connected. Hence, S = ϕ or X.

(2) $\Rightarrow$ (1). Let X = A $\cup$ B where A $\cap$ B = ϕ, A $\neq \phi$, B $\neq \phi$ and A, B are (1,2)*-fwg-open. Since A = X $\setminus$ B, A is (1,2)*-fwg-closed. According to the assumption A = ϕ, which is a contradiction.

(2) $\Rightarrow$ (3). Let f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1,σ_2) be a (1,2)*-fwg-continuous map where Y is a discrete fuzzy bitopological space with at least two points. Then f^{-1}({y}) is (1,2)*-fwg-closed and (1,2)*-fwg-open for each y $\in$Y and X = $\cup$ { f^{-1}({y}) y $\in$ Y}. According to the assumption, f^{-1}({y}) = ϕ or f^{-1}({y}) = X. If f^{-1}({y}) = ϕ for all y $\in$ Y, f will not be a map. Also there is no exist more than one y $\in$ Y such that f^{-1}({y}) = X. Hence, there exists only one y $\in$ Y such that f^{-1}({y}) = X and f^{-1}({y_1}) = ϕ where y $\neq$ y_1 $\in$ Y. This shows that f is a constant map.

(3) $\Rightarrow$ (2). Let S $\neq \phi$ be both (1,2)*-fwg-open and (1,2)*-fwg-closed in X. Let f : X $\rightarrow$ Y be a (1,2)*-fwg-continuous map defined by f(S) = {a} and f(X $\setminus$ S) = {b} where a $\neq$ b. Since f is constant map we get S = X. □

Theorem 4.22. *Let f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1,σ_2) be a (1,2)*-fwg-continuous surjective map. If X is (1,2)*-fwg-connected, then Y is (1,2)*-fuzzy connected.*

Proof. We suppose that Y is not (1,2)*-fuzzy connected. Then Y = A $\cup$ B where A $\cap$ B = ϕ, A $\neq \phi$, B $\neq \phi$ and A, B are $\sigma_{1,2}$-open sets in Y. Since f is (1,2)*-fwg-continuous surjective map, X = f^{-1}(A) $\cup$ f^{-1}(B) are disjoint union of two non-empty (1,2)*-fwg-open subsets. This is contradiction with the fact that X is (1,2)*-fwg-connected. □

5. Weakly (1,2)*-fg-Open and Weakly (1,2)*-fg-Closed Maps

Definition 5.1. *Let (X, τ_1, τ_2) and (Y, σ_1,σ_2) be fuzzy bitopological spaces. A map f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1,σ_2) is called weakly (1,2)*-fg-open (briefly, (1,2)*-fwg-open) if f(V) is a (1,2)*-fwg-open set in Y for each $\tau_{1,2}$-open set V of X.*

Remark 5.2. *Every (1,2)*-fg-open map is (1,2)*-fwg-open but not conversely.*

Definition 5.3. *Let (X, τ_1, τ_2) and (Y, σ_1,σ_2) be bitopological spaces. A map f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1,σ_2) is called weakly (1,2)*-fg-closed (briefly, (1,2)*-fwg-closed) if f(V) is a (1,2)*-fwg-closed set in Y for each $\tau_{1,2}$-closed set V of X.*
It is clear that an (1,2)-fuzzy open map is (1,2)*-fwg-open and a (1,2)*-fuzzy closed map is (1,2)*-fwg-closed.*

Theorem 5.4. *Let (X, τ_1, τ_2) and (Y, σ_1,σ_2) be fuzzy bitopological spaces. A map f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1,σ_2) is (1,2)*-fwg-closed if and only if for each subset B of Y and for each $\tau_{1,2}$-open set G containing f^{-1}(B) there exists a (1,2)*-fwg-open set F of Y such that B $\leq$ F and f^{-1}(F) $\leq$ G.*

Proof. Let B be any subset of Y and let G be an $\tau_{1,2}$-open subset of X such that f^{-1}(B) $\leq$ G. Then F = Y $\setminus$ f(X $\setminus$ G) is (1,2)*-fwg-open set containing B and f^{-1}(F) $\leq$ G.
Conversely, let U be any $\tau_{1,2}$-closed subset of X. Then f^{-1}(Y $\setminus$ f(U)) $\leq$ X $\setminus$ U and X $\setminus$ U is $\tau_{1,2}$-open. According to the assumption, there exists a (1,2)*-fwg-open set F of Y such that Y $\setminus$ f(U) $\leq$ F and f^{-1}(F) $\leq$ X $\setminus$ U. Then U $\leq$ X $\setminus$ f^{-1}(F). From Y $\setminus$ F $\leq$ f(U) $\leq$ f(X $\setminus$ f^{-1}(F)) $\leq$Y $\setminus$ F, it follows that f(U) = Y $\setminus$ F, so f(U) is (1,2)*-fwg-closed in Y. Therefore f is a (1,2)*-fwg-closed map. □

Remark 5.5. *The composition of two (1,2)*-fwg-closed maps need not be a (1,2)*-fwg-closed as we can see from the following example.*

Example 5.6. *Let (X, τ_1, τ_2) be a fuzzy bitopological space where $X = \{a, b, c\}$, $\tau_1 = 0, 1$, $\lambda = \frac{1}{a} + \frac{0}{b} + \frac{0}{c}$, $\mu = \frac{1}{a} + \frac{0}{b} + \frac{1}{c}$ and $\tau_2 = \{0, 1\}$. $\tau_{1,2}$-closed are 0, 1, $\lambda' = \frac{0}{a} + \frac{1}{b} + \frac{1}{c}$, $\mu = \frac{0}{a} + \frac{1}{b} + \frac{0}{c}$. Then (1,2)*-fg closed are 0, 1, $\lambda' = \frac{0}{a} + \frac{1}{b} + \frac{1}{c}$, $\mu = \frac{0}{a} + \frac{1}{b} + \frac{0}{c}$, $\frac{\alpha_1}{a} + \frac{\alpha_2}{b} + \frac{\alpha_3}{c}$, where $0 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1$, $\alpha_2 \neq 0$. Let (Y, σ_1, σ_2) be a fuzzy bitopological space where $Y = \{a, b, c\}$. $\sigma_1 = 0, 1$, $\lambda = \frac{1}{a} + \frac{0}{b} + \frac{0}{c}$ and $\sigma_2 = \{0, 1\}$. τ_{12}-closed are 0, 1, $\lambda' = \frac{0}{a} + \frac{1}{b} + \frac{1}{c}$. Then (1,2)*-fg closed are 0, 1, $\lambda' = \frac{0}{a} + \frac{1}{b} + \frac{1}{c}$, $\frac{\alpha_1}{a} + \frac{\alpha_2}{b} + \frac{\alpha_3}{c}$, where $0 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1$. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is an (1,2)*-fg-closed map.*

Let (Z, η_1, η_2) be a fuzzy bitopological space where $Z = \{a, b, c\}$. $\eta_1 = 0, 1$, $\lambda = \frac{1}{a} + \frac{0.5}{b} + \frac{0}{c}$ and $\eta_2 = \{0, 1\}$. η_{12}-closed are 0, 1, $\lambda' = \frac{0}{a} + \frac{0.5}{b} + \frac{1}{c}$. Then (1,2)-fg closed are 0, 1, $\lambda' = \frac{0}{a} + \frac{0.5}{b} + \frac{1}{c}$, $\frac{\alpha_1}{a} + \frac{\alpha_2}{b} + \frac{\alpha_3}{c}$, where $0 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1$, $\alpha_3 \neq 0$*

Let $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ be the identity map. Then both f and g are (1,2)-fwg-closed maps but their composition g o f : $(X, \tau_1, \tau_2) \to (Z, \eta_1, \eta_2)$ is not an (1,2)*-fwg-closed map, since for the τ_{12} closed set $\frac{0}{a} + \frac{1}{b} + \frac{0}{c}$ in X, (g o f) $\left(\frac{0}{a} + \frac{1}{b} + \frac{0}{c}\right) = \frac{0}{a} + \frac{1}{b} + \frac{0}{c}$ which is not (1,2)*-fwg-closed set in Z.*

Theorem 5.7. *Let (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2) be fuzzy bitopological spaces. If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a (1,2)*-fuzzy closed map and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ is a (1,2)*-fwg-closed map, then gof : $(X, \tau_1, \tau_2) \to (Z, \eta_1, \eta_2)$ is a (1,2)*-fwg-closed map.*

Definition 5.8. *A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called a weakly (1,2)*-fg-irresolute (briefly, (1,2)*-fwg-irresolute) map if $f^{-1}(U)$ is a (1,2)*-fwg-open set in X for each (1,2)*-fwg-open set U of Y.*

Theorem 5.9. *The composition of two (1,2)*-fwg-irresolute maps is also (1,2)*-fwg-irresolute.*

Theorem 5.10. *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ be maps such that gof : $(X, \tau_1, \tau_2) \to (Z, \eta_1, \eta_2)$ is (1,2)*-fwg-closed map. Then the following statements hold:*

(1). if f is (1,2)-fuzzy continuous and injective, then g is (1,2)*-fwg-closed.*

(2). if g is (1,2)-fwg-irresolute and surjective, then f is (1,2)*-fwg-closed.*

Proof.

(1). Let F be a $\sigma_{1,2}$-closed set of Y. Since f^{-1}(F) is $\tau_{1,2}$-closed in X, we can conclude that (g o f)(f^{-1}(F)) is (1,2)*-fwg-closed in Z. Hence g(F) is (1,2)*- (1,2)*-fwg-closed in Z. Thus g is a (1,2)*-fwg-closed map.

(2). It can be proved in a similar manner as (1).

□

Theorem 5.11. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an (1,2)*-fwg-irresolute map, then it is (1,2)*-fwg-continuous.*

Theorem 5.12. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is surjective (1,2)*-fwg-irresolute map and X is (1,2)*-fwg-compact, then Y is (1,2)*-fwg-compact.*

Theorem 5.13. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is surjective (1,2)*-fwg-irresolute map and X is (1,2)*-fwg-connected, then Y is (1,2)*-fwg-connected.*

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References

[1] R.Devi, *Studies on generalizations of closed maps and homeomorphisms in topological spaces*, Ph.D Thesis, Bharathiar University, Coimbatore, (1994).

[2] N.Levine, *Generalized closed sets in topology*, Rend. Circ. Math. Palermo, 19(2)(1970), 89-96.

[3] S.R.Malghan, *Generalized closed maps*, J. Karnataka Univ. Sci., 27(1982), 82-88.

[4] O.Ravi and S.Ganesan, *$\ddot{g}$-closed sets in topology*, International Journal of Computer Science and Emerging Technologies, 2(2011), 330-337.

[5] O.Ravi, M.L.Thivagar and A.Nagarajan, *(1,2)*-αg-closed sets and (1,2)*-$g\alpha$-closed sets*, (submitted).

[6] M.Sheik John, *A study on generalizations of closed sets and continuous maps in topological and bitopological spaces*, Ph.D Thesis, Bharathiar University, Coimbatore, (2002).