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Strong Split Independent Domination in Fuzzy Graphs

Research Article

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Abstract: An independent dominating set D of a fuzzy graph $G = (\sigma, \mu)$ is a split dominating set if the induced subgraph $H = (\langle V - D\rangle, \sigma', \mu')$ is disconnected. The minimum of the fuzzy cardinalities of a split independent dominating sets of G is called the split independent domination number $\gamma_{sif}(G)$ of G. An independent dominating set D of a fuzzy graph $G = (\sigma, \mu)$ is a strong split independent dominating set if the induced subgraph $H = (\langle V - D\rangle, \sigma', \mu')$ is totally disconnected. The minimum of the fuzzy cardinalities of a strong split independent dominating sets of G is called the strong split independent domination number $\gamma_{ssif}(G)$ of G. In this paper we study a strong split independent dominating sets of fuzzy graphs and investigate the relationship of $\gamma_{ssif}(G)$ or γ_{ssif} with other known parameter of G.

Keywords: Fuzzy split dominating set, fuzzy split independent dominating set, fuzzy strong split dominating set, fuzzy strong split independent dominating set.

1. Introduction

The study of dominating sets in graphs was begun by orge Berge V.R.Kulli wrote on theory of domination in graphs. The domination number and independent domination number are introduced by cockayne and Hedetniemi. A.Somasundram , S.Somasundram presented the concepts of independent domination, connected domination in fuzzy graphs. Q.M.Mahyoub and N.D.Soner initiate the split dominating set and split domination number in fuzzy graphs. Nagoorgani and vadivel discussed domination, Independent domination and Irredundant in fuzzy graph using strong arcs. In this paper we introduce the concept of split independent dominating set in fuzzy graph and strong split independent dominating set in fuzzy graph and obtain some interesting results for this new parameter in strong split independent fuzzy graphs.

2. Preliminaries

Let V be a finite non empty set. A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \rightarrow [0, 1]$ and $\mu : V \times V \rightarrow [0, 1]$ where $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for $u, v \in V$. The scalar cardinality of a fuzzy set V is the sum of the membership grades of the elements of the fuzzy set and is denoted by $|V|$. The order p and size of q of a fuzzy graph $G = (\sigma, \mu)$ are defined to be $p = \sum_{v \in V} (v)$ and $q = \sum_{u,v \in E} (uv)$. An edge $(u, v) = e$ is called an effective edge if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$. If (u ,v) is an effective edge, then u and v are adjacent effective edges. A fuzzy graph is said to be strong fuzzy graph if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ for all u , v in V. A vertex u is said to be isolated vertex $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for all $v \in V - \{u\}$. $N(u) = \{v \in V / \mu(u, v) = \sigma(u) \wedge \sigma(v)\}$

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is called the open neighborhood of u and $N[u] = N(u) \cup \{u\}$ is the closed neighborhood of u. If an edge (u ,v) is an effective edge, then its incident is said to be effective incident with both the vertices.

The effective incident degree of a vertex u of a fuzzy graph is defined to be the sum of the scalar cardinality of the effective edges incident at u and is denoted by $d_E(u)$. The minimum effective incident degree is $\delta_E(G) = \min\{d_E(u)/u \in V\}$ and the maximum effective incident degree is $\Delta_E(G) = \max\{d_E(u)/u \in V\}$. Let $G(\sigma, \mu)$ be a fuzzy graph on V and let $u, v \in V$. If $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ then u dominates v in G . A subset D of V is called a dominating set in G if for every $v \in V - D$ then there exist $u \in D$ such that u dominates v. A dominating set S of a fuzzy graph G is said to be a minimal dominating set if there is no dominating set S' of G such that $S' \subset S$. A dominating set D of a fuzzy graph G is said to be a minimum dominating set, if there is no dominating set D' of G such that $|D'| < |D|$. The minimum fuzzy cardinality of minimum dominating set of G is called the domination number of G and is denoted by $\gamma(G)$ or γ. A set $S \subseteq V$ in a fuzzy graph G is said to be independent if $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for all $u, v \in S$. A dominating set is called an independent dominating set if D is independent. An independent dominating set S of a fuzzy graph G is said to be a maximum independent dominating set if there is no independent dominating set S' of G such that $S' \subset S$. An independent dominating set S of a fuzzy graph G is said to be a maximum independent dominating set if there is no independent dominating set S' of G such that $|S'| > |S|$. The minimum scalar cardinality of an maximum independent dominating set G is called the independent domination number of G and is denoted by i(G).The vertex covering number $\alpha_o(G)$ denotes the minimum cardinality of a vertex covering of G and $\beta_o(G)$ denotes the maximum cardinality among the independent sets of G.

3. Split Independent Domination in Fuzzy Graphs

Definition 3.1. *An independent dominating set D of a fuzzy graph $G = (\sigma, \mu)$ is a split independent dominating set if the induced sub graph $H = (\langle V - D \rangle, \sigma, \mu')$ is disconnected. The split independent domination number $\gamma_{sif}(G)$ of a fuzzy graph G equals the minimum cardinality of a split independent dominating set.That is $\gamma_{sif}(G) = \min |D|$, where the minimum is taken over D in $\mathcal{D}$ where $\mathcal{D}$ is the set of all minimal split dominating set of G.*

Example 3.2.

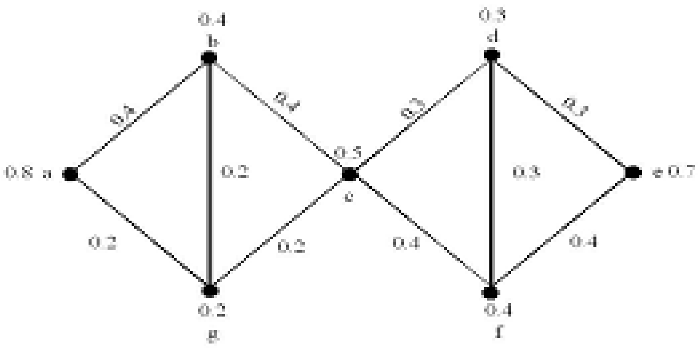

Figure 1. $\gamma_{sif}(G) = 2.0$

Theorem 3.3. *An independent dominating set D of a fuzzy graph G is a split independent dominating set if and only if there exists two vertices $u, v \in V - D$ such that every $u - v$ path contains a fuzzy vertex of D.*

Proof. Let D be a split independent dominating set of an fuzzy graph G. Then induced fuzzy subgraph $\langle V - D \rangle$ is disconnected. Hence there exists two vertices $u, v \in V - D$ such that every $u - v$ path contain a fuzzy vertex of D. Suppose there exists two vertices $u, v \in V - D$ such that every $u - v$ path contains a fuzzy vertex of D.Let D be an independent dominating set then induced subgraph $V - D$ is connected or disconnected. If it is connected then there exists u, v two vertices in $V - D$ such that some $u - v$ path does not contain a fuzzy vertex of D, which is a contradictions. Hence $V - D$ is disconnected, which implies D is a split independent dominating set of a fuzzy graph G. □

Theorem 3.4. *For any fuzzy graph $G = (\sigma, \mu)$, $\gamma_{sif}(G) \leq \alpha_o(G)$. where $\alpha_o(G)$ is a fuzzy vertex covering number of G.*

Proof. Let D be a maximal independent set of a fuzzy vertex in G, then D has at least two fuzzy vertices and every fuzzy vertex in D is adjacent to some vertex in $V - D$. This implies that $V - D$ is a split dominating set of G. Thus $\gamma_{sif}(G) \leq \alpha_o(G)$. □

Theorem 3.5. *For any connected fuzzy graph $G = (\sigma, \mu)$, $\gamma_f(G) + \gamma_{sif}(G) \leq P$.*

Proof. Since $\gamma_f(G) \leq \beta_o(G)$ and $\gamma_{sif}(G) \leq \alpha_o(G)$,

$$\gamma_f(G) + \gamma_{sif}(G) \leq \alpha_o(G) + \beta_o(G)$$
$$\gamma_f(G) + \gamma_{sif}(G) \leq P$$

□

Theorem 3.6. *For any connected fuzzy graph $G = (\sigma, \mu)$, $\gamma_{sif}(G) \leq P.\Delta(G)/(\Delta(G) + 1)$.*

Theorem 3.7. *For any connected fuzzy graph $G = (\sigma, \mu)$, then $\gamma_{sif}(G) \geq P/1 + \Delta(G)$.*

Proof. Let D be a independent dominating set in a fuzzy graph G with $|D| = \gamma_{sif}(G)$. Since every vertex in $V - D$ is adjacent to some vertices in D, we have

$$\begin{aligned} |V - D| &\leq \gamma_{sif}(G)\Delta(G) \\ P - \gamma_{sif}(G) &\leq \gamma_{sif}(G)\Delta(G) = \gamma_{sif}(G)\Delta(G) \\ P &\leq \gamma_{sif}(G) + \gamma_{sif}(G)\Delta(G) \\ &= \gamma_{sif}(G)(1 + \Delta(G)) \\ P/1 + \Delta(G) &\leq \gamma_{sif}(G) \end{aligned}$$

Thus $\gamma_{sif}(G) \geq P/1 + \Delta(G)$. □

Theorem 3.8. *For any fuzzy graph $G = (\sigma, \mu)$, then $p - q \leq \gamma_{sif}(G) \leq p - \Delta(G)$.*

4. Strong Split Independent Domination in Fuzzy Graphs

Definition 4.1. *An independent dominating set D of a fuzzy graph $G = (\sigma, \mu)$ is a strong split independent dominating set if the induced subgraph $H = (\langle V - D \rangle, \sigma', \mu')$ is totally disconnected.*
The strong split independent domination number $\gamma_{ssif}(G)$ of a fuzzy graph $G = (\sigma, \mu)$ equals the minimum cardinality of a strong split independent dominating set. That is $\gamma_{ssif}(G) = \min |D|$, where the minimum is taken over D in $\mathcal{D}$ where $\mathcal{D}$ is the set of all minimal strong split independent dominating set of G.

Observations 4.2.

(a). For any connected fuzzy graph $G = (\sigma, \mu)$, $\gamma_{sif}(G) \leq \gamma_{ssif}(G)$.

(b). For any connected fuzzy graph $G = (\sigma, \mu)$, $\gamma_f(G) \leq \gamma_{sif}(G) \leq \gamma_{ssif}(G)$.

(c). For any connected fuzzy graph $G = (\sigma, \mu)$, $\gamma_{if}(G) \leq \gamma_{sif}(G) \leq \gamma_{ssif}(G)$.

Example 4.3.

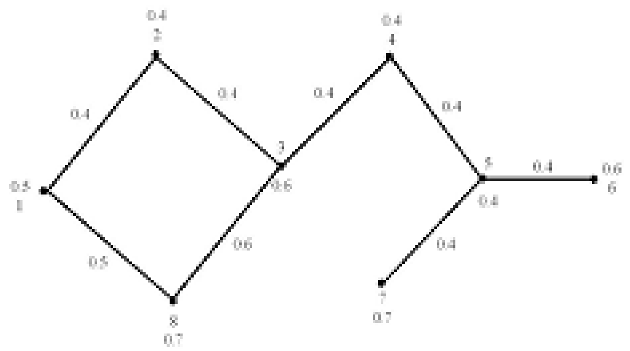

Figure 2. $\gamma_{ssif}(G) = 2.2$

Observations 4.4. *For any connected fuzzy graph* $G = (\sigma, \mu)$, $\gamma_{ssif}(G) \leq p - 1$.

Proposition 4.5. *For any fuzzy graph* $G = (\sigma, \mu)$ *of order* $p \geq 3$ *then* $\gamma_{ssif}(G) \leq q + p_0$ *where* p_0 *is the number of isolated vertices of G.*

Proposition 4.6. *If* $G = (\sigma, \mu)$ *is a fuzzy graph without isolated vertices and* $p \geq 3$ *then,* $\gamma_{ssif}(G) = \alpha_o(G)$.

Theorem 4.7. *Let* $G = (\sigma, \mu)$ *be a fuzzy graph of order p, then* $\gamma_f(G) \leq \gamma_{ssif}(G) \leq p - \Delta(G)$.

Proof. Every strong split independent fuzzy dominating set is a split independent fuzzy dominating set of a fuzzy graph G. $\gamma_f(G) \leq \gamma_{ssif}(G)$. If $d_G(u) = \Delta(G)$. Clearly $V - N(u)$ is a strong split independent fuzzy dominating set. Therefore, $\gamma_{ssif}(G) \leq |V - N(u)|$. That is $\gamma_{ssif}(G) \leq p - \Delta(G)$. □

Theorem 4.8. *For any fuzzy graph* $G = (\sigma, \mu)$, $\lceil P/1 + -\Delta(G) \rceil \leq \gamma_f(G) \leq p - \Delta(G)$.

Theorem 4.9. *For any fuzzy graph* $G = (\sigma, \mu)$ *with* $p \geq 2$ *then* $\lceil P/1 + \Delta(G) \rceil \leq \gamma_{ssif}(G) \leq 2q - p + 1$.

Proof. Using the above theorems result we get, $\lceil P/1 + \Delta(G) \rceil \leq \gamma_f(G) \leq \gamma_{ssif}(G)$ and the above observations, we get

$$\begin{aligned} \gamma_{ssif}(G) &\leq p - 1 = 2(p-1) - (p-1) \\ &\leq 2q - p + 1. \text{ Therefore} \\ \gamma_{ssif}(G) &\leq 2q - p + 1. \end{aligned}$$

□

Theorem 4.10. *If T is a fuzzy tree with* $p \geq 3$ *vertices then* $p - m \leq \gamma_{ssif}(T)$. *Where m denote the number of vertices adjacent to end vertices.*

Theorem 4.11. *Let T be any fuzzy tree of order* $p \geq 3$. *Then* $\gamma_{ssif}(T) \leq p - \epsilon(T)$ *where* $\epsilon(T)$ *denotes the number of pendant vertices of T.*

Theorem 4.12. *For any fuzzy graph* $G = (\sigma, \mu)$, $\gamma_{ssif}(G) \leq \beta_o(G)$ *where* $\beta_o(G)$ *is a maximum independent set.*

Theorem 4.13. *Let $G = (\sigma, \mu)$ be a fuzzy graph such that both G and $\overline{G}$ are connected then $\gamma_{ssif}(G) + \gamma_{ssif}(\overline{G}) \leq p$.*

Proof. By the previous theorem $\gamma_{ssif}(G) \leq \beta_o(G)$. Since both G and $\overline{G}$ are connected then $\Delta(G)$, $\Delta(\overline{G}) \leq p$. This implies $\alpha_o(G)$, $\alpha_o(\overline{G}) \geq 0$. Hence $\gamma_{ssif}(G) \leq p$. Similarly $\gamma_{ssif}(\overline{G}) \leq p$. Thus $\gamma_{ssif}(G) + \gamma_{ssif}(\overline{G}) \leq p + p = 2p$. □

Theorem 4.14. *Let T be any tree then $\gamma_{ssif}(G) = p - \Delta(T)$ if and only if T is a Wounded spider.*

Proof. Suppose T is a Wounded spider. Then it is easy to check that $\gamma_{ssif}(G) = p - \Delta(T)$. Conversely, suppose T is a tree with $\gamma_{ssif}(G) = p - \Delta(T)$. Let $v \in V(T)$ be a vertex of maximum degree. If $V(T) \backslash N[v] = \phi$ then T is a star $K_{1,t}$, $t \geq 1$, which is a wounded spider. We assume that there is at least one vertex in $V(T) \backslash N[v]$. Since $\langle V(T) \backslash N[v] \rangle$ is a bipartite graph, we may take a partition (X, Y) of $V(T) \backslash N[v]$ such that $|X|$ is maximum. Then X is a maximal independent set of the induced subgraph $\langle V(T) \backslash N[v] \rangle$, and $X \cup \{v\}$ is an independent $\gamma_{ssif}(T)$. Thus $p = \gamma_{ssif}(T) + \Delta(T) \leq 1 \times 1 + 1 + \Delta(T)$, which implies that $Y = \phi$, and so $V(T) \backslash N[v]$ is an independent set. The connecting of T implies that each vertex in $V(T) \backslash N[v]$ is adjacent to exactly one vertex in $N(v)$. Therefore T is a spider and at least one vertex in $N(v)$ is not adjacent to any vertex in $V(T) \backslash N[v]$, that is T is a wounded spider. □

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