International Journal of Mathematics And its Applications
Volume 4, Issue 1–A (2016), 171–174.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Fuzzy Meet-Semidistributive Lattice

Research Article

K.Nithya[1]* and V.Vinoba[2]

1 Research Scholar, Department of Mathematics, Bharathidhasan University, Trichy, TamilNadu, India.

2 Department of Mathematics, K.N.Government Arts College for Women, Thanjavur, TamilNadu, India.

Abstract: In this Paper, Fuzzy Meet-Semidistributive LatticeDefinition of Fuzzy meet-semidistributive Lattice-Characterization theorem are given.

Keywords: Fuzzy Lattice, Fuzzy Modular Lattice, Fuzzy Distributive Lattice, Fuzzy Meet-Semidistributive Lattice.
© JS Publication.

1. Introduction

The Concept of Fuzzy Lattice was already introduced by G.Gratzer [5], G.H.BarDalo and E.Rodrigues [7], M.Stern [6] explained semimodular Lattices, M.Mullai and B.Chellappa [3] explained Fuzzy L-ideal, N.Ajmal [1], S.Nanda [2] and L.R.WilCox [4] explained modularity in the theory of Lattices., V.Vinoba and K.Nithya [8] Explained fuzzy modular pairs in Fuzzy Distributive Lattice. A few definitions and results are listed that the fuzzy Meet-semidistributive lattice using in this paper we explain fuzzy Meet-semidistributive lattice, Definition of fuzzy Meet-semidistributive lattice, Characterization theorem of Fuzzy Meet-semidistributive lattice and some examples are given, L is a Fuzzy distributive Lattice iff L does not contain a Fuzzy sublattice isomorphic to S_7.

Definition 1.1. *A Fuzzy lattice L is called Fuzzy meet- semi distributive if* $\mu(a \wedge b) = \mu(a \wedge c) \Rightarrow \mu(a \wedge b) = \mu(a) \wedge (b \vee c)$*, for all* $\mu(a)$*,* $\mu(b)$*,* $\mu(c) \in L$*.*

2. Main Section

Theorem 2.1. *Every meet-semi distributive lattice is fuzzy lattice and the converse is not true.*

Proof. Given L is a meet-semi distributive lattice $\Rightarrow \mu(a \wedge b) = \mu(a) \wedge (b \vee c)$, for all $\mu(a)$, $\mu(b)$, $\mu(c) \in L$. To prove L is a Fuzzy lattice. That is to prove if $\mu(a \wedge b) = \mu(a \wedge c)$, for all $\mu(a)$, $\mu(b)$, $\mu(c) \in L$.

$$\begin{aligned}\mu(a \wedge b) &= \mu(a) \wedge (b \vee c)\\ &\geq \min\{\mu(a), \mu(b \vee c)\}\\ &\geq \min\{\mu(a), \min\{\mu(b), \mu(c)\}\}\end{aligned}$$

* E-mail: nithya8183@gmail.com

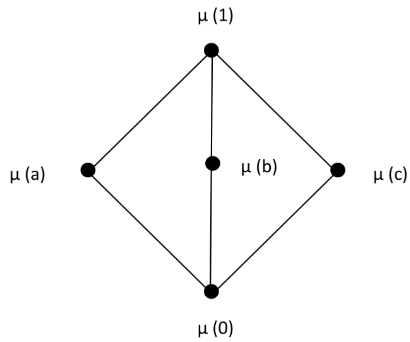
$$\geq \min\{\mu(a), \min\{\mu(c), \mu(b)\}\}, \quad \text{by commutative law}$$

$$\geq \min\{\mu(a), \mu(c \vee b)\}$$

$$= \mu(a) \wedge (c \vee b)$$

$$= \mu(a \wedge c)$$

Hence L is a Fuzzy Lattice. The converse is not true. That is every Fuzzy lattice need not be Fuzzy meet-semi distributive. Consider the Fuzzy lattice L of following figure



This Fuzzy lattice is not Fuzzy meet-semi distributive. Here

$$\mu(a) \wedge \mu(b) = \mu(0)$$

$$\mu(a) \wedge \mu(c) = \mu(0)$$

$$\mu(b) \vee \mu(c) = \mu(1)$$

$$\begin{aligned} \mu(a) \wedge \mu(b \vee c) &\geq \min\{\mu(a), \mu(b \vee c)\} \\ &\geq \min\{\mu(a), \mu(b) \vee \mu(c)\} \\ &\geq \min\{\mu(a), \mu(1)\} \\ &\geq \mu(1) \wedge \mu(a) \\ &= \mu(a) \end{aligned}$$

Thus $\mu(a) \wedge \mu(b) = \mu(a) \wedge \mu(c)$. But $\mu(a) \wedge \mu(b) \neq \mu(a) \wedge \mu(b \vee c) \Rightarrow$ L is not Fuzzy meet-semi distributive. □

Theorem 2.2. *Every Fuzzy distributive is Fuzzy meet-semi distributive and the converse is not true.*

Proof. Given L is a Fuzzy distributive lattice

$$\Rightarrow \mu(a) \vee \mu(b \wedge c) = \mu(a \vee b) \wedge \mu(a \vee c) \text{ for all } \mu(a), \mu(b), \mu(c) \in L \tag{1}$$

To prove: L is Fuzzy meet- semi distributive lattice. That is to prove if $\mu(a) \wedge \mu(b) = \mu(a) \wedge \mu(c)$ implies $\mu(a) \wedge \mu(b) = \mu(a) \wedge \mu(b \vee c)$, for all $\mu(a)$, $\mu(b)$, $\mu(c) \in L$. First to claim that $\mu(a) \wedge \mu(b \vee c) = \mu(a \wedge b) \vee \mu(a \wedge c)$ for all $\mu(a)$, $\mu(b)$, $\mu(c) \in L$. For let $\mu(a)$, $\mu(b)$, $\mu(c) \in L$ be an arbitrary. Then

$$\mu(a) \wedge \mu(b \vee c) = \mu(a \wedge b) \vee \mu(a \wedge c) \text{ for all } \mu(a), \mu(b), \mu(c) \in L \tag{2}$$

By Theorem, In a Fuzzy distributive lattice, the duality of the Fuzzy distributive condition is holds.

Given L is a Fuzzy distributive lattice $\Rightarrow \mu(a) \vee \mu(b \wedge c) = \mu(a \vee b) \wedge \mu(a \vee c)$, for all $\mu(a)$, $\mu(b)$, $\mu(c) \in L$.

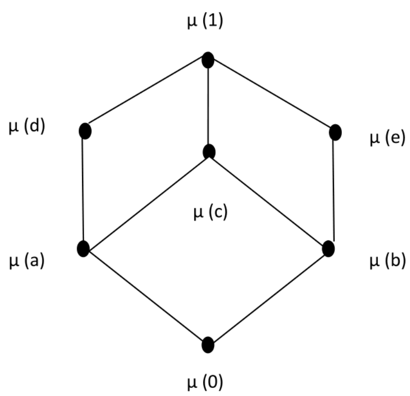
To prove: $\mu(a) \wedge \mu(b \vee c) = \mu(a \wedge b) \vee \mu(a \wedge c)$, for all $\mu(a)$, $\mu(b)$, $\mu(c) \in L$. Let $\mu(a)$, $\mu(b)$, $\mu(c) \in L$ be arbitrary. Then

$$\begin{aligned}
\mu(a \wedge b) \vee \mu(a \wedge c) &\geq \min\{\mu(a \wedge b), \mu(a \wedge c)\} \\
&\geq \min\{\mu(a \wedge b) \vee \mu(a), \mu(a \wedge b) \vee \mu(c)\} \\
&\geq \min\{\mu(a) \vee \mu(a \wedge b), \mu(c) \vee \mu(a \wedge b)\} \\
&\geq \min\{\mu(a), \mu(c \vee a) \wedge \mu(c \vee b)\} \\
&\geq \min\{\mu(a) \wedge \mu(c \vee a), \mu(c \vee b)\} \\
&\geq \min\{\mu(a) \wedge \mu(a \vee c), \mu(b \vee c)\} \\
&= \mu(a) \wedge \mu(b \vee c), forall \mu(a), \mu(b), \mu(c) \in L
\end{aligned}$$

Suppose $\mu(a) \wedge \mu(b) = \mu(a) \wedge \mu(c)$. Then

$$\begin{aligned}
\mu(a) \wedge \mu(b \vee c) &= \mu(a \wedge b) \vee \mu(a \wedge c), \quad \text{by } (2) \\
&= \mu(a \wedge b) \vee \mu(a \wedge b) \\
&= \mu(a \wedge b)
\end{aligned}$$

Thus $\mu(a \wedge b) = \mu(a \wedge c) \Rightarrow \mu(a \wedge b) = \mu(a) \wedge \mu(b \vee c)$ for all $\mu(a)$, $\mu(b)$, $\mu(c) \in L \Rightarrow$ L is Fuzzy meet- semi distributive lattice. The converse need not be true. That is every Fuzzy meet-semi distributive lattice need not be a Fuzzy distributive lattice. We shall verify it by the following example. Consider the Fuzzy lattice S_7 of following figure.



This Fuzzy lattice is Fuzzy meet – semi distributive but not Fuzzy distributive. Here

$$\begin{aligned}
\mu(a) \vee \mu(d \wedge e) &\geq \min\{\mu(a), \mu(d \wedge e)\} \\
&\geq \min\{\mu(a), \mu(0)\} \\
&= \mu(a) \vee \mu(0) \\
&= \mu(a) \\
\mu(a \vee d) \wedge \mu(a \vee e) &\geq \min\{\mu(a \vee d) \wedge \mu(a \vee e)\} \\
&\geq \min\{\mu(d), \mu(1)\} \\
&= \mu(d) \wedge \mu(1) \\
&= \mu(d)
\end{aligned}$$

Therefore $\mu(a) \vee \mu(d \wedge e) \neq \mu(a \vee d) \wedge \mu(a \vee e) \Rightarrow S_7$ is not Fuzzy distributive. □

Theorem 2.3. *A Fuzzy meet-semi distributive lattice L is Fuzzy distributive if and only if L does not contain a Fuzzy sublattice isomorphic to S_7.*

Proof. Assume that a Fuzzy meet-semi distributive lattice L is Fuzzy distributive.

To Prove: L does not contain a Fuzzy sublattice isomorphic to S_7. Suppose L contain a Fuzzy sublattice isomorphic to S_7. Thus L is not Fuzzy distributive. This is a Contradiction. Hence L does not contain a Fuzzy sublattice isomorphic to S_7.

Conversely Assume that a Fuzzy meet- semi distributive lattice L does not contain a Fuzzy sublattice is isomorphic to S_7.

To Prove: L is Fuzzy distributive.

Suppose L is not Fuzzy distributive. Then L contain a Fuzzy sublattice isomorphic to S_7. This is Contradiction. Hence L is a Fuzzy distributive Lattice. □

Theorem 2.4. *Every Fuzzy modular lattice need not be Fuzzy meet-semi distributive lattice.*

Proof. Given L is Fuzzy modular lattice $\Rightarrow$ L contain a Fuzzy sublattice isomorphic to M_3. A Fuzzy lattice L is Fuzzy modular if and only if does not contain a Fuzzy sublattice isomorphic to N_5. Assume that a Fuzzy lattice L is Fuzzy modular. To Prove L does not contain a Fuzzy sublattice isomorphic to N_5.

Suppose L contain a Fuzzy sublattice isomorphic to $N_5 \Rightarrow$ L is not Fuzzy modular. This is a Contradiction. Hence L does not contain a Fuzzy sublattice isomorphic to N_5.

Conversely Assume that a Fuzzy lattice L does not contain a Fuzzy sublattice isomorphic to N_5.

To Prove: L is Fuzzy modular.

Suppose L is not Fuzzy modular. L contain a Fuzzy sublattice isomorphic to N_5. This is a Contradiction to our assumption $\Rightarrow$L is Fuzzy modular $\Rightarrow$ L is not Fuzzy meet-semi distributive, by Theorem 2.1. □

3. Conclusion

This paper is proved that Every meet-semi distributive lattice is fuzzy lattice and the converse is not true, Every Fuzzy distributive is Fuzzy meet-semi distributive and the converse is not true, A Fuzzy meet- semi distributive lattice L is Fuzzy distributive if and only if L does not contain a Fuzzy sublattice isomorphic to S_7 and Every Fuzzy modular lattice need not be Fuzzy meet-semi distributive lattice.

References

[1] N.Ajmal, *Fuzzy lattices*, Inform. Sci, 79(1994), 271-291.

[2] S.Nanda, *Fuzzy Lattice*, Bull. Cal. Math. Soc., 81(1989).

[3] M.Mullai and B.Chellappa, *Fuzzy L-ideal*, Acta Ciencia India, XXXVM(2)(2009).

[4] L.R.Wilcox, *Modularity in the theory of Lattices*, Bull. Amer. Math. Soc., (1938), 44-50.

[5] G.Gratzer, *General Lattice Theory*, Academic Press Inc, (1978).

[6] M.Sterin, *Semimodular Lattices*, Teubner-Text Zur Mathematik, Stuttgart-Leipzig, (1991).

[7] G.H.Bar Dalo and E.Rodrigues, *Complements in Modular and Semimodular Lattices*, Portugaliae Mathematice, 55(3)(1998).

[8] V.Vinoba and K.Nithya, *Fuzzy Modular Pairs in Fuzzy Distributive Lattice*, IJFSRS, 8(1)(2015), 79-81.