International Journal of Mathematics And its Applications
Volume 4, Issue 1–B (2016), 29–35.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Weaker Form of δ-open Sets via Ideals

Research Article

A.Anis Fathima[1]*, V.Inthumathi[2] and M.Maheswari[2]

1 Department of Mathematics(CA), Sri GVG Visalakshi College for Women, Udumalpet, Tamil Nadu, India.

1 Department of Mathematics, N.G.M College, Pollachi, Tamil Nadu, India.

Abstract: In this paper, the notions of $\delta_{\mathcal{I}}$-semi-open sets and $\delta_{\mathcal{I}}$-semi-closed sets are introduced and investigated in ideal topological spaces.

MSC: 54A05.

Keywords: $\delta_{\mathcal{I}}$-semi-open sets and $\delta_{\mathcal{I}}$-semi-closed sets.

1. Introduction

The concept of ideals in topological spaces has been introduced and studied by Kuratowski [12] and Vaidyanathasamy [19]. An ideal $\mathcal{I}$ on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies (i) $A \in \mathcal{I}$ and $B \subset A$ implies $B \in \mathcal{I}$ and (ii) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$. For a subset A of X, $A^*(\mathcal{I}, \tau) = \{x \in X : A \cap U \notin \mathcal{I}$ for every $U \in \tau(x)\}$ is called the *local function* [12] of A with respect to $\mathcal{I}$ and τ. We simply write A^* in case there is no chance for confusion. A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(\mathcal{I}, \tau)$ finer than τ is defined by $cl^*(A)$=$A \cup A^*$ [19]. Throughout this paper, $(X, \tau, \mathcal{I})$ (or simply X), always mean ideal topological space on which no separation axiom is assumed. In this paper we introduce weaker form of δ-open sets in ideal topological spaces.

2. Preliminaries

Definition 2.1 ([18]). *A subset A of a topological space (X, τ) is said to be*

(1). regular open if $A = int(cl(A))$,

(2). regular closed if $A = cl(int(A)))$.

A is called δ-open [20] if for each $x \in A$, there exists a regular open set G such that $x \in G \subset A$. The complement of a δ-open set is called δ-closed. A point $x \in X$ is called a δ-cluster point of A if $int(cl(V)) \cap A \neq \emptyset$ for each open set V containing X. The set of all δ-cluster points of A is called the δ-closure of A and is denoted by $\delta cl(A)$. The δ-interior of A is the union of all regular open sets of X contained in A and it is denoted by $\delta int(A)$.

* *E-mail: anisnazer2009@gmail.com*

Definition 2.2. *A subset A of a topological space (X, τ) is said to be*

(1). semi-open [13] if $A \subseteq cl(int(A))$,

(2). pre-open [14] if $A \subseteq int(cl(A))$,

(3). α-open [15] if $A \subseteq int(cl(int(A)))$,

(4). β-open [2] if $A \subseteq cl(int(cl(A)))$,

(5). b-open [4] if $A \subseteq int(cl(A)) \cup cl(int(A))$,

(6). δ-semi-open [16] if $A \subseteq cl(int_\delta(A))$,

(7). δ-pre-open [17] if $A \subseteq int(cl_\delta(A))$,

(8). δ-β-open [10] if $A \subseteq cl(int(cl_\delta(A)))$,

(9). a-open [8] if $A \subseteq int(cl(int_\delta(A)))$.

Definition 2.3. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be*

(1). $\mathcal{I}$-open [3] if $A \subseteq int(A^*)$,

(2). δ-$\mathcal{I}$-open [1] if $int(cl^*(A)) \subseteq cl^*(int(A))$,

(3). pre-$\mathcal{I}$-open [6] if $A \subseteq int(cl^*(A))$,

(4). semi-$\mathcal{I}$-open [9] if $A \subseteq cl^*(int(A))$,

(5). α-$\mathcal{I}$-open [9] if $A \subseteq int(cl^*(int(A)))$,

(6). β-$\mathcal{I}$-open [9] if $A \subseteq cl(int(cl^*(A)))$,

(7). b-$\mathcal{I}$-open [5] if $A \subseteq int(cl^*(A)) \cup cl^*(int(A))$,

(8). α_*-$\mathcal{I}$-open [7] if $A \subseteq int(cl^*(int_\delta(A)))$,

(9). $\beta^*_{\mathcal{I}}$-open [7] if $A \subseteq cl^*(int(cl_\delta(A)))$,

(10). t-$\mathcal{I}$-set [10] if $int(cl^*(A)) = int(A)$,

(11). δ_β-t-set [10] if $cl(int(cl_\delta(A))) = int(A)$.

Lemma 2.4. *[[11]] Let $(X, \tau, \mathcal{I})$ be an ideal topological space and let $A \subseteq X$. Then $U \in \tau \Rightarrow U \cap A^* \subseteq (U \cap A)^*$.*

3. $\delta_{\mathcal{I}}$-semi-open Sets

Definition 3.1. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be $\delta_{\mathcal{I}}$-semi-open if $A \subseteq cl^*(int_\delta(A))$.*

The family of all $\delta_{\mathcal{I}}$-semi-open sets of $(X, \tau, \mathcal{I})$ is denoted by $\delta_{\mathcal{I}}SO(X)$.

Theorem 3.2. *Every δ-open set is $\delta_{\mathcal{I}}$-semi-open.*

Theorem 3.3. *For a space $(X, \tau, \mathcal{I})$, the following hold:*

(1). Every $\delta_{\mathcal{I}}$-semi-open set is semi-open, β-open and b-open.

(2). Every $\delta_{\mathcal{I}}$-semi-open set is δ-semi-open and δ-β-open.

(3). Every $\delta_{\mathcal{I}}$-semi-open set is δ-$\mathcal{I}$-open, semi-$\mathcal{I}$-open, β-$\mathcal{I}$-open, b-$\mathcal{I}$-open and $\beta^*_{\mathcal{I}}$-open.

Remark 3.4. *The converses of the above theorems need not be true as seen from the following examples.*

Example 3.5. *Let $X=\{a,b,c\}$ with topology $\tau=\{\emptyset,\{a\},\{b\},\{a,b\},\{b,c\},X\}$, and $\mathcal{I}=\{\emptyset,\{a\},\{b\},\{a,b\}\}$. Then $A=\{b\}$ is semi-open, β-open, b-open, δ-$\mathcal{I}$-open, δ-β-open, semi-$\mathcal{I}$-open, β-$\mathcal{I}$-open, b-$\mathcal{I}$-open and $\beta^*_{\mathcal{I}}$-open but it is not $\delta_{\mathcal{I}}$-semi-open.*

Example 3.6. *Let $X=\{a,b,c,d\}$ with topologies $\tau=\{\emptyset,\{d\},\{a,c\},\{a,c,d\},X\}$, and $\mathcal{I}=\{\emptyset,\{a\},\{d\},\{a,d\}\}$. Then the set $A=\{b,d\}$ is δ-semi-open but not $\delta_{\mathcal{I}}$-semi-open.*

Example 3.7. *Let $X=\{a,b,c\}$ with topology $\tau=\{\emptyset,\{b\},\{c\},\{b,c\},X\}$, and $\mathcal{I}=\{\emptyset,\{a\}\}$. Then $A=\{a,b\}$ is $\delta_{\mathcal{I}}$-semi-open but not δ-open.*

Remark 3.8. *From the following examples, we see that in a space $(X,\tau,\mathcal{I})$,*

(1). The notions of $\delta_{\mathcal{I}}$-semi-open sets and open (resp.$\mathcal{I}$-open)sets are independent.

(2). The notions of $\delta_{\mathcal{I}}$-semi-open sets and pre open (resp.δ-pre open and pre-$\mathcal{I}$-open)sets are independent.

(3). The notions of $\delta_{\mathcal{I}}$-semi-open sets and α-open (resp.α-$\mathcal{I}$-open)sets are independent.

(4). The notions of $\delta_{\mathcal{I}}$-semi-open sets and α_*-$\mathcal{I}$-open sets (resp.t-$\mathcal{I}$-sets and δ_β-t-sets)are independent.

Example 3.9. *In Example 3.5, the set $A=\{b\}$ is open and $\mathcal{I}$-open but not $\delta_{\mathcal{I}}$-semi-open. In Example 3.7, the set $B=\{a,c\}$ is $\delta_{\mathcal{I}}$-semi-open but not open, $\mathcal{I}$-open, pre open, δ-pre open, pre-$\mathcal{I}$-open and δ_β-t-set.*

Example 3.10. *In Example 3.5, the set $A=\{b\}$ is pre open, δ-pre open, pre-$\mathcal{I}$-open α-open and α-$\mathcal{I}$-open but not $\delta_{\mathcal{I}}$-semi-open. In Example 3.7, the set $B=\{a\}$ is δ_β-t-set and t-$\mathcal{I}$-set but not $\delta_{\mathcal{I}}$-semi-open.*

Example 3.11. *In Example 3.7, the set $B=\{a,b\}$ is $\delta_{\mathcal{I}}$-semi-open but not α-open and α-$\mathcal{I}$-open. Moreover, the set $C=\{b,c\}$ is $\delta_{\mathcal{I}}$-semi-open but not t-$\mathcal{I}$-set.*

Remark 3.12. *If A is $\delta_{\mathcal{I}}$-semi-open and open, then it is a-open and α_*-$\mathcal{I}$-open.*

Theorem 3.13. *If $A_\alpha \in \delta_{\mathcal{I}}SO(X)$ for each $\alpha \in \Delta$, then $\bigcup_{\alpha\in\Delta}\{A_\alpha : \alpha \in \Delta\} \in \delta_{\mathcal{I}}SO(X)$.*

Proof. Let A_α be $\delta_{\mathcal{I}}$-semi-open for each $\alpha \in \Delta$.
Then, we have $A_\alpha \subseteq cl^*(int_\delta(A_\alpha))$. Thus
$\bigcup_{\alpha\in\Delta} A_\alpha \subseteq \bigcup_{\alpha\in\Delta}[cl^*(int_\delta(A_\alpha))] = \bigcup_{\alpha\in\Delta}[int_\delta(A_\alpha) \cup (int_\delta(A_\alpha))^*] \subseteq int_\delta(\bigcup_{\alpha\in\Delta} A_\alpha) \cup (int_\delta(\bigcup_{\alpha\in\Delta} A_\alpha))^* = cl^*(int_\delta(\bigcup_{\alpha\in\Delta} A_\alpha))$
This shows that $\bigcup_{\alpha\in\Delta}\{A_\alpha : \alpha \in \Delta\} \in \delta_{\mathcal{I}}SO(X)$. □

Remark 3.14. *From the following example, we observe that the intersection of two $\delta_{\mathcal{I}}$-semi-open sets need not be $\delta_{\mathcal{I}}$-semi-open.*

Example 3.15. *In Example 3.7, the sets $A=\{a,b\}$ and $B=\{a,c\}$ are $\delta_{\mathcal{I}}$-semi-open sets but $A\cap B=\{a\}$ is not $\delta_{\mathcal{I}}$-semi-open.*

Theorem 3.16. *For a subset A of a space $(X,\tau,\mathcal{I})$,* *(1). If $\mathcal{I} = \emptyset$, then A is $\delta_{\mathcal{I}}$-semi-open if and only if A is δ-semi-open.*

(2). If $\mathcal{I} = P(X)$, then A is $\delta_{\mathcal{I}}$-semi-open if and only if A is δ-open.

Proof. The proof of **(1)** follows from the fact that $A^*(\{\emptyset\}) = cl(A)$ and **(2)** follows from the fact that $A^*(P(X)) = \{\emptyset\}$. $\square$

Corollary 3.17. *If $\mathcal{I} = P(X)$ and A be $\delta_{\mathcal{I}}$-semi-open then A is pre open (resp.δ-pre open and pre-$\mathcal{I}$-open.)*

Theorem 3.18. *A subset A of a space $(X, \tau, \mathcal{I})$ is $\delta_{\mathcal{I}}$-semi-open if and only if $cl^*(A) = cl^*(int_\delta(A))$.*

Proof. Let A be $\delta_{\mathcal{I}}$-semi-open, we have $A \subseteq cl^*(int_\delta(A))$. Then $cl^*(A) \subseteq cl^*(int_\delta(A))$. Hence $cl^*(A) = cl^*(int_\delta(A))$. Conversely, $A \subseteq cl^*(A) = cl^*(int_\delta(A))$. Thus A is $\delta_{\mathcal{I}}$-semi-open. $\square$

Theorem 3.19. *A subset A of a space $(X, \tau, \mathcal{I})$ is $\delta_{\mathcal{I}}$-semi-open if and only if there exists a δ-open set U such that $U \subseteq A \subseteq cl^*(U)$.*

Proof. Suppose that A is $\delta_{\mathcal{I}}$-semi-open. Then we have $A \subseteq cl^*(int_\delta(A))$. Put $U = int_\delta(A)$. We have U is δ-open and so $U \subseteq A \subseteq cl^*(U)$. Conversely, let U be δ-open set such that $U \subseteq A \subseteq cl^*(U)$. Thus $cl^*(int_\delta(U)) \subseteq cl^*(int_\delta(A))$ and so $A \subseteq cl^*(U) \subseteq cl^*(int_\delta(A))$. Therefore A is $\delta_{\mathcal{I}}$-semi-open. $\square$

Corollary 3.20. *If a set A is $\delta_{\mathcal{I}}$-semi-open, then there exists a δ-open set U such that $U \subseteq A \subseteq cl(A)$.*

Proposition 3.21. *If U and V are δ-open sets and A is $\delta_{\mathcal{I}}$-semi-open set such that $U \cap V = \emptyset$ then $A \cap U = \emptyset$.*

Proof. Since U is δ-open and $U \cap V = \emptyset$, we have $cl^*(V) \subseteq U^c$. Thus $A \subseteq U^c$. Hence $A \cap U = \emptyset$. $\square$

Theorem 3.22. *If A is an $\delta_{\mathcal{I}}$-semi-open set in $(X, \tau, \mathcal{I})$ and $A \subseteq B \subseteq cl^*(A)$ then B is $\delta_{\mathcal{I}}$-semi-open.*

Proof. Since A is $\delta_{\mathcal{I}}$-semi-open, then there exists a δ-open set U such that $U \subseteq A \subseteq cl^*(U)$. Then, we have $U \subseteq A \subseteq B \subseteq cl^*(A) \subseteq cl^*(U)$ and hence $U \subseteq B \subseteq cl^*(U)$. By Proposition 3.19, we obtain B is $\delta_{\mathcal{I}}$-semi-open. $\square$

Theorem 3.23. *If $A \in \delta_{\mathcal{I}}SO(X)$ and B is δ-open then $A \cap B \in \delta_{\mathcal{I}}SO(X)$.*

Proof. Let $A \in \delta_{\mathcal{I}}SO(X)$ and B be δ-open. Then $A \subseteq cl^*(int_\delta(A))$. By Lemma 2.4, we have $A \cap B \subseteq cl^*(int_\delta(A)) \cap B$ $= [int_\delta(A) \cap B] \cup [(int_\delta(A))^* \cap B] \subseteq [int_\delta(A) \cap B] \cup [(int_\delta(A) \cap B)^*] = [int_\delta(A \cap B)] \cup [(int_\delta(A \cap B))^*] = cl^*(int_\delta(A \cap B))$ Thus $A \cap B \in \delta_{\mathcal{I}}SO(X)$. $\square$

Definition 3.24. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be $\delta_{\mathcal{I}}$-semi-closed if its complement is $\delta_{\mathcal{I}}$-semi-open.*

Theorem 3.25. *A subset A of a space $(X, \tau, \mathcal{I})$ is $\delta_{\mathcal{I}}$-semi-closed if and only if $int^*(cl_\delta(A)) \subseteq A$.*

Theorem 3.26. *If a subset A of a space $(X, \tau, \mathcal{I})$ is $\delta_{\mathcal{I}}$-semi-closed then $int_\delta(cl^*(A)) \subseteq A$.*

Proof. Since A is $\delta_{\mathcal{I}}$-semi-closed, $X - A \in \delta_{\mathcal{I}}SO(X)$. Now, we have
$X - A \subseteq cl^*(int_\delta(X - A)) \subseteq cl(int_\delta(X - A)) = X - int(cl_\delta(A)) \subseteq X - int_\delta(cl^*(A))$. Therefore, $int_\delta(cl^*(A)) \subseteq A$. $\square$

Corollary 3.27. *Let A be a subset of a space $(X, \tau, \mathcal{I})$ such that $X - (int_\delta(cl^*(A))) = cl^*(int_\delta(X - A))$. Then A is $\delta_{\mathcal{I}}$-semi-closed if and only if $int_\delta(cl^*(A)) \subseteq A$.*

Proof. **Necessity.** This is an immediate consequence of Theorem 3.26. **Sufficiency.** Let $int_\delta(cl^*(A)) \subseteq A$. Then $X - A \subseteq X - [int_\delta(cl^*(A))] = cl^*(int_\delta(X - A))$. Thus $X - A$ is $\delta_{\mathcal{I}}$-semi-open and so A is $\delta_{\mathcal{I}}$-semi-closed. $\square$

Theorem 3.28. *A set A is $\delta_{\mathcal{I}}$-semi-closed if and oniy if threr exists a δ-closed set C such that $int^*(C) \subseteq A \subseteq C$.*

Proof. Obvious from definition and Theorem 3.19. □

Theorem 3.29. *If U is δ-open and $V \in \delta_{\mathcal{I}}SO(X, \tau, \mathcal{I})$ then $U \cap V \in \delta_{\mathcal{I}}SO(U, \tau|U, \mathcal{I}|U)$.*

Proof. Since U is δ-open, we have $\delta int_U(A) = int_\delta(A)$ for any subset A of U.
Now, $U \cap V \subseteq U \cap cl^*(int_\delta(V)) = U \cap [int_\delta(V) \cup (int_\delta(V))^*] = ([U \cap int_\delta(V)] \cup [U \cap (int_\delta(V))^*]) \cap U \subseteq [U \cap (int_\delta(V)) \cap U] \cup [U \cap (U \cap int_\delta(V))^*] = [U \cap (\delta int_U(U \cap V))] \cup [U \cap (\delta int_U(U \cap V))^*] = [\delta int_U(U \cap V)] \cup [\delta int_U(U \cap V)]^*(\tau|U, \mathcal{I}|U)$ $= cl^*_U(\delta int_U(U \cap V))$. Thus $U \cap V \in \delta_{\mathcal{I}}SO(U, \tau|U, \mathcal{I}|U)$. □

Remark 3.30. *Intersection of two $\delta_{\mathcal{I}}$-semi-closed sets is $\delta_{\mathcal{I}}$-semi-closed in $(X, \tau, \mathcal{I})$.*

Example 3.31. *Union of two $\delta_{\mathcal{I}}$-semi-closed sets need not be $\delta_{\mathcal{I}}$-semi-closed as seen from this example. In Example 3.7, the sets A=$\{b\}$ and B=$\{c\}$ are $\delta_{\mathcal{I}}$-semi-closed sets but $A \cup B$=$\{b, c\}$ is not $\delta_{\mathcal{I}}$-semi-closed.*

Definition 3.32. *Let A be a subset of an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ and x be a point of X. Then*

(1). x is called an $\delta_{\mathcal{I}}$-semi-cluster point of A if $A \cap U \neq \emptyset$ for every $U \in \delta_{\mathcal{I}}SO(X)$,

(2). the family of all $\delta_{\mathcal{I}}$-semi-cluster points of A is called $\delta_{\mathcal{I}}$-semi-closure of A and is denoted by $scl_{\delta_{\mathcal{I}}}(A)$.

Theorem 3.33. *For subsets $A, B \subseteq (X, \tau, \mathcal{I})$, the following hold:*

(1). $scl_{\delta_{\mathcal{I}}}(A)$=$\bigcap\{F \subseteq X : A \subseteq F$ and F is $\delta_{\mathcal{I}}$-semi-closed$\}$.

(2). $scl_{\delta_{\mathcal{I}}}(A)$ is the smallest $\delta_{\mathcal{I}}$-semi-closed subset of X containing A.

(3). If $A \subseteq B$, then $scl_{\delta_{\mathcal{I}}}(A) \subseteq scl_{\delta_{\mathcal{I}}}(B)$.

(4). A is $\delta_{\mathcal{I}}$-semi-closed if and only if $A = scl_{\delta_{\mathcal{I}}}(A)$.

(5). $scl_{\delta_{\mathcal{I}}}(scl_{\delta_{\mathcal{I}}}(A)) = scl_{\delta_{\mathcal{I}}}(A)$.

(6). $scl_{\delta_{\mathcal{I}}}(A \cap B) \subseteq scl_{\delta_{\mathcal{I}}}(A) \cap scl_{\delta_{\mathcal{I}}}(B)$.

(7). $scl_{\delta_{\mathcal{I}}}(A) \cup scl_{\delta_{\mathcal{I}}}(B) \subseteq scl_{\delta_{\mathcal{I}}}(A \cup B)$.

Proof. (1). Suppose that $x \notin scl_{\delta_{\mathcal{I}}}(A)$. Then there exists $U \in \delta_{\mathcal{I}}SO(X)$ such that $U \cap A = \emptyset$. Then, we have U^c is $\delta_{\mathcal{I}}$-semi-closed set containing A and $x \notin U^c$. Thus $x \notin \bigcap\{F \subseteq X : A \subseteq F$ and F is $\delta_{\mathcal{I}}$-semi-closaed$\}$. Conversely, suppose there exists $F \in \delta_{\mathcal{I}}$-SC(X) such that $A \subseteq F$ and $x \notin F$. Then F^c is $\delta_{\mathcal{I}}$-semi-open set containing x, we have $F^c \cap A = \emptyset$. Thus $x \notin scl_{\delta_{\mathcal{I}}}(A)$. Hence $scl_{\delta_{\mathcal{I}}}(A)$=$\bigcap\{F \subseteq X : A \subseteq F$ and F is $\delta_{\mathcal{I}}$-semi-closed$\}$. The other proofs are obvious. □

Definition 3.34. *Let A be a subset of an ideal topological space $(X, \tau, \mathcal{I})$ and x be a point of X. Then*

(1). x is called an $\delta_{\mathcal{I}}$-semi-interior point of A if there exists $U \in \delta_{\mathcal{I}}SO(X)$ such that $x \in U \subseteq A$.

(2). the family of all $\delta_{\mathcal{I}}$-semi-interior points of A is called $\delta_{\mathcal{I}}$-semi-interior of A and is denoted by $sint_{\delta_{\mathcal{I}}}(A)$.

Theorem 3.35. *For subsets $A, B \subseteq (X, \tau, \mathcal{I})$, the following hold:*

(1). $sint_{\delta_{\mathcal{I}}}(A)$=$\bigcup\{F \subseteq X : F \subseteq A$ and F is $\delta_{\mathcal{I}}$-semi-open$\}$.

(2). $sint_{\delta_{\mathcal{I}}}(A)$ is the largest $\delta_{\mathcal{I}}$-semi-open subset of X contained in A.

(3). If $A \subseteq B$, then $sint_{\delta_{\mathcal{I}}}(A) \subseteq sint_{\delta_{\mathcal{I}}}(B)$.

(4). A is $\delta_{\mathcal{I}}$-semi-open if and only if $A = sint_{\delta_{\mathcal{I}}}(A)$.

(5). $sint_{\delta_{\mathcal{I}}}(sint_{\delta_{\mathcal{I}}}(A)) = sint_{\delta_{\mathcal{I}}}(A)$.

(6). $sint_{\delta_{\mathcal{I}}}(A \cap B) \subseteq sint_{\delta_{\mathcal{I}}}(A) \cap sint_{\delta_{\mathcal{I}}}(B)$.

(7). $sint_{\delta_{\mathcal{I}}}(A) \cup sint_{\delta_{\mathcal{I}}}(B) \subseteq sint_{\delta_{\mathcal{I}}}(A \cup B)$.

Proof. (1). Let x $\in \bigcup\{F \subseteq X : F \subseteq A$ and F is $\delta_{\mathcal{I}}$-semi-open$\}$. Then, there exists $F \in \delta_{\mathcal{I}}$-SO(X) such that $x \in F \subseteq A$ and hence $x \in sint_{\delta_{\mathcal{I}}}(A)$. This shows that $\bigcup\{F \subseteq X : F \subseteq A$ and F is $\delta_{\mathcal{I}}$-semi-open$\} \subseteq sint_{\delta_{\mathcal{I}}}(A)$. Let $x \in sint_{\delta_{\mathcal{I}}}(A)$. Then there exists $F \in \delta_{\mathcal{I}}$-SO(X) such that $x \in F \subseteq A$, we obtain x $\in \bigcup\{F \subseteq X : F \subseteq A$ and F is $\delta_{\mathcal{I}}$-semi-open)$\}$. This shows that $sint_{\delta_{\mathcal{I}}}(A) \subseteq \bigcup\{F \subseteq X : F \subseteq A$ and F is $\delta_{\mathcal{I}}$-semi-open)$\}$. Therefore, we obtain $sint_{\delta_{\mathcal{I}}}(A)$=$\bigcup\{F \subseteq X : F \subseteq A$ and F is $\delta_{\mathcal{I}}$-semi-open$\}$. The other proofs are obvious. □

Theorem 3.36. *For a subset $A \subseteq (X, \tau, \mathcal{I})$, the following hold:*

(1). $scl_{\delta_{\mathcal{I}}}(X - A) = X - sint_{\delta_{\mathcal{I}}}(A)$.

(2). $sint_{\delta_{\mathcal{I}}}(X - A) = X - scl_{\delta_{\mathcal{I}}}(A)$.

References

[1] A.Acikgoz, T.Noiri and S.Yuksel, *On δ-$\mathcal{I}$-open sets and decomposition of α-$\mathcal{I}$-continuity*, Acta Math. Hungar., 102(4)(2004), 349-357.

[2] M.E.Abd El-Monsef, S.N.El-Deeb and R.A.Mahmoud , *β-open sets and β-continuous mappings*, Bull. Fac. Sci. Assiut Univ., 12(1983), 77-90.

[3] M.E.Abd El-Monsef, E.F.Lashien and A.A.Nasef , *On $\mathcal{I}$-open sets and $\mathcal{I}$-continuous functions*, Kyungpook Math. J., 32(1992), 21-30.

[4] D.Andrijevic, *On b-open sets*, Mathematichki Vesnik, 48(1-2)(1996), 59-64.

[5] A.Caksu Guler and G.Aslim, *b-I-open sets and decomposition of continuity via idealization*, Proceedings of Institute of Mathematics and Mechanics. National Acedemy of Sciences of Azerbaijan, 22(2005), 27-32.

[6] J.Dontchev, *On Pre-$\mathcal{I}$-open sets and a decomposition of $\mathcal{I}$-continuity*, Banyan Math.J., 2(1996).

[7] E.Ekici and T.Noiri. *On subsets and decompositions of continuity in ideal topological spaces*, The Arabian Journal for science and engineering, 34(1A)(2009), 165-177.

[8] E.Ekici, *On a-open sets, A^*-sets and decompositions of continuity and super-continuity*, Annales Univ. Sci. Budapest., 51(2008), 39-51.

[9] E.Hatir and T.Noiri, *On decompositions of continuity via idealization*, Acta Math. Hungar., 96(4)(2002), 341-349.

[10] E.Hatir and T.Noiri, *Decomposition of continuity and complete continuity*, Acta Math. Hungar., 113(4)(2006), 281-287.

[11] D.Jankovic and T.R.Hamlett, *New topologies from old via ideals*, Amer. Math. Monthly, 97(1990), 295-310.

[12] K.Kuratowski, *Topology*, Vol. I, Academic Press, New York, (1966).

[13] N.Levine, *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly, 70(1963), 36-41.

[14] A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb, *On precontinuous and weak precontinuous mappings*, Proc. Math. Phys. Soc. Egypt, 53(1982), 47-53.

[15] O.Njastad, *On some classes of nearly open sets*, Pacific J. Math., 15(1965), 961-970.

[16] T.Noiri, *Remarks on δ-semiopen sets and δ-preopen sets*, Demonstratio Math, 36(2003), 1007-1020.

[17] S.Raychaudhuri and M.N.Mukherjee, *On δ-almost continuity and δ-preopen sets*, Bull. Inst. Math. Acad, Sinica., 21(1993), 357-366.

[18] M.H.Stone, *Applications of the Theory of Boolean Rings to General Topology*, Trans. Amer. Math. Soc., 41(1937), 375381.

[19] R.Vaidyanathaswamy, *The localisation theory in set topology*, Proc. Indian Acad. Sci., 20(1945), 51-61.

[20] N.V.Velicko, *H-closed Topological Spaces*, Amer. Math. Soc. Transl., 78(2)(1968), 103-118.