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On Relaxed Skolem Mean Labling For Four Star

Research Article

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Abstract: In this paper, we prove that if $\ell \leq m < n$, the four star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a relaxed skolem mean graph if $|m-n| \leq 2\ell+6$ for $\ell = 2, 3, 4, \ldots$; m = 2, 3, 4, . . . and $2\ell+m \leq n \leq 2\ell+m+6$.

MSC: 05C78.

Keywords: Skolem mean graph and star.

1. Introduction

All graphs in this chapter are finite, simple and undirected. Terms not defined here are used in the sense of Harary [8]. In [2], we proved that the three star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m-n| = 4+\ell$ for $\ell = 1, 2, 3, \ldots$, $m = 1, 2, 3, \ldots$, $n = \ell+m+4$ and $\ell \leq m < n$; the three star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m-n| > 4+\ell$ for $\ell = 1, 2, 3, \ldots$, $m = 1, 2, 3, \ldots$, $n \geq \ell+m+5$ and $\ell \leq m < n$; the four star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m-n| = 4+2\ell$ for $\ell = 2, 3, 4, \ldots$, $m = 2, 3, 4, \ldots$, $n = 2\ell+m+4$ and $\ell \leq m < n$; the four star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m-n| > 4+2\ell$ for $\ell = 2, 3, 4, \ldots$, $m = 2, 3, 4, \ldots$, $n = 2\ell+m+5$ and $\ell \leq m < n$; the four star $K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m-n| = 7$ for $m = 1, 2, 3, \ldots$, $n = m+7$ and $1 \leq m < n$. Also, the four star $K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m-n| > 7$ for $m = 1, 2, 3, \ldots$, $n \geq m+8$ and $1 \leq m < n$. In [4], the three star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a relaxed skolem mean graph if $|m-n| > 6+\ell$ for $\ell = 1, 2, 3, \ldots$, $m = 1, 2, 3, \ldots$, and $\ell+m \leq n \leq \ell+m+6$. Also,If $\ell \leq m < n$ the four star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a relaxed skolem mean graph if $|m-n| \leq 2\ell+6$ for $\ell = 2, 3, 4, \ldots$, $m = 2, 3, 4, \ldots$, and $2\ell+m \leq n \leq 2\ell+m+6$. In [3], the necessary condition for a graph to be relaxed skolem mean is that $p \geq q$.

2. Relaxed Skolem Mean Labeling

Definition 2.1. *The four star is the disjoint union of $K_{1,a}$, $K_{1,b}$, $K_{1,c}$ and $K_{1,d}$. It is denoted by $K_{1,a} \cup K_{1,b} \cup K_{1,c} \cup K_{1,d}$.*

Definition 2.2. [2] *A graph $G = (V, E)$ with p vertices and q edges is said to be a skolem mean graph if there exists a function f from the vertex set of G to $\{1, 2, 3, \ldots, p+1\}$ such that the induced map f^* from the edge set of G to $\{2, 3, 4, \ldots, p+1\}$*

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defined by

$$f^*(e=uv)=\begin{cases}\dfrac{f(u)+f(v)}{2} & \textit{if } f(u)+f(v) \textit{ is even}\\ \dfrac{f(u)+f(v)+1}{2} & \textit{if } f(u)+f(v) \textit{ is odd}\end{cases}$$

then the resulting edges get distinct labels from the set $\{2,3,4,\dots,p+1\}$.

Note 2.3. *In a relaxed skolem mean graph,* $p \geq q$.

Theorem 2.4. *If* $\ell \leq m < n$*, the four star* $K_{1,\ell}\cup K_{1,\ell}\cup K_{1,m}\cup K_{1,n}$ *is a relaxed skolem mean graph if* $|m-n| \leq 2\ell+6$ *for* ℓ*= 2, 3, 4, . . . ; m = 2, 3, 4 . . . and* $2\ell+m \leq n \leq 2\ell+m+6$.

Proof.

Case (a): Consider the graph G = $K_{1,\ell}\cup K_{1,\ell}\cup K_{1,m}\cup K_{1,n}$ where n = 2ℓ + m + 6. We have to prove that G is a relaxed skolem mean graph. We have V(G) = {u, v, w, x} ∪ {u_i: $1 \leq i \leq \ell$} ∪ {v_j: $1 \leq j \leq \ell$} ∪ {w_k: $1 \leq k \leq m$} ∪ {x_h: $1 \leq h \leq n$} and $E(G)=\{uu_i : 1\leq i\leq \ell\}\cup\{vv_j : 1\leq j\leq \ell\}\cup\{ww_k : 1\leq k\leq m\}\cup\{xx_h : 1\leq h\leq n\}$. Then G has 2ℓ + m + n + 4 vertices and 2ℓ + m + n edges. The required vertex labeling f: V (G) → {1, 2, 3, 4 . . . 2ℓ + m + n + 4} is defined as follows:

$$\begin{aligned}
&f(u)=1;\quad f(v)=3;\quad f(w)=5;\quad f(x)=2\ell+m+n+4;\\
&f(u_i)=2i+5 \qquad for \quad 1\leq i\leq \ell\\
&f(v_j)=2\ell+2j+5 \qquad for \quad 1\leq j\leq \ell\\
&f(w_k)=4\ell+2k+5 \qquad for \quad 1\leq k\leq m\\
&f(x_h)=2h \qquad for \quad 1\leq h\leq n-2\\
&f(x_{n-1})=2\ell+m+n+3\\
&f(x_n)=2\ell+m+n+5
\end{aligned}$$

The corresponding edge labels are as follows: The edge label of uu_i is $i+3$ for $1 \leq i \leq \ell$; vv_j is $\ell+j+4$ for $1 \leq j \leq \ell$; ww_k is $2\ell+k+5$ for $1 \leq k \leq m$ and xx_h is $\dfrac{2h+2\ell+m+n+4}{2}$ for $1 \leq h \leq n-2$. Also, the edge label of xx_n-1 is $2\ell+m+n+4$ and xx_n is $2\ell+m+n+5$. Therefore, the required edge labels of $G=\{4,\dots,\ell+3,\ell+5,\dots,2\ell+4,2\ell+6,\dots,2\ell+m+5,2\ell+m+6,\dots,4\ell+2m+9,4\ell+2m+10,4\ell+2m+11\}$ is $\{(\ell+3)-4+1+(2\ell+4)-(\ell+5)+1+(4\ell+2m+11)-(2\ell+6)+1\}=4\ell+2m+6$ distinct edges. Hence the induced edge labels of G are distinct. Hence the graph G is a relaxed skolem mean graph if $|m-n|\leq 2\ell+6$ for $\ell=2,3,4,\dots$; $m=2,3,4,\dots$ when $n=2\ell+m+6$.

Case (b): Consider the graph $G=K_{1,\ell}\cup K_{1,\ell}\cup K_{1,m}\cup K_{1,n}$ where $n=2\ell+m+5$. We have to prove that G is a relaxed skolem mean graph. We have $V(G)=\{u,v,w,x\}\cup\{u_i : 1\leq i\leq \ell\}\cup\{v_j : 1\leq j\leq \ell\}\cup\{w_k : 1\leq k\leq m\}\cup\{x_h : 1\leq h\leq n\}$ and $E(G)=\{uu_i : 1\leq i\leq \ell\}\cup\{vv_j : 1\leq j\leq \ell\}\cup\{ww_k : 1\leq k\leq m\}\cup\{xx_h : 1\leq h\leq n\}$. Then G has $2\ell+m+n+4$ vertices and $2\ell+m+n$ edges. The required vertex labeling $f: V(G)\to\{1,2,3,4,\dots,2\ell+m+n+4\}$ is defined as follows:

$$\begin{aligned}
&f(u)=1;\quad f(v)=3;\quad f(w)=5;\quad f(x)=2\ell+m+n+4;\\
&f(u_i)=2i+5 \qquad for \quad 1\leq i\leq \ell\\
&f(v_j)=2\ell+2j+5 \qquad for \quad 1\leq j\leq \ell\\
&f(w_k)=4\ell+2k+5 \qquad for \quad 1\leq k\leq m\\
&f(x_h)=2h \qquad for \quad 1\leq h\leq n
\end{aligned}$$

The corresponding edge labels are as follows: The edge label of uu_i is $i+3$ for $1 \leq i \leq \ell$; vv_j is $\ell+j+4$ for $1 \leq j \leq \ell$; ww_k is $2\ell+k+5$ for $1 \leq k \leq m$ and xx_h is $\frac{2h+2\ell+m+n+4}{2}$ for $1 \leq h \leq n$. Therefore, the required edge labels of $G = \{4,\ldots,\ell+3,\ell+5,\ldots,2\ell+4,2\ell+6,\ldots,2\ell+m+5,2\ell+m+6,\ldots,4\ell+2m+10\}$ is $\{(\ell+3)-4+1+(2\ell+4)-(\ell+5)+1+(4\ell+2m+10)-(2\ell+6)+1\} = 4\ell+2m+5$ distinct edges. Hence the induced edge labels of G are distinct. Hence the graph G is a relaxed skolem mean graph if $|m-n| \leq 2\ell+6$ $\ell = 2,3,4,\ldots$; $m = 2,3,4,\ldots$ when $n = 2\ell+m+5$.

Case (c): Consider the graph $G = K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ where $n = 2\ell+m+4$. We have to prove that G is a relaxed skolem mean graph. We have $V(G) = \{u,v,w,x\} \cup \{u_i : 1 \leq i \leq \ell\} \cup \{v_j : 1 \leq j \leq \ell\} \cup \{w_k : 1 \leq k \leq m\} \cup \{x_h : 1 \leq h \leq n\}$ and $E(G) = \{uu_i : 1 \leq i \leq \ell\} \cup \{vv_j : 1 \leq j \leq \ell\} \cup \{ww_k : 1 \leq k \leq m\} \cup \{xx_h : 1 \leq h \leq n\}$. Then G has $2\ell+m+n+4$ vertices and $2\ell+m+n$ edges. The required vertex labeling $f : V(G) \to \{1,2,3,4,\ldots,2\ell+m+n+4\}$ is defined as follows:

$$f(u) = 1; \quad f(v) = 2; \quad f(w) = 4; \quad f(x) = 2\ell+m+n+4;$$
$$f(u_i) = 2i+6 \qquad for \quad 1 \leq i \leq \ell$$
$$f(v_j) = 2\ell+2j+6 \qquad for \quad 1 \leq j \leq \ell$$
$$f(w_k) = 4\ell+2k+6 \qquad for \quad 1 \leq k \leq m$$
$$f(x_h) = 2h+1 \qquad for \quad 1 \leq h \leq n.$$

The corresponding edge labels are as follows: The edge label of uu_i is $i+4$ for $1 \leq i \leq \ell$; vv_j is $\ell+j+4$ for $1 \leq j \leq \ell$; ww_k is $2\ell+k+5$ for $1 \leq k \leq m$ and xx_h is $\frac{2h+2\ell+m+n+5}{2}$ for $1 \leq h \leq n$. Therefore, the required edge labels of $G = \{5,\ldots,\ell+4,\ell+5,\ldots,2\ell+4,2\ell+6,\ldots,2\ell+m+5,2\ell+m+6,\ldots,4\ell+2m+9\}$ is $\{(\ell+4)-5+1+(2\ell+4)-(\ell+5)+1+(4\ell+2m+9)-(2\ell+6)+1\} = 4\ell+2m+4$ distinct edges. Hence the induced edge labels of G are distinct. Hence the graph G is a relaxed skolem mean graph if $|m-n| \leq 2\ell+6$ for $\ell = 2,3,4,\ldots$; when $n = 2\ell+m+4$.

Case (d): Consider the graph $G = K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$. Where $n = 2\ell+m+3$. We have to prove that G is a relaxed skolem mean graph. We have $V(G) = \{u,v,w,x\} \cup \{u_i : 1 \leq i \leq \ell\} \cup \{v_j : 1 \leq j \leq \ell\} \cup \{w_k : 1 \leq k \leq m\} \cup \{x_h : 1 \leq h \leq n\}$ and $E(G) = \{uu_i : 1 \leq i \leq \ell\} \cup \{vv_j : 1 \leq j \leq \ell\} \cup \{ww_k : 1 \leq k \leq m\} \cup \{xx_h : 1 \leq h \leq n\}$. Then G has $2\ell+m+n+4$ vertices and $2\ell+m+n$ edges. The required vertex labeling $f : V(G) \to \{1,2,3,4,\ldots,2\ell+m+n+4\}$ is defined as follows:

$$f(u) = 1; \quad f(v) = 2; \quad f(w) = 3; \quad f(x) = 2\ell+m+n+4;$$
$$f(u_i) = 2i+5 \qquad for \quad 1 \leq i \leq \ell$$
$$f(v_j) = 2\ell+2j+5 \qquad for \quad 1 \leq j \leq \ell$$
$$f(w_k) = 4\ell+2k+5 \qquad for \quad 1 \leq k \leq m$$
$$f(x_h) = 2h+2 \qquad for \quad 1 \leq h \leq n.$$

The corresponding edge labels are as follows: The edge label of uu_i is $i+3$ for $1 \leq i \leq \ell$; vv_j is $\ell+j+4$ for $1 \leq j \leq \ell$; ww_k is $2\ell+k+4$ for $1 \leq k \leq m$ and xx_h is $\frac{2h+2\ell+m+n+6}{2}$ for $1 \leq h \leq n$. Therefore, the required edge labels of $G = \{4,\ldots,\ell+3,\ell+5,\ldots,2\ell+4,2\ell+5,\ldots,2\ell+m+4,2\ell+m+6,\ldots,4\ell+2m+8\}$ is $\{(\ell+3)-4+1+(2\ell+4)-(\ell+5)+1+(4\ell+2m+8)-(2\ell+6)+1\} = 4\ell+2m+3$ distinct edges. Hence the induced edge labels of G are distinct. Hence the graph G is a relaxed skolem mean graph if $|m-n| \leq 2\ell+6$ for $\ell = 2,3,4,\ldots$; $m = 2,3,4,\ldots$ when $n = 2\ell+m+3$.

Case (e): Consider the graph $G = K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ where $n = 2\ell + m + 2$. We have to prove that G is a relaxed skolem mean graph. We have $V(G) = \{u, v, w, x\} \cup \{u_i : 1 \leq i \leq \ell\} \cup \{v_j : 1 \leq j \leq \ell\} \cup \{w_k : 1 \leq k \leq m\} \cup \{x_h : 1 \leq h \leq n\}$ and $E(G) = \{uu_i : 1 \leq i \leq \ell\} \cup \{vv_j : 1 \leq j \leq \ell\} \cup \{ww_k : 1 \leq k \leq m\} \cup \{xx_h : 1 \leq h \leq n\}$. Then G has $2\ell + m + n + 4$ vertices and $2\ell + m + n$ edges. The required vertex labeling $f : V(G) \to \{1, 2, 3, 4, \ldots, 2\ell + m + n + 5\}$ is defined as follows:

$$\begin{aligned}
&f(u) = 1; \quad f(v) = 2; \quad f(w) = 3; \quad f(x) = 2\ell + m + n + 4;\\
&f(u_i) = 2i + 4 \qquad for \quad 1 \leq i \leq \ell\\
&f(v_j) = 2\ell + 2j + 4 \qquad for \quad 1 \leq j \leq \ell\\
&f(w_k) = 4\ell + 2k + 4 \quad for \quad 1 \leq k \leq m\\
&f(x_h) = 2h + 3 \qquad for \quad 1 \leq h \leq n.
\end{aligned}$$

The corresponding edge labels are as follows: The edge label of uu_i is $i + 3$ for $1 \leq i \leq \ell$; vv_j is $\ell + j + 3$ for $1 \leq j \leq \ell$; ww_k is $2\ell + k + 4$ for $1 \leq k \leq m$ and xx_h is $\dfrac{2h + 2\ell + m + n + 7}{2}$ for $1 \leq h \leq n$. Therefore, the required edge labels of $G = \{4, \ldots, \ell + 3, \ell + 4, \ldots, 2\ell + 3, 2\ell + 5, \ldots, 2\ell + m + 4, 2\ell + m + 6, \ldots, 4\ell + 2m + 7\}$ is $\{(\ell + 3) - 4 + 1 + (2\ell + 3) - (\ell + 4) + 1 + (4\ell + 2m + 7) - (2\ell + 5) + 1\} = 4\ell + 2m + 2$ distinct edges. Hence the induced edge labels of G are distinct. Hence the graph G is a relaxed skolem mean graph if $|m - n| \leq 2\ell + 6$ for $\ell = 2, 3, 4, \ldots$; $m = 2, 3, 4, \ldots$ when $n = 2\ell + m + 2$.

Case (f): Consider the graph $G = K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ where $n = 2\ell + m + 1$. We have to prove that G is a relaxed skolem mean graph. We have $V(G) = \{u, v, w, x\} \cup \{u_i : 1 \leq i \leq \ell\} \cup \{v_j : 1 \leq j \leq \ell\} \cup \{w_k : 1 \leq k \leq m\} \cup \{x_h : 1 \leq h \leq n\}$ and $E(G) = \{uu_i : 1 \leq i \leq \ell\} \cup \{vv_j : 1 \leq j \leq \ell\} \cup \{ww_k : 1 \leq k \leq m\} \cup \{xx_h : 1 \leq h \leq n\}$. Then G has $2\ell + m + n + 4$ vertices and $2\ell + m + n$ edges. The required vertex labeling $f : V(G) \to \{1, 2, 3, 4, \ldots, 2\ell + m + n + 4\}$ is defined as follows:

$$\begin{aligned}
&f(u) = 1; \quad f(v) = 2; \quad f(w) = 4; \quad f(x) = 2\ell + m + n + 4;\\
&f(u_i) = 2i + 3 \qquad for \quad 1 \leq i \leq \ell\\
&f(v_j) = 2\ell + 2j + 3 \qquad for \quad 1 \leq j \leq \ell\\
&f(w_k) = 4\ell + 2k + 3 \quad for \quad 1 \leq k \leq m\\
&f(x_h) = 2h + 4 \qquad for \quad 1 \leq h \leq n.
\end{aligned}$$

The corresponding edge labels are as follows: The edge label of uu_i is $i + 2$ for $1 \leq i \leq \ell$; vv_j is $\ell + j + 3$ for $1 \leq j \leq \ell$; ww_k is $2\ell + k + 4$ for $1 \leq k \leq m$ and xx_h is $\dfrac{2h + 2\ell + m + n + 8}{2}$ for $1 \leq h \leq n$. Therefore, the required edge labels of $G = \{3, \ldots, \ell + 2, \ell + 4, \ldots, 2\ell + 3, 2\ell + 5, \ldots, 2\ell + m + 4, 2\ell + m + 6, \ldots, 4\ell + 2m + 6\}$ is $\{(\ell + 2) - 3 + 1 + (2\ell + 3) - (\ell + 4) + 1 + (2\ell + m + 4) - (2\ell + 5) + 1(4\ell + 2m + 6) - (2\ell + m + 6) + 1\} = 4\ell + 2m + 1$ distinct edges. Hence the induced edge labels of G are distinct. Hence the graph G is a relaxed skolem mean $|m - n| \leq 2\ell + 6$ for $\ell = 2, 3, 4, \ldots$; $m = 2, 3, 4, \ldots$. when $n = 2\ell + m + 1$.

Case (g): Consider the graph $G = K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ where $n = 2\ell + m$. We have to prove that G is a relaxed skolem mean graph. We have $V(G) = \{u, v, w, x\} \cup \{u_i : 1 \leq i \leq \ell\} \cup \{v_j : 1 \leq j \leq \ell\} \cup \{w_k : 1 \leq k \leq m\} \cup \{x_h : 1 \leq h \leq n\}$ and $E(G) = \{uu_i : 1 \leq i \leq \ell\} \cup \{vv_j : 1 \leq j \leq \ell\} \cup \{ww_k : 1 \leq k \leq m\} \cup \{xx_h : 1 \leq h \leq n\}$. Then G has $2\ell + m + n + 4$ vertices and $2\ell + m + n$ edges. The required vertex labeling $f : V(G) \to \{1, 2, 3, 4, \ldots, 2\ell + m + n + 4\}$ is defined as follows:

$$f(u) = 1; \quad f(v) = 3; \quad f(w) = 5; \quad f(x) = 2\ell + m + n + 4;$$
$$f(u_i) = 2i + 2 \qquad for \quad 1 \leq i \leq \ell$$
$$f(v_j) = 2\ell + 2j + 2 \qquad for \quad 1 \leq j \leq \ell$$
$$f(w_k) = 4\ell + 2k + 2 \quad for \quad 1 \leq k \leq m$$
$$f(x_h) = 2h + 5 \qquad for \quad 1 \leq h \leq n.$$

The corresponding edge labels are as follows: The edge label of uu_i is $i + 2$ for $1 \leq i \leq \ell$; vv_j is $\ell + j + 3$ for $1 \leq j \leq \ell$; ww_k is $2\ell + k + 4$ for $1 \leq k \leq m$ and xx_h is $\dfrac{2h + 2\ell + m + n + 9}{2}$ for $1 \leq h \leq n$. Therefore, the required edge labels of $G = \{3, \ldots, \ell + 2, \ell + 4, \ldots, 2\ell + 3, 2\ell + 5, \ldots, 2\ell + m + 4, 2\ell + m + 6, \ldots, 4\ell + 2m + 5\}$ is $\{(\ell + 2) - 3 + 1 + (2\ell + 3) - (\ell + 4) + 1 + (2\ell + m + 4) - (2\ell + 5) + 1 + (4\ell + 2m + 5) - (2\ell + m + 6) + 1\} = 4\ell + 2m$ distinct edges. Hence the induced edge labels of G are distinct. Hence the graph G is a relaxed skolem mean graph if $|m - n| \leq 2\ell + 6$ for $\ell = 2, 3, 4, \ldots$; $m = 2, 3, 4, \ldots$ when $n = 2\ell + m$. □

3. Application of Graph Labeling in Communication Networks

The Graph Theory plays a vital role in various fields. One of the important area is Graph (Relaxed Skolem mean) Labeling, used in many applications like coding theory, X - ray crystallography, radar, astronomy, circuit design, communication network addressing and data base management. Applications of labeling (Relaxed Skolem Mean) of graphs extends to heterogeneous fields but here we mainly focus on the communication networks. Communication network is of two types 'Wired Communication' and 'Wireless Communication'. Day by day wireless networks have been developed to **ease** communication between any two systems, results more efficient communication. To explore the role of labeling in expanding the utility of this channel assignment process in communication networks. Also, graph labeling has been observed and identified its usage towards communication networks. We address how the concept of graph labeling can be applied to network security, network addressing, channel assignment process and social networks. Network representations play an important role in many domains of computer science, ranging from data structures and graph algorithms, to parallel and communication networks. Geometric representation of the graph structure imposed on these data sets provides a powerful aid to visualizing and understanding the data. The graph labeling is one of the most widely used labeling methods of graphs. While the labeling of graphs is perceived to be a primarily theoretical subject in the field of graph theory and discrete mathematics, it serves as models in a wide range of applications as listed below.

- The coding theory.
- The x-ray crystallography.
- The communication network addressing.
- Fast Communication in Sensor Networks Using Graph Labeling.
- Automatic Channel Allocation for Small Wireless Local Area Network.
- Graph Labeling in Communication Relevant to Adhoc Networks.
- Effective Communication in Social Networks by Using Graphs.
- Secure Communication in Graphs.

4. Conclusion

Researchers may get some information related to graph labeling and its applications in communication field and work on some ideas related to their field of research.

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