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Some Results For Semi Derivations of Prime Near Rings

Research Article

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Abstract: Let R be a prime near ring. By a semi-derivation associated with a function $g : R \to R$ we mean an additive mapping $f : R \to R$ such that $f(xy) = f(x)g(y) + xf(y) = f(x)y + g(x)f(y)$ and $f(g(x)) = g(f(x))$ for all $x, y \in R$. In this paper we try to generalize some properties of prime rings with derivations to the prime near rings with semi-derivations.

MSC: 16BXX.

Keywords: Semi derivation, Prime ring, Commuting mapping, Characteristic of a ring.

1. Introduction

In [1], I.S. Chang, K.W. Jun and Y.S. Jung prove that if there exist a derivation D on a non-commutative 2-torsion free prime ring R such that the mapping $x \to [aD(x), x]$ is commuting on R, then $a = 0$ or $D = 0$. We proved that this conclusion for semi-derivations of prime near rings as follows in Theorem 2.1. In [2], K. Kaya, O.Golbasi, N. Aydin proved that if R is a prime ring of characteristic different from 2, D is a nonzero derivation of R, then $(D(R), a) = 0$, if and only if, $((R, a)) = 0$. We also proved that this conclusion for semi- derivations of prime near rings as follows in Theorem 2.2.

1.1. Left Near Ring

A Left near ring is a set R with two operations $+$ and $\cdot$ such that $(R, +)$ is a group and $(R, \cdot)$ is a semi group satisfying the left distributive law: $x(y + z) = xy + xz$ for all x, y, z in R.

1.2. Derivation

An additive mapping D from R to R is called a derivation if $D(xy) = D(x)y + xD(y)$ holds for all $x, y \in R$.

1.3. Semi-Derivation

Let R be an associative ring. An additive mapping $f : R \to R$ is called a semi-derivation associated with a function $g : R \to R$ if, for all $x, y \in R$:

(1). $f(xy) = f(x)g(y) + xf(y) = f(x)y + g(x)f(y)$;

(2). $f(g(x)) = g(f(x))$.

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1.4. Prime

A ring R is prime if $aRb = 0$ implies that $a = 0$ or $b = 0$.

1.5. Center

Let R be a ring with center $C(R)$. For any $x, y \in R; [x, y], (x, y)$ will denote$xyyx, xy + yx$ respectively.

1.6. Characteristic of a Ring

A ring R is to be n torsion free if $nx = 0, x \in R$ implies $x = 0$. A mapping $F : R \to R$ is said to be commuting on R if $[F(x), x] = 0$ holds for all $x \in R$, and is said to be centralizing on R if $[F(x), x] \in C(R)$ holds for all $x \in R$. We make extensive use of the basic commutator identities: $(xy, z) = (x, z)y + x[y, z] = [x, z]y + x(y, z), [xy, z] = [x, z]y + x[y, z]$.

2. Main Section

Theorem 2.1. *Let R be a non-commutative 2-torsion free prime near ring and f is a semi-derivation of R with $g : R \to R$ is an onto endomorphism. If the mapping $x \to [af(x), x]$ is commuting on R, then $a = 0$ $orf = 0$.*

Proof. we assume that a be a non zero element of R. Then by [[2], Theorem 1], the mapping $x \to af(x)$ is commuting on R. Thus we have

$$[af(x), x] = 0, \tag{1}$$

for all $x \in R$. By linearizing (1), we have

$$[af(x), y] + [af(y), x] = 0 \tag{2}$$

for all $x, y \in R$. From this relation it follows that

$$a[f(x), y] + [a, y]f(x) + a[f(y), x] + [a, x]f(y) = 0, \tag{3}$$

for all $x, y \in R$. Replacing y by yx in (2) and using (1), we get

$$\begin{aligned} &= a[f(x), y]x + [a, y]f(x)x + a[f(y), x]x + [a, x]f(y)x + ag(y)[f(x), x] \\ &= a[g(y), x]f(x) + [a, x]g(y)f(x) \end{aligned} \tag{4}$$

for all $x, y \in R$. Right multiplication of (3) by x gives

$$a[f(x), y]x + [a, y]f(x)x + a[f(y), x]x + [a, x]f(y)x = 0 \tag{5}$$

for all $x, y \in R$. Subtracting (5) from (4), we obtain

$$ag(y)[f(x), x] + a[g(y), x]f(x) + [a, x]g(y)f(x) = 0 \tag{6}$$

for all $x, y \in R$. Taking $ag(y)$ instead of $g(y)$ in (6), we have

$$0 = a^2g(y)[f(x), x] + a^2[g(y), x]f(x) + a[a, x]g(y)f(x) + [a, x]ag(y)f(x) \tag{7}$$

for all $x, y \in R$. Left multiplication of (6) by a leads to

$$a^2 g(y)[f(x), x] + a^2[g(y), x]f(x) + a[a, x]g(y)f(x) = 0 \tag{8}$$

for all $x, y \in R$. Subtracting (8) from (7), we get

$$[a, x]ag(y)f(x) = 0 \tag{9}$$

for all $x, y \in R$. Since R is prime, we obtain that for any $x \in R$ either $f(x) = 0$ or $[a, x] = 0$. It means that R is the union of its additive subgroups $P = x \in R : f(x) = 0$ and $Q = x \in R : [a, x]a = 0$. Since a group cannot be the union of two proper subgroups, we find that either $P = R$ or $Q = R$. If $P = R$, then $f = 0$. If $Q = R$, then this implies that $[a, x]a = 0$, for all $x \in R$. Let us take xy instead of x in this relation. Then we get $[a, x]ya = 0$, for all $x \in R$. Since $a \in R$ is nonzero and R is prime, we obtain $a \in C(R)$. Thus by this and (1), the relation (6) reduces to $a[g(y), x]f(x) = 0$, for all $x, y \in R$. Since g is onto, we see that $az[u, x]f(x) = 0$, for all $x, u, z \in R$. Now by primeness of R, we obtain that $[u, x]f(x) = 0$, for all $x, u \in R$. Replacing u by uw, we get $[u, x]wf(x) = 0$, for all $x, u, w \in R$. Again using the fact that a group cannot be the union of two proper sub-groups, it follows that $f = 0$, since R is noncommutative. Hence we see that, in any case, $f = 0$. This completes the proof. □

Theorem 2.2. *Let R be a prime near ring of characteristic different from 2, f is a nonzero semi-derivation of R, with associated endomorphism g and $a \in R$. If $g \mp I$ (I is an identity map of R), then $(f(R), a) = 0$ if, and only, if $f((R, a)) = 0$.*

Proof. Suppose $(f(R), a) = 0$. Firstly, we will prove that $f(a) = 0$. If $a = 0$ then $f(a) = 0$. So we assume that $a \in 0$. By our hypothesis, we have $(f(x), a) = 0$, for all $x \in R$. From this relation, we get

$$= (f(xa), a) = (f(x)g(a) + xf(a), a)$$
$$= f(x)[g(a), a] + (f(x), a)a + x(f(a), a)[x, a],$$

and so,

$$[x, a]f(a) = 0 \tag{10}$$

for all $x \in R$.cNow, replacing x by xy in (9), we get

$$[R, a]Rf(a) = (0) \tag{11}$$

The primeness of R implies that $a \in C(R)$ or $f(a) = 0$. Now suppose that $a \in C(R)$. Then we obtain that

$$0 = (f(a), a) = f(a)a + af(a) = 2af(a).$$

Since the characteristic of R is different from 2, $af(a) = 0$. Since we assumed that $0 \in a$ and R is prime ring, we get $f(a) = 0$. Hence we have

$$f((r, a)) = f(ra + ar) = (f(r), a) + (g(r), f(a)),$$

for all $x \in R$. This yields that $f((R, a)) = 0$.

Conversely, for all $x \in R$,

$$= f((ax, a)) = f(a(x, a) + [a, a]x)$$
$$= f(a(x, a)) = f(a)(x, a) + g(a)f((x), a).$$

By hypothesis we have

$$f(a)(x,a)=0 \tag{12}$$

for all $x\in R$. Replacing x by xy in (11), we get

$$0=f(a)(xy,a)=f(a)x[y,a]+f(a)(x,a)y=f(a)x[y,a].$$

This implies that $f(a)R[R,a]=0$. For the primeness of R, we have either $f(a)=0$ or $a\in C(R)$. If $f(a)=0$, then we have

$$0=f((r),a)=(f(r),a)+(f(a),g(r))=(f(r),a),$$

for all $r\in R$. This yields that $(f(R),a)=0$. If $a\in C(R)$, then we have

$$0=f((a,a))=2f(a)(a+g(a)).$$

Since the characteristic of R is different from 2, we obtain $f(a)(a+g(a))=0$. Since R is prime we have $f(a)=0$ or $a+g(a)=0$. But since g is different from $\mp I$ we find that $f(a)=0$. Finally, $(f(R),a)=0$ implies the required result. □

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