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A New Approach For Solving All Integer Linear Fractional Programming Problem

Research Article

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Abstract: In this paper we present a new approach for solving all integer linear fractional programming problem (ILFPP) in which the objective function is a linear fractional function, and where the constraint functions are in the form of linear inequalities. The approach adopted is based mainly upon simplex method as well as dual simplex method. A simple example is given to clarify the theory of this new approach.

Keywords: Fractional programming, Simplex method, Dual simplex method, Gomorys fractional cut method.

1. Introduction

Linear fractional programming problem (i.e. ratio objective that have numerator and denominator) have attracted considerable research and interest, since they are useful in production planning, financial and corporate planning, health care and hospital planning. Several methods to solve this problem are proposed (1962), Charnes and Kooper [1] have proposed their method depends on transforming this (LFP) to an equivalent linear program, and in 1965 Swarup [3] developed a simplex technique for the same problem. J.K. Sharma, A.K. Gupta at all [2] solves the linear fractional functional programming problem by the simplex method in which three basic variables are replaced by three non basic variables at a time. R.E. Gomory [5] fined an algorithm for integer solution to linear programs and in 1960 [4] he solved the integer linear programming problem in which an integer solution of linear programming problem is obtained. Recently S.C. Sharma and Abha Bansal [6] solve an integer fractional programming problem. Here our task is to find the optimal integer solution of linear fractional programming problem. For it, we use simplex method, dual simplex method and gomorys fractional cut method. In Section 2 preliminaries are given while in Section 3 we give the steps of the proposed algorithms. Section 4 provides a numerical example and finally in section 5 we present the references.

2. Preliminaries

A maximization integer linear fractional programming problem may be stated as:

$$\max\ Z = \frac{(\boldsymbol{c}^T\boldsymbol{X} + \alpha)}{(\boldsymbol{d}^T\boldsymbol{X} + \beta)}$$

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Subject to:

$$AX \leq b$$

$$X \geq 0 \text{ and an integer}$$

Where X, c and d are $n \times 1$ vectors, b is an $m \times 1$ vector, $\boldsymbol{c}^T$, $\boldsymbol{d}^T$ denote transpose of vectors, A is an $m \times n$ matrix and α, β are scalars. It is assumed that the constraint $S = \{x : AX \leq b, X \geq 0 \text{ and integer}\}$ is non empty and bounded.

3. Algorithms For Solving AILFPP

3.1. Simplex Algorithm

Step: 1 First, we observe whether all the right side constants of the constraints are non-negative. If not, it can be changed into positive value on multiplying both the sides of the constraints by -1.

Step: 2 Next convert the inequality constraints to equality by introducing the non negative slack or surplus variables. The coefficients of slack or surplus variables are always taken zero in the objective function.

Step: 3 Construct the simplex table by using the following notations. Let X_B be the initial basic feasible solution of the given problem such that $BX_B = b$

$$X_B = bB^{-1}$$

Where $B = (b_1, b_2, b_3, \ldots, b_r, b_s, \ldots, b_m)$. Further suppose that

$$Z_1 = \boldsymbol{c}^T{}_B X_B + \alpha$$

$$Z_2 = \boldsymbol{d}^T{}_B X_B + \beta$$

Where $\boldsymbol{c}^T{}_B$ and $\boldsymbol{d}^T{}_B$ are the vectors having their components as the coefficients associated with the basic variables n the numerator and denominator of the objective function respectively.

Step: 4 Now, compute the net evaluation Δ_j for each variable X_j (column vector X_j) by the formula

$$\Delta_j = Z_2\left(Z^{(1)}{}_j - c_j\right) - Z_1(Z^{(2)}{}_j - d_j)$$

Step: 5 If all $\Delta_j \geq 0$, the optimal solution is obtained, and

Step: 6 If optimal solution is an integer solution then we get the required solution otherwise use Gomory's fractional cut method.

3.2. Gomory's Fractional Cut Algorithm

Step: 1 Choose the largest fractional value of the basic variables. Let it be f_{k0}.

Step: 2 Express the negative fraction, if any, in the k^{th} row of the optimum simplex table as the sum of a negative integer and a non-negative fraction.

Step: 3 Generate the Gomorian constraint (fractional cut) in the form

$$G_1 = -f_{k0} + f_{k1}x_1 + f_{k2}x_2 + \cdots + f_{kn}x_n,$$

Where $0 = f_{kj} < 1$ and $0 < f_{k0} < 1$.

Step: 4 Add the Gomorain constraint generated in step III at the bottom of the optimum simplex table. Use dual simplex method to find an improved optimum solution.

3.3. Dual Simplex algorithm

Step: 1 First we convert the problem into maximization if it is initially in the minimization form.

Step: 2 Next we convert $\geq$ type constraints, if any, into $\leq$ type by multiplying both sides of such constraints by -1.

Step: 3 Then convert the inequality constraints into equalities by addition of slack variables and obtain the initial solution.

Step: 4 Construct the dual simplex table by using the above information.

Step: 5 Now, compute the net evaluation Δ_j for each X_j (column vector X_j) by the formula

$$\Delta_j = Z_2\left(Z^{(1)}{}_j - c_j\right) - Z_1(Z^{(2)}{}_j - d_j).$$

Step: 6 If all $\Delta_j \geq 0$ and also $X_{Bi} \geq 0$, the optimal solution is obtained.

4. Numerical Example

Example 4.1. *Find the integer solution of following linear fractional programming problem.*

$$\max \quad Z = \frac{2x_1 + x_2}{3x_1 + x_2 + 6}$$

Subject to:

$$5x_1 + 3x_2 \leq 6$$

$$7x_1 + x_2 \leq 6$$

$$x_1,\ x_2 \geq 0 \quad \textit{and an integer.}$$

			c_j	2	1	0	0	Min. ratio
			d_j	3	1	0	0	x_{ij}/y_{ij}
d_B	c_B	X_B	B	x_1	x_2	x_3	x_4	
0 0	0 0	x_3 x_4	6 6	5 7	3 1	1 0	0 1	6/5 6/7
$Z_2 = 6$	Z_1=0	Z = 0	$Z^{(1)}{}_j - c_j$	-2	-1	0	0	
			$Z^{(2)}{}_j - d_j$	-3	-1	0	0	
			Δ_j	-12 $\uparrow$	-6	0	0 $\downarrow$	
0 3	0 2	x_3 x_1	12/7 6/7	0 1	16/7 1/7	1 0	-5/7 1/7	3/4 6
$Z_2 = 60/7$	Z_1=12/7	Z =1/5	$Z^{(1)}{}_j - c_j$	0	-5/7	0	2/7	
			$Z^{(2)}{}_j - d_j$	0	-4/7	0	3/7	
			Δ_j	0	-252/49 $\uparrow$	0 $\downarrow$	12/7	
1 3	1 2	x_2 x_1	3/4 3/4	0 1	1 0	7/16 -1/16	-5/16 3/16	
$Z_2 = 9$	Z_1=9/4	Z =1/4	$Z^{(1)}{}_j - c_j$	0	0	5/16	1/16	
			$Z^{(2)}{}_j - d_j$	0	0	1/4	1/4	
			Δ_j	0	0	9/4	0	

Table 1.

Here $x_1 = \frac{3}{4}$, $x_2 = \frac{3}{4}$ *and* $Z = \frac{Z_1}{Z_2} = \frac{1}{4}$. *Since all* $\Delta_j \geq 0$, *therefore the current solution is the optimal basic feasible solution. But* x_1 *and* x_2 *are not integer value so make them integer, we use Gomory's fractional cut method.*

			c_j	2	1	0	0	0
			d_j	3	1	0	0	0
d_B	c_B	X_B	B	x_1	x_2	x_3	x_4	G_1
1	1	x_2	3/4	0	1	7/16	-5/16	0
3	2	x_1	3/4	1	0	-1/16	3/16	0
0	0	G_1	-3/4	0	0	-15/16	-3/16	1
$Z_2 =$ 9	Z_1=9/4	Z =1/4	$Z^{(1)}{}_j - c_j$	0	0	5/16	1/16	0
			$Z^{(2)}{}_j - d_j$	0	0	1/4	1/4	0
			Δ_j	0	0	9/4 $\uparrow$	0	0 $\downarrow$
1	1	x_2	2/5	0	1	0	-2/5	7/15
3	2	x_1	4/5	1	0	0	1/5	-1/15
0	0	x_3	4/5	0	0	1	1/5	-16/15
$Z_2 =$ 44/5	Z_1=2	Z =5/22	$Z^{(1)}{}_j - c_j$	0	0	0	0	1/3
			$Z^{(2)}{}_j - d_j$	0	0	0	1/5	4/15
			Δ_j	0	0	0 $\downarrow$	-2/5 $\uparrow$	388/15
1	1	x_2	2	0	1	2	0	-2/15
3	2	x_1	0	1	0	-1	0	1
0	0	x_4	4	0	0	5	1	-16/3
$Z_2 =$ 8	Z_1=2	Z =1/4	$Z^{(1)}{}_j - c_j$	0	0	0	0	28/15
			$Z^{(2)}{}_j - d_j$	0	0	-1	0	43/15
			Δ_j	0	0	2	0	128/15

Table 2.

Hence we get the integer solution of the given problem. The optimal solution is $x_1 = 0$, $x_2 = 2$ *and* max $Z = \frac{1}{4}$.

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