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On Equitable Total Minimal Dominating Signed Graphs

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Abstract: In this paper we introduced the new notion called equitable total minimal dominating signed graph of a signed and its properties are studied. Also, we obtained the structural characterization of this new notion and presented some switching equivalent characterizations.

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Keywords: Signed graphs, Balance, Switching, equivalent total minimal domination inating signed graph.

1. Introduction

Oystein Ore a well known lattice theorist, introduced the concept of domination in graphs in his famous book 'Theory of Graphs' published in 1962 and this concept lived almost in hybernation until 1975, when E. J. Cockayne and S. T. Hedetniemi unfolded its diverse aspects, by surveying all the available results, bringing to light new ideas and citing its application potential in a variety of scientific areas in their paper 'Towards a Theory of Domination in Graphs' which appeared later on in 'Networks' in 1977. Earlier to their joint venture, it was only Claude Berge's pioneering book 'Graphs and Hypergraphs' in 1973 which included a bound on the domination number of a graph, Vizing's bound on the size of a graph with given order and domination number and very interesting applications of the idea of domination to surveillance networks and to Game Theory. Before it was shaped to become a theory in itself, it had acquired several disguised forms under the heading 'graph coverings' in a number of research papers. Today, Ore's concept of domination in graphs has indeed become an independent theory.In mathematics, this concept deals with various fields such as topological indices [9–13, 13–18]. In view of this great history on domination theory, we make an attempt to study the equitable total minimal dominating signed graph and proved some results.

If each vertex $u_1 \in V - \mathcal{D}$ there exists a vertex $u_2 \in \mathcal{D}$ such that u_1u_2 in an edge in G and $|deg(u_1) - deg(u_2)| \leq 1$, where $D \subseteq V$, then $\mathcal{D}$ is said to be an equitable total dominating set of G (See [8]).

If $\mathcal{D}$ is an equitable total dominating set and its induced subgraph $< \mathcal{D} >$ has no isolated vertices, then $\mathcal{D}$ is said to be an equitable total dominating set of G. Let $G = (V, E)$ and its equitable total minimal dominating (e.t.m.d) graph is denoted by $\mathcal{ETMD}(G)$ is a graph whose vertices are minimal equitable total dominating sets of a graph G. Any two vertices D_1 and D_2 in $\mathcal{ETMD}(G)$ are adjacent, if $D_1 \cap D_2 \neq 0$ (See [2]). The following figure represents G and its $\mathcal{ETMD}(G)$.

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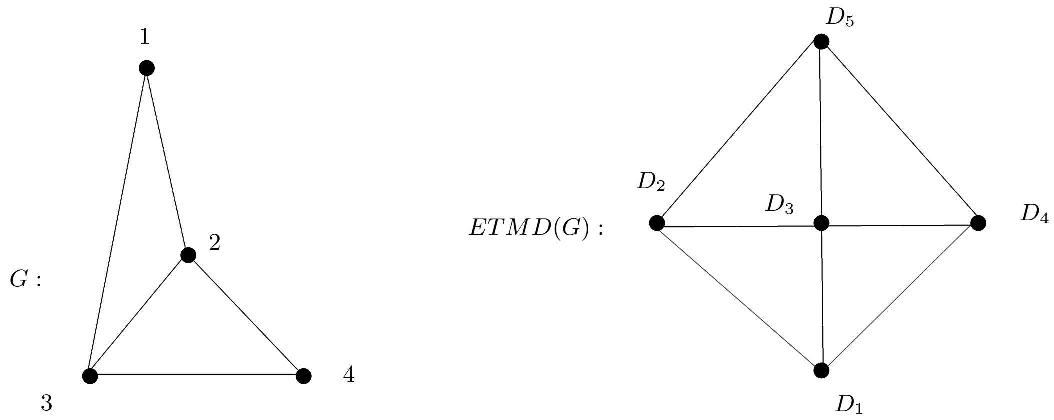

Figure 6.1 : *graph G and its ETMD of G*

2. Equitable Total Minimal Dominating Signed Graph of a Signed Graph

By the motivation of e.t.m.d of a graph defined in the above, in this section we defined the new notion called equitable total minimal dominating signed graph (e.t.m.d.s) of a signed graph as: the e.t.m.d.s $\mathcal{ETMD}(S) = (\mathcal{ETMD}(G), \sigma')$ of $S = (G, \sigma)$ is a signed graph, the labeling of any edge $pq \in E(\mathcal{ETMD}(S))$ is the product of canonical marking of the vertices p and q. If any signed graph S is isomorphic to e.t.m.d.s of some signed S' (i.e., $\mathcal{ETMD}(S) \cong S'$), then S is called an e.t.m.d.s.
In general signed graphs can be partitioned into groups as: positive signed graphs (i.e., balanced signed graphs) and negative signed graphs (i.e., unbalanced signed graphs). Given signed graph $S = (G, \sigma)$ is either positive or negative, the e.t.m.d.s is always positive.

Theorem 2.1. *The e.t.m.d.s $\mathcal{ETMD}(S)$ is positive, for any $S = (G, \sigma)$.*

Proof. Let $S = (G, \sigma)$ be any signed graph and S_ζ is a signed marked graph subsequently employ the canonical marking. Through the elucidation of e.t.m.d.s $\mathcal{ETMD}(S)$, we examined in order that the marking of each line $e = pq$ in $\mathcal{ETMD}(S)$ is $\sigma(pq) = \zeta(p)\zeta(q)$. From Theorem 1.2.2, e.t.m.d.s $\mathcal{ETMD}(S)$ is balanced. □

Consider the $\mathbb{Z}^+$ and $k \in \mathbb{Z}^+$, the k^{th} iterated e.t.m.d.s $\mathcal{ETMD}(S)$ of S is defined as follows:

$$\mathcal{ETMD}^0(S) = S,\ \mathcal{ETMD}^k(S) = \mathcal{ETMD}(\mathcal{ETMD}^{k-1}(S)).$$

Corollary 2.2. *The k^{th} iterated e.t.m.d.s $\mathcal{ETMD}^k(S)$ is always positive, for any $S = (G, \sigma)$.*

Theorem 2.3. *The equitable total minimal dominating signed graphs of $S_1 = (G_1, \sigma)$ and $S_2 = (G_2, \sigma)$ are switching equivalent (i.e., $\mathcal{ETMD}(S_1) \sim \mathcal{ETMD}(S_2)$), if G_1 and G_2 are isomorphic.*

Proof. Consider two signed graphs S_1 and S_2 with $G_1 \cong G_2$. Thereupon, the corresponding equitable total minimal dominating signed graphs $\mathcal{ETMD}(S_1)$ and $\mathcal{ETMD}(S_2)$ are positive. From Theorem 1.2.3, it follows that $\mathcal{ETMD}(S_1)$ and $\mathcal{ETMD}(S_2)$ are switching equivalent. □

In [2], the authors remarked that $\mathcal{ETMD}(G)$ and G are isomorphic iff G is C_3 or C_4. We now characterize the signed graphs such that the e.t.m.d.s and its corresponding signed graph are cycle isomorphic.

Theorem 2.4. *For any $S = (G, \sigma)$, the e.t.m.d.s $\mathcal{ETMD}(S)$ and S are cycle isomorphic if and only if the underlying of S is isomorphic to C_3 or C_4 and S is balanced.*

Proof. Consider S is positive and its underlying graph G is either C_3 or C_4. Then, G and $\mathcal{ETMD}(G)$ are isomorphic. Now the e.t.m.d.s $\mathcal{ETMD}(S)$ of a signed graph S with underlying graph is either C_3 or or C_4, is positive. From the hypothesis, S is positive and just now we have seen that $\mathcal{ETMD}(S)$ is also positive and hence S and $\mathcal{ETMD}(S)$ are cycle isomorphic, from the Theorem 1.2.3.

Conversely, suppose that signed graph and its e.t.m.d.s are cycle isomorphic. Then $G \cong \mathcal{ETMD}(G)$. Therefore G is either C_3 or C_4. Since $\mathcal{ETMD}(S)$ and S are cycle isomorphic. This satisfies only when S is positive. □

We now give the structural characterization of equitable total minimal dominating signed graphs.

Theorem 2.5. *Suppose $S = (G, \sigma)$ be any signed graph. Then S is positive and its underlying graph is equitable total minimal dominating graph if and only if S is an e.t.m.d.s $\mathcal{ETMD}(S)$.*

Proof. Let us consider that S is an e.t.m.d.s $\mathcal{ETMD}(S)$. Then the signed graph S and the e.t.m.d.s of some signed graph S_1 (i.e., $\mathcal{ETMD}(S_1)$) are isomorphic. Since, the e.t.m.d.s of any signed graph is positive and we have $S \cong \mathcal{ETMD}(S_1)$. Consequently, S is positive and its underlying graph is an e.t.m.d.g.

Conversely, suppose that S is positive and its underlying graph is an e.t.m.d. Since, the signed graph S is positive, then establish the S_ζ. With the evidence of Sampathkumar's result (Theorem 1.2.2), every edge pq in S_ζ amuse $\sigma(pq) = \zeta(p)\zeta(q)$. Deliberate, the signed graph $\Sigma_1 = (G_1, \sigma_1)$ in which each edge $e = (pq)$ in G_1, $\sigma_1(e) = \zeta(p)\zeta(q)$. Therefore, the signed graph S and the e.t.m.d.s of S_1 are isomorphic. Hence, S is a c.m.n.s $\mathcal{ETMD}(S)$. □

Behzad and Chartrand [3] introduced the notion of line signed graph $L(S)$ of a given signed graph S as follows: Given a signed graph $S = (G, \sigma)$ its *line signed graph* $L(S) = (L(G), \sigma')$ is the signed graph whose underlying graph is $L(G)$, the line graph of G, where for any edge e_ie_j in $L(S)$, $\sigma'(e_ie_j)$ is negative if, and only if, both e_i and e_j are adjacent negative edges in S. Another notion of line signed graph introduced in [4], is as follows: The *line signed graph* of a signed graph $S = (G, \sigma)$ is a signed graph $L(S) = (L(G), \sigma')$, where for any edge ee' in $L(S)$, $\sigma'(ee') = \sigma(e)\sigma(e')$. In this paper, we follow the notion of line signed graph defined by M. K. Gill [4] (See also E. Sampathkumar et al. [6, 7]).

Theorem 2.6. (**M. Acharya** [1]) *For any signed graph $S = (G, \sigma)$, its line signed graph $L(S) = (L(G), \sigma')$ is balanced.*

In [2], the authors remarked that $\mathcal{ETMD}(G) \cong L(G)$ if and only if G is a $(p-2)$ regular. We now characterize the signed graphs such that $\mathcal{ETMD}(S)$ and $L(S)$ are cycle isomorphic.

Theorem 2.7. *For any $S = (G, \sigma)$, $\mathcal{ETMD}(S)$ and $L(S)$ are cycle isomorphic if and only if the underlying graph of S is a $(p-2)$ regular.*

Proof. Suppose that $\mathcal{ETMD}(S)$ and $L(S)$ are cycle isomorphic. Then, $\mathcal{ETMD}(G)$ and $L(G)$ are isomorphic. Then G is a $(p-2)$ regular.

Conversely, suppose that S is any signed graph whose underlying graph G is a $(p-2)$ regular. Then, $\mathcal{ETMD}(G)$ and $L(G)$ are isomorphic. Since for any signed graph S, both $\mathcal{ETMD}(S)$ and $L(S)$ are positive, the result follows by Theorem 1.2.3. □

The concept negation of a signed graph introduced by Harary [5] as follows: Consider a signed graph $S = (G, \sigma)$, the negation of S is denoted by $\eta(S)$ and the underlying graph of S and $\eta(S)$ are isomorphic. Further, the marking of each line $e = pq$ in $\eta(S)$ is $+$ $(-)$, if the marking of the line $e = pq$ in S is $-$ $(+)$.

In view of the negation operator introduced by Harary [5], we have the following cycle isomorphic characterizations:

Corollary 2.8. *The negation of equitable total minimal dominating signed graphs of $S_1 = (G_1, \sigma)$ and $S_2 = (G_2, \sigma)$ are cycle isomorphic (i.e., $\eta(\mathcal{ETMD}(S_1)) \sim \eta(\mathcal{ETMD}(S_2))$), if G_1 and G_2 are isomorphic.*

Corollary 2.9. *For any two signed graphs $S_1 = (G_1, \sigma)$ and $S_2 = (G_2, \sigma)$, $\mathcal{ETMD}(\eta(S_1))$ and $\mathcal{ETMD}(\eta(S_2))$ are cycle isomorphic, if G_1 and G_2 are isomorphic.*

Corollary 2.10. *For any $S = (G, \sigma)$, the e.t.m.d.s $\mathcal{ETMD}(\eta(S))$ and S are cycle isomorphic if and only if the underlying of S is isomorphic to C_3 or C_4 and S is positive.*

Corollary 2.11. *For any $S = (G, \sigma)$, the e.t.m.d.s $\mathcal{ETMD}(S)$ and $\eta(S)$ are cycle isomorphic if and only if the underlying of S is isomorphic to C_4 and S is positive.*

Corollary 2.12. *For any $S = (G, \sigma)$, $\mathcal{ETMD}(\eta(S))$ and $L(S)$ are cycle isomorphic if and only if the underlying graph of S is a $(p-2)$ regular.*

Corollary 2.13. *For any $S = (G, \sigma)$, $\mathcal{ETMD}(S)$ and $L(\eta(S))$ are cycle isomorphic if and only if the underlying graph of S is a $(p-2)$ regular.*

Corollary 2.14. *For any $S = (G, \sigma)$, $\mathcal{ETMD}(\eta(S))$ and $L(\eta(S))$ are cycle isomorphic if and only if the underlying graph of S is a $(p-2)$ regular.*

We have observed that, the signed graph is either positive or negative but the e.t.m.d.s of one such signed graph is always positive. Using the concept negation in signed graphs introduced by Harary [5], we have the following result to $\mathcal{ETMD}(S)$.

Theorem 2.15. *Suppose the e.t.m.d.g $\mathcal{ETMD}(G)$ is bipartite. Then $\eta(\mathcal{ETMD}(S))$ is positive, where S is any signed graph.*

Proof. Since, by Theorem 6.2.1, $\mathcal{ETMD}(S)$ is positive. Then all the cycles in $\mathcal{ETMD}(S)$ is positive. By the hypothesis, the $\mathcal{ETMD}(G)$ is bipartite. Then each cycle C_n (where n is even) in $\mathcal{ETMD}(S)$ is positive. Therefore, $\eta(\mathcal{ETMD}(S))$ is positive. □

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