International Journal of Mathematics And its Applications

Volume 4, Issue 1–C (2016), 93–98.

ISSN: 2347-1557

Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Super Root Square Mean Labeling Of Disconnected Graphs

Research Article

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Abstract: Let G be a graph with p vertices and q edges. Let $f : V(G) \to \{1, 2, 3, \ldots, p+q\}$ be an injective function. For a vertex labeling f, the induced edge labeling $f^*(e = uv)$ is defined by $f^*(e) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$ then f is called a super root square mean if $f(v(G)) \bigcup \{f(e)/e \in E(G)\} = \{1, 2, 3, \ldots, p+q\}$. A graph which admits super root square mean labeling is called super root square mean graph. In this paper, we investigate super root square mean labeling of disconnected graphs.

MSC: 05C78.

Keywords: Super root square mean, $P_n \bigcup P_m$, $P_n \bigcup P_m^+$, $P_n^+ \bigcup P_m^+$.

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [7]. The symbols V(G) and E(G) will denote the vertex set and edge set of a graph. Graph labeling were first introduced in the late 1960's. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions.

If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling). For a detail survey of graph labeling we refer to Gallian [1]. Mean labeling of graphs was discussed in [3, 8], the concept of k-even mean and (k, d)-even mean labeling are introduced and discussed in [4, 5], the concept of root square mean labeling was introduced and discussed in [9] and the concept of super root square mean labeling was introduced and discussed in [10].

In this paper, we investigate super root square mean labeling of disconnected graphs.

2. Main Result

Theorem 2.1. *The path $P_n \bigcup P_m$ $(m, n \geq 3)$ is a super root square mean graph.*

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Proof. Let $\{v_i, v_i', 1 \le i \le n\}$ be the vertices and $\{e_i, e_i', 1 \le i \le n-1\}$ be the edges which are denoted as in Fig. 1

Figure 1. **Ordinary labeling of $P_n \bigcup P_m$**

First we label that vertices as follows: Define f:V→ {1,2,...,p+q} by for $1 \le i \le n$, $f(v_i) = 2i-1$, $f(v_i') = 2(i-1) + 2n$. Then the induced edge labels are: For $1 \le i \le n-1$, $f^*(e_i) = 2i$. For $1 \le i \le n-1$, $f^*(e_i') = 2(n+i) - 1$. Thus the vertices and edges together get distinct labels. Hence the graph $P_n \bigcup P_m\,(m, n \ge 3)$ is a super root square mean graph. Super root square mean labeling of $P_3 \bigcup P_7$ and $P_4 \bigcup P_6$ is given in Fig. 2 and Fig. 3 respectively.

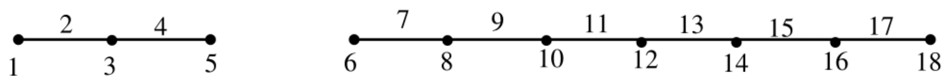

Figure 2. **Super root square mean labeling of $P_3 \bigcup P_7$**

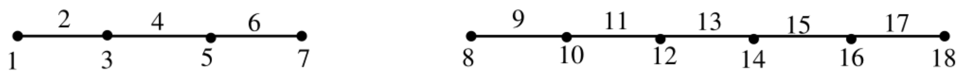

Figure 3. **Super root square mean labeling of $P_4 \bigcup P_6$**

□

Theorem 2.2. *The graph in $P_n \bigcup P_m^+\,(m, n \ge 3)$ is a super root square mean graph.*

Proof. Let $\{v_i, 1 \le i \le n, u_i, u_i', 1 \le i \le m\}$ be the vertices and $\{e_i, 1 \le i \le n, a_i, a_i', 1 \le i \le m\}$ be the edges which are denoted as in Fig. 4.

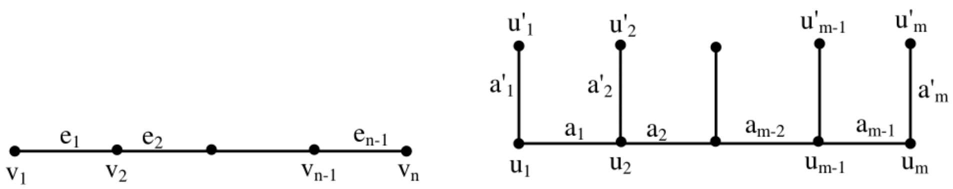

Figure 4. **Ordinary labeling of $P_n \bigcup P_m^+$**

First we label the vertices as follows: Define $f : V \to \{1, 2, ..., p+q\}$ by for $1 \le i \le n$, $f(v_i) = 2i - 1$. For $1 \le i \le m$,

$$f(u_i) = \begin{cases} 2n + 4(i-1) & \text{i is odd} \\ 4i + 2(n-1) & \text{i is even} \end{cases}$$

For $1 \le i \le m$,

$$f(u_i') = \begin{cases} 4i + 2(n-1) & \text{i is odd} \\ 2n + 4(i-1) & \text{i is even} \end{cases}$$

Then the induced edge labels are: For $1 \leq i \leq n-1$, $f^*(e_i) = 2i$. For $1 \leq i \leq m-1$, $f^*(a_i) = 4i + 2n - 1$, *For* $1 \leq i \leq m$, $f^*(a_i') = 4i + 2n - 3$. Thus the vertices and edges together get distinct labels. Hence the graph $P_n \bigcup P_m^+ \, (m, n \geq 3)$ is a super root square mean graph. Super root square mean labeling of $P_5 \bigcup P_7^+$ and $P_6 \bigcup P_8^+$ are given in Fig. 5 and Fig. 6 respectively.

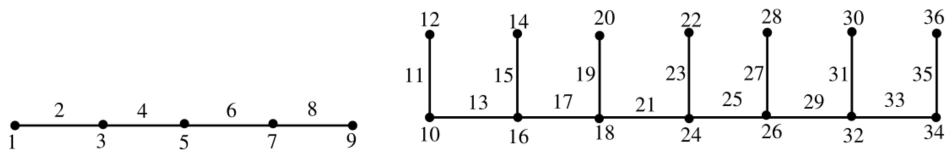

Figure 5. **Super root square mean labeling of $P_5 \bigcup P_7^+$**

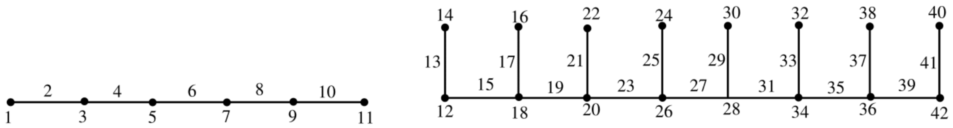

Figure 6. **Super root square mean labeling of $P_6 \bigcup P_8^+$**

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Theorem 2.3. *The graph $P_n^+ \bigcup P_m^+$ is a super root square mean graph.*

Proof. Let $\{V_i, V_i', 1 \leq i \leq n, u_i, u_i', 1 \leq i \leq m\}$ be the vertices and $\{a_i\,,\ 1 \leq i \leq n-1,\ a_i',\ 1 \leq i \leq n,\ b_i,\ 1 \leq i \leq m-1,\ b_i',\ 1 \leq i \leq m\}$ be the edges which are denoted as in Fig. 7

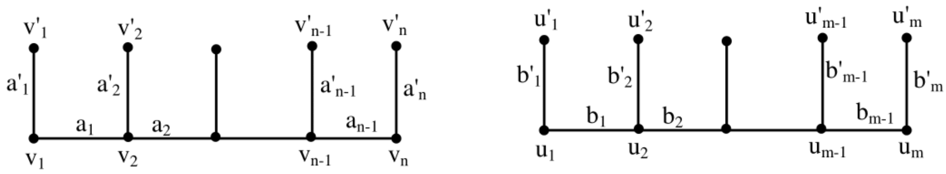

Figure 7. **Ordinary labeling of $P_n^+ \bigcup P_m^+$**

First we label the vertices as follows: Define $f : V \to \{1, 2, \ldots, p+q\}$ by for $1 \leq i \leq n$

$$f(v_i) = \begin{cases} 4i - 1 & \text{i is odd} \\ 4i - 3 & \text{i is even} \end{cases}$$

$$f(v_i') = \begin{cases} 4i - 3 & \text{i is odd} \\ 4i - 1 & \text{i is even} \end{cases}$$

For $1 \leq i \leq m$

$$f(u_i) = \begin{cases} 4n + 4(i-1) & \text{i is odd} \\ 4n + 4i - 2 & \text{i is even} \end{cases}$$

$$f(u_i') = \begin{cases} 4n + 4i - 2 & \text{i is odd} \\ 4n + 4(i-1) & \text{i is even} \end{cases}$$

Then the induced edge labels are: For $1 \leq i \leq n-1$, $f^*(a_i) = 4i$. For $1 \leq i \leq n$, $f^*(a_i') = 4i-2$. For $1 \leq i \leq m-1$, $f^*(b_i) = 4n+4i-3$. For $1 \leq i \leq m$, $f^*(b_i') = 4n+4i-1$.

Thus the vertices and edges together get distinct labels. Hence the graph $P_n^+ \bigcup P_m^+$ is a super rood square mean graph. Super root square mean labeling of $P_3^+ \bigcup P_7^+$ *and* $P_4^+ \bigcup P_6^+$ are given in Fig. 8 and Fig. 9 respectively.

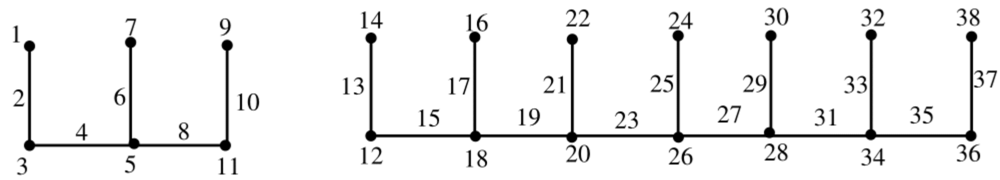

Figure 8. **Super root square mean labeling of** $P_3^+ \bigcup P_7^+$

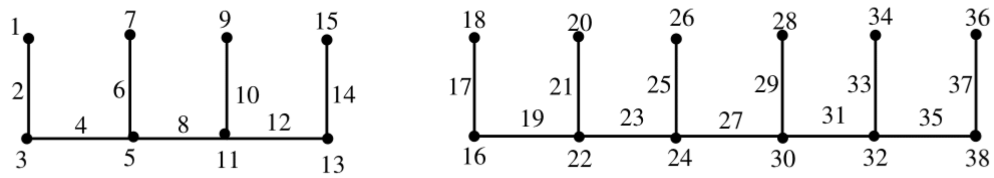

Figure 9. **Super root square mean labeling of** $P_4^+ \bigcup P_6^+$

□

Theorem 2.4. *The graph* $P_n \bigcup T_m$ *is a super root square mean graph.*

Proof. Let $\{V_i, 1 \leq i \leq n, u_i, 1 \leq i \leq m, u_i', 1 \leq i \leq m-1\}$ be the vertices and $\{e_i, 1 \leq i \leq n-1, a_i, b_i, e_i', 1 \leq i \leq m-1\}$ be the edges which are denoted as in Fig. 10.

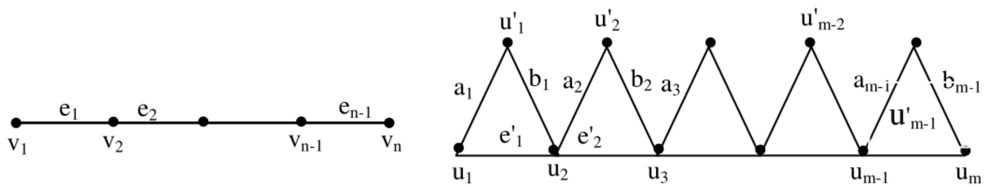

Figure 10. **Ordinary labeling of** $P_n \bigcup T_m$

First we label the vertices as follows: Define $f : V \to \{1, 2, \ldots, p+q\}$ by for $1 \leq i \leq n$, $f(V_i) = 2i-1$; $f(u_i) = 2n+2$. For $2 \leq i \leq m$, $f(u_i) = 2n+5(i-1)$; $f(u_1') = 2n$. For $2 \leq i \leq m-1$, $f(u_i) = 2n+5i-3$.

Then the induced edge labels are: For $1 \leq i \leq n-1$, $f^*(e_i) = 2i$; $f^*(e_i') = 2n+4$. For $2 \leq i \leq m-1$, $f^*(e_i') = 5i+2n+3$. For $1 \leq i \leq m-1$, $f^*(a_i) = 2n+5i-4$; $f(b_1) = 2n+3$. For $2 \leq i \leq m-1$, $f^*(b_i) = 2n+5i-1$.

Thus the vertices and edges together get distinct labels. Hence the graph $P_n \bigcup T_m$ is a super root square mean graph. Super root square mean labeling of $P_5 \bigcup T_4$ and $P_4 \bigcup T_6$ are given in Fig. 11 and Fig. 12 respectively.

□

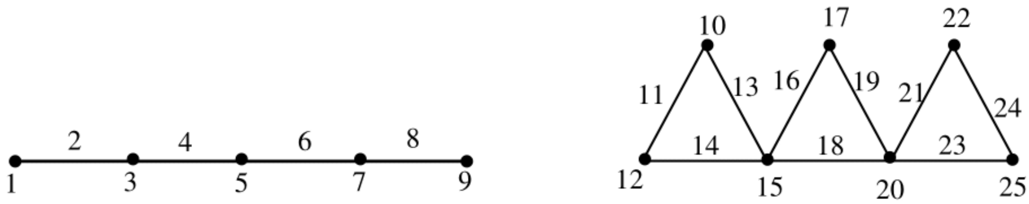

Figure 11. **Super root square mean labeling of $P_5 \bigcup T_4$**

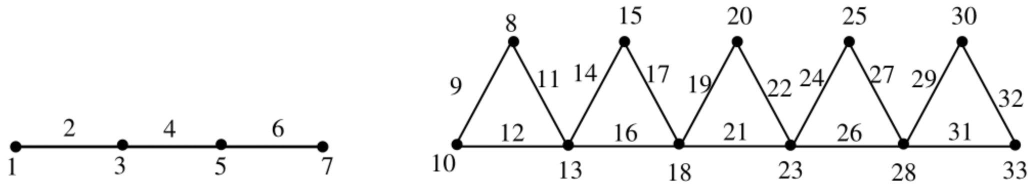

Figure 12. **Super root square mean labeling of $P_4 \bigcup T_6$**

Theorem 2.5. *The graph $P_n^+ \bigcup T_m$ is a super root square mean graph.*

Proof. Let $\{v_i, v_i', 1 \le i \le n, u_i', 1 \le i \le m, u_i, 1 \le i \le m-1\}$ be the vertices and $\{e_i\,,\ 1 \le i \le n-1,\ e_i',\ 1 \le i \le n,\ a_i,\ b_i,\ c_i,\ 1 \le i \le m-1\}$ be the edges which are denoted as in Figure 13.

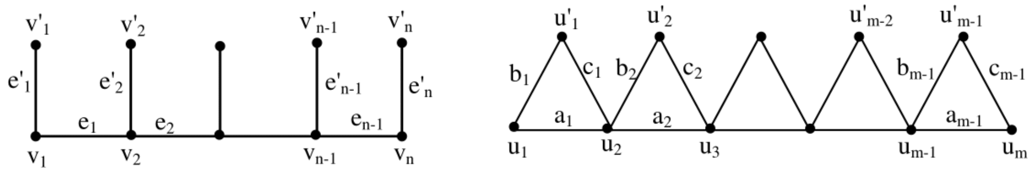

Figure 13. **Ordinary labeling of $P_n^+ \bigcup T_m$**

First we label the vertices as follows: Define $f : V \to \{1, 2, \ldots, p+q\}$ by for $1 \le i \le n$

$$f(v_i) = \begin{cases} 4i-1 & \text{i is odd} \\ 4i-3 & \text{i is even} \end{cases}$$

$$f(v_i') = \begin{cases} 4i-3 & \text{i is odd} \\ 4i-1 & \text{i is even} \end{cases}$$

$$f(u_1) = 4n+2$$

For $2 \le i \le m,\ f(u_i) = 4n+5(i-1)$; $f(u_1') = 4n$. For $2 \le i \le m-1,\ f(u_i') = 4n+5i-3$. Then the induced edge labels are: For $1 \le i \le n-1,\ \ f^*(e_i) = 4i$. For $1 \le i \le n,\ \ f^*(e_i') = 4i-2$; $f(a_1) = 4n+4$. For $2 \le i \le m-1,\ \ f^*(a_i) = 4n+5i-2$. For $1 \le i \le m-1,\ \ f^*(b_i) = 4n+5i-4$; $f(c_1) = 4n+3$. For $2 \le i \le m-1,\ \ f^*(c_i) = 4n+5i-1$.

Thus the vertices and edges together get distinct labels. Hence the graph $P_n^+ \bigcup T_n$ is a super root square mean graph. Super root square mean labeling of $P_5^+ \bigcup T_4$ and $P_6^+ \bigcup T_5$ are given in Fig. 14 and Fig. 15 respectively.

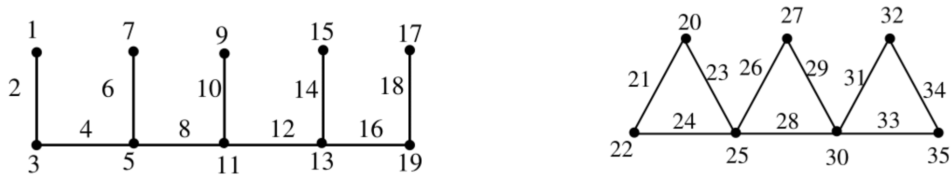

Figure 14. **Super root square mean labeling of $P_5^+ \bigcup T_4$**

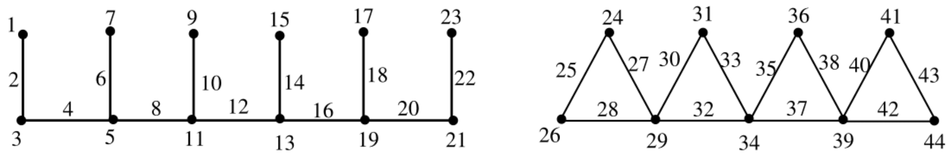

Figure 15. **Super root square mean labeling of $P_6^+ \bigcup T_5$**

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