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Lucky Edge Labeling of K_n and Special Types of Graphs

Research Article

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Abstract: Let G be a simple graph with vertex set V(G) and edge set E(G) respectively. Vertex set V(G) is labeled arbitrary by positive integers and E(e) denote the edge label such that it is the sum of labels of vertices incident with edge e. The labeling is said to be lucky edge labeling if the edge E(G) is a proper coloring of G, that is, if we have $E(e_1) \neq E(e_2)$ whenever e_1 and e_2 are adjacent edges. The least integer k for which a graph G has a lucky edge labeling from the set $\{1, 2, \ldots, k\}$ is the lucky number of G denoted by $\eta(G)$. A graph which admits lucky edge labeling is the lucky edge labeled graph. In this paper, we proved that complete graph K_n, tadpole graph $T_{m,n}$ and rectangular book B_p^4 are lucky edge labeled graphs.

MSC: 05C78.

Keywords: Lucky edge labeled graph, Lucky edge labeling, Lucky number.

1. Introduction

In 1967, Rosa [4] introduced the concept of labeling the edges and Golomb [2] gave the name graceful for such labelings. Gallian [1] has given a dynamic survey of graph labeling. Many graphs are constructed from standard graphs by using various operations. Nellai Murugan [3] introduced the concept of lucky edge labeling. In this paper, lucky edge labeling of K_n, $K_{m,n}$, $T_{m,n}$ and B_p^4 are discussed.

2. Preliminaries

Definition 2.1. *Let G be a simple graph with vertex set V(G)and edge set E(G) respectively. Vertex set V(G) is labeled arbitrary by positive integers and E(e) denote the edge label such that it is the sum of labels of vertices incident with edge e. The labeling is said to be lucky edge labeling if the edge E(G) is a proper coloring of G, that is, if we have* $E(e_1) \neq E(e_2)$ *whenever* e_1 *and* e_2 *are adjacent edges. The least integer k for which a graph G has a lucky edge labeling from the set* $\{1, 2, \ldots, k\}$ *is the lucky number of G denoted by* $\eta(G)$*. A graph which admits lucky edge labeling is the lucky edge labeled graph.*

Definition 2.2. *A graph G in which any two distinct vertices are adjacent is called complete graph* K_n*. A complete graph with n vertices is denoted by* ${}_nC_2$*.*

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Definition 2.3. *A graph G is called a complete bipartite graph $K_{m,n}$ with bipartition $V(G) = V_1 \cup V_2$ where $V_1 = \{x_1, x_2, \ldots, x_m\}$ and $V_2 = \{y_1, y_2, \ldots, y_n\}$ and all vertices in V_1 are adjacent to all vertices in V_2 but no vertices in V_1 and V_2.*

Definition 2.4. *The tadpole graph $T_{m,n}$ are also called dragon graph is the graph obtained by joining a cycle C_m to a path P_n with a bridge.*

Definition 2.5. *One edge union of cycles of same length is called a book. The common edge is called base of the book. If we consider t copies of cycles of length $n \geq 3$, the book is denoted by B_n^t. If $n = 4$, then the book B is called book with rectangular (or quadrilateral).*

3. Main Results

Theorem 3.1. *Lucky number of complete graph K_n is $\eta(K_n) = 2n - 1$.*

Proof. Let V_1, V_2,..., V_n be the vertices of K_n. Then $|V(K_n)| = n$ and $|E(K_n)| = \begin{pmatrix} n \\ 2 \end{pmatrix}$.
The vertex and edge labeling are defined as follows:

$$f(v_i) = i, \;\; 1 \leq i \leq n.$$

$$f*(v_iv_j) = i + j, \;\; 1 \leq i, j \leq n.$$

Therefore, lucky number of K_n is $\eta(K_n) = 2n - 1$. □

Illustration 3.2. *Lucky edge labeling of K_5 is shown in the Figure 1 and $\eta(K_5) = 9$.*

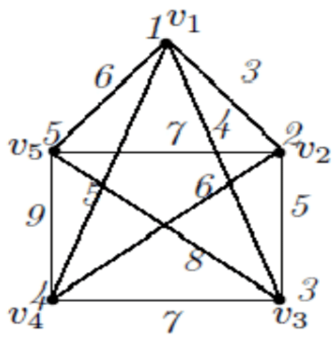

Figure 1.

Remark 3.3. *Lucky number of complete bipartite graph $K_{m,n}$ is $\eta(K_{m,n}) = m + n$.*

Theorem 3.4. *$T_{m,n}$ has $\{a, b, c\}$ lucky edge labeling graph, for any $a, b, c \in N$.*

Proof. Let v_1, v_2,..., v_n be the vertices of the path P_n and v_{n+1}, v_{n+2},..., $v_{n+m} = v_p$ be the vertices of cycle C_m. So that the length of tadpole graph $T_{m,n}$ is $p = m + n$.

Case(i): Both m and n are even or odd

Subcase(i): When $m \equiv 0\ (mod\ 4)$, $n \equiv 0, 2\ (mod\ 4)$ and $m \equiv 3\ (mod\ 4)$, $n \equiv 1, 3\ (mod\ 4)$. Let $f : V[T_{m,n}] \to \{1, 2, 3\}$ be defined by

$$f(v_{2i}) = \begin{cases} 1 & i \equiv 1\ mod\ 2 \\ 2 & i \equiv 0\ mod\ 2 \end{cases} \quad 1 \le i \le \left(\frac{p}{2}\right) - 1$$

$$f(v_{2i-1}) = \begin{cases} 1 & i \equiv 1\ mod\ 2 \\ 2 & i \equiv 0\ mod\ 2 \end{cases} \quad 1 \le i \le \left(\frac{p}{2}\right)$$

$$f(v_i) = 3, \quad i = p.$$

Then the induced edge labeling are

$$f^*(v_i v_{i+1}) = \begin{cases} 2 & i \equiv 1\ mod\ 4 \\ 3 & i \equiv 0, 2\ mod\ 4 \\ 4 & i \equiv 3\ mod\ 4 \end{cases} \quad 1 \le i \le p - 2$$

$f^*(v_{p-1}v_p) = 5$ and $f^*(v_p v_{n+1}) = 4$, when $m, n \equiv 0\ mod\ 4$ and $m \equiv 3 mod\ 4$, $n \equiv 1 mod\ 4$. $f^*(v_{p-1}v_p) = 4$ and $f^*(v_p v_{n+1}) = 5$, when $m \equiv 0 mod\ 4$, $n \equiv 2 mod\ 4$ and $m, n \equiv 3 mod\ 4$. It is clear that the lucky edge labeling of $T_{m,n}$ is $\{2, 3, 4, 5\}$. Therefore, Lucky number is $\eta(T_{m,n}) = 5$. For example, lucky edge labeling of $T_{8,2}$ is shown in the Figure 2.

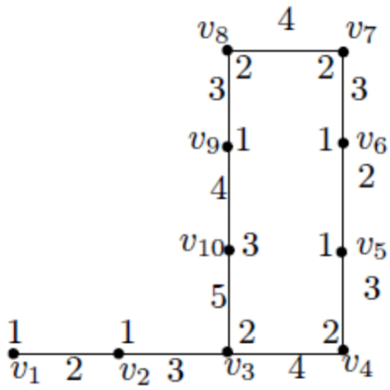

Figure 2.

Subcase(ii): When $m \equiv 2\ (mod\ 4)$ and $n \equiv\ 0, 2\ (mod\ 4)$. Let $f : V[T_{m,n}] \to \{1, 2, 3\}$ be defined by

$$f(v_{2i}) = \begin{cases} 1 & i \equiv 1\ mod\ 2 \\ 2 & i \equiv 0\ mod\ 2 \end{cases} \quad 1 \le i \le \left(\frac{p}{2}\right) - 1$$

$$f(v_{2i-1}) = \begin{cases} 1 & i \equiv 1\ mod\ 2 \\ 2 & i \equiv 0\ mod\ 2 \end{cases} \quad 1 \le i \le \left(\frac{p}{2}\right) - 1$$

$$f(v_i) = 3, \quad i = p - 1, p.$$

Then the induced edge labeling are

$$f^*(v_i v_{i+1}) = \begin{cases} 2 & i \equiv 1\ mod\ 4 \\ 3 & i \equiv 0, 2\ mod\ 4 \\ 4 & i \equiv 3\ mod\ 4 \end{cases} \quad 1 \le i \le p - 3$$

$f^*(v_{p-2}v_{p-1}) = 5$, $f^*(v_{p-1}v_p) = 6$, and $f^*(v_pv_{n+1}) = 4$, when $n \equiv 0$ *mod* 4. $f^*(v_{p-2}v_{p-1}) = 4$, $f^*(v_{p-1}v_p) = 6$ and $f^*(v_pv_{n+1}) = 5$, when $n \equiv 2$ *mod* 4. It is clear that the lucky edge labeling of $T_{m,n}$ is $\{2,3,4,5,6\}$. Therefore, Lucky number is $\eta(T_{m,n}) = 6$. For example, lucky edge labeling of $T_{10,2}$ is shown in the Figure 3.

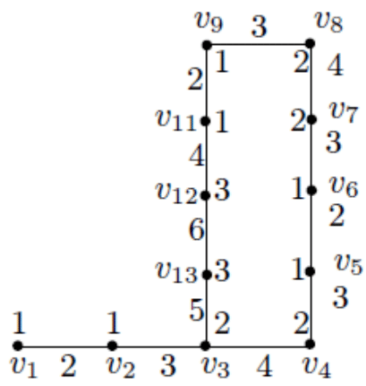

Figure 3.

Subcase(iii): When $m \equiv 1$ $(mod\ 4)$ and $n \equiv 1,3$ $(mod\ 4)$. Let $f : V[T_{m,n}] \to \{1,2,3\}$ be defined by

$$f(v_{2i}) = \begin{cases} 1 & i \equiv 1 \ mod\ 2 \\ 2 & i \equiv 0 \ mod\ 2 \end{cases} \quad 1 \leq i \leq \left(\frac{p}{2}\right) - 1$$

$$f(v_{2i-1}) = \begin{cases} 1 & i \equiv 0 \ mod\ 2 \\ 2 & i \equiv 1 \ mod\ 2 \end{cases} \quad 1 \leq i \leq \left(\frac{p}{2}\right)$$

$$f(v_i) = 3, \ i = p.$$

Then the induced edge labeling are

$$f^*(v_iv_{i+1}) = \begin{cases} 2 & i \equiv 2 \ mod\ 4 \\ 3 & i \equiv 1,3 \ mod\ 4 \\ 4 & i \equiv 0 \ mod\ 4 \end{cases} \quad 1 \leq i \leq p-3$$

$f^*(v_{p-2}v_{p-1}) = 4$, $f^*(v_{p-1}v_p) = 5$ and $f^*(v_pv_{n+1}) = 4$, when $n \equiv 1$ *mod* 4. $f^*(v_{p-2}v_{p-1}) = 2$, $f^*(v_{p-1}v_p) = 4$ and $f^*(v_pv_{n+1}) = 5$, when $n \equiv 3$ *mod* 4. It is clear that the lucky edge labeling of $T_{m,n}$ is $\{2,3,4,5\}$. Therefore, Lucky number is $\eta(T_{m,n}) = 5$. For example, Lucky edge labeling of $T_{5,3}$ is shown in the Figure 4.

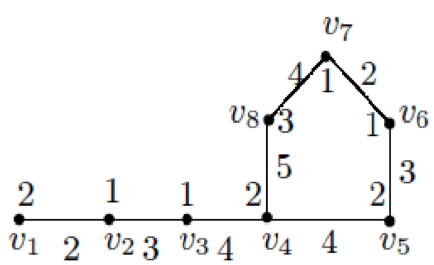

Figure 4.

Case(ii): When m-even, n-odd and m-odd, n-even

Subcase(i): When $m \equiv 0\ (mod\ 4)$, $n \equiv 1,3\ (mod\ 4)$ and $m \equiv 1\ (mod\ 4)$, $n \equiv 0,2\ (mod\ 4)$. Let $f : V[T_{m,n}] \to \{1,2,3\}$ be defined by

$$f(v_{2i}) = \begin{cases} 1 & i \equiv 1\ mod\ 2 \\ 2 & i \equiv 0\ mod\ 2 \end{cases} \quad 1 \le i \le \left(\frac{p-1}{2}\right)$$

$$f(v_{2i-1}) = \begin{cases} 1 & i \equiv 1\ mod\ 2 \\ 2 & i \equiv 0\ mod\ 2 \end{cases} \quad 1 \le i \le \left(\frac{p-1}{2}\right)$$

$$f(v_i) = 3,\ i = p.$$

Then the induced edge labeling are

$$f^*(v_i v_{i+1}) = \begin{cases} 2 & i \equiv 1\ mod\ 4 \\ 3 & i\ \equiv 0,2\ mod\ 4 \\ 4 & i\ \equiv 3\ mod\ 4 \end{cases} \quad 1 \le i \le p-3$$

$f^*(v_{p-2}v_{p-1}) = 4$, $f^*(v_{p-1}v_p) = 5$ and $f^*(v_p v_{n+1}) = 4$, when $m \equiv 0\ mod\ 4$, $n \equiv 1\ mod\ 4$ and $m \equiv\ 1\ mod\ 4$, $n \equiv 0\ mod\ 4$. $f^*(v_{p-2}v_{p-1}) = 2$, $f^*(v_{p-1}v_p) = 4$ and $f^*(v_p v_{n+1}) = 5$, when $m \equiv 1\ mod\ 4$, $n \equiv 2\ mod\ 4$ and $m \equiv 0\ mod\ 4$, $n \equiv 3\ mod\ 4$. It is clear that the lucky edge labeling of $T_{m,n}$ is {2,3,4,5}. Therefore, Lucky number is $\eta(T_{m,n}) = 5$. For example, lucky edge labeling of $T_{4,5}$ is shown in the Figure 5.

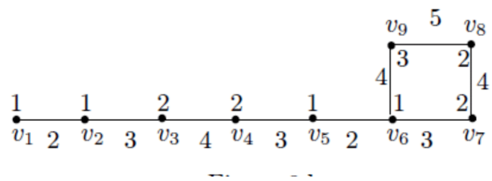

Figure 5.

Subcase(ii): When $m \equiv 2\ (mod\ 4)$ and $n \equiv 1,3\ (mod\ 4)$. Let $f : V[T_{m,n}] \to \{1,2,3\}$ be defined by

$$f(v_{2i}) = \begin{cases} 1 & i \equiv 1\ mod\ 2 \\ 2 & i \equiv 0\ mod\ 2 \end{cases} \quad 1 \le i \le \left(\frac{p-1}{2}\right) - 1$$

$$f(v_{2i-1}) = \begin{cases} 1 & i \equiv 1\ mod\ 2 \\ 2 & i \equiv 0\ mod\ 2 \end{cases} \quad 1 \le i \le \left(\frac{p-1}{2}\right)$$

$$f(v_i) = 3,\ i = p-1,\ p.$$

Then the induced edge labeling are

$$f^*(v_i v_{i+1}) = \begin{cases} 2 & i \equiv 1\ mod\ 4 \\ 3 & i\ \equiv 0,2\ mod\ 4 \\ 4 & i\ \equiv 3\ mod\ 4 \end{cases} \quad 1 \le i \le p-3$$

$f^*(v_{p-2}v_{p-1}) = 4$, $f^*(v_{p-1}v_p) = 6$ and $f^*(v_p v_{n+1}) = 4$, when $n \equiv 1\ mod\ 4$, $f^*(v_{p-2}v_{p-1}) = 5$, $f^*(v_{p-1}v_p) = 6$ and $f^*(v_p v_{n+1}) = 5$, when $n \equiv 3\ mod\ 4$. It is clear that the lucky edge labeling of $T_{m,n}$ is $\{2,3,4,5,6\}$. Therefore, Lucky number is $\eta(T_{m,n}) = 6$. For example, lucky edge labeling of $T_{10,1}$ is shown in the Figure 6.

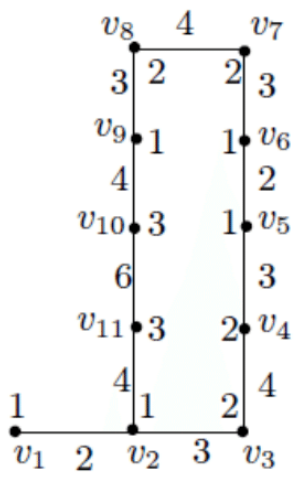

Figure 6.

Subcase(iii): When $m \equiv 3\ (mod\ 4)$ and $n \equiv 0, 2\ (mod\ 4)$. Let $f : V[T_{m,n}] \to \{1, 2, 3\}$ be defined by

$$f(v_{2i}) = \begin{cases} 1 & i \equiv 1\ mod\ 2 \\ 2 & i \equiv 0\ mod\ 2 \end{cases} \quad 1 \le i \le \left(\frac{p-1}{2}\right)$$

$$f(v_{2i-1}) = \begin{cases} 1 & i \equiv 0\ mod\ 2 \\ 2 & i \equiv 1\ mod\ 2 \end{cases} \quad 1 \le i \le \left(\frac{p-1}{2}\right)$$

$$f(v_i) = 3, \ \ i = p.$$

Then the induced edge labeling are

$$f^*(v_i v_{i+1}) = \begin{cases} 2 & i \equiv 2\ mod\ 4 \\ 3 & i \equiv 1, 3\ mod\ 4 \\ 4 & i \equiv 0\ mod\ 4 \end{cases} \quad 1 \le i \le p-2$$

$f^*(v_{p-1}v_p) = 4$ and $f^*(v_p v_{n+1}) = 5$, when $n \equiv 0\ mod\ 4$. $f^*(v_{p-1}v_p) = 5$ and $f^*(v_p v_{n+1}) = 4$, when $n \equiv 2\ mod\ 4$. It is clear that the lucky edge labeling of $T_{m,n}$ is $\{2, 3, 4, 5\}$. Therefore, Lucky number is $\eta(T_{m,n}) = 5$. For example, lucky edge labeling of $T_{3,8}$ is shown in the Figure 7.

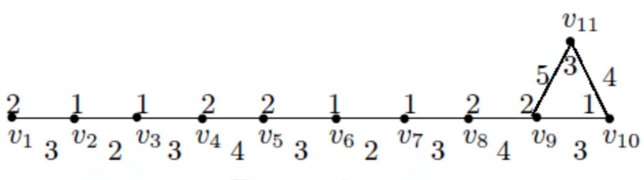

Figure 7.

□

Theorem 3.5. *Lucky number of rectangular book is $2p + 2$.*

Proof. Let u and v be the vertices that connecting common edges of rectangular book and u_i and $v_i (1 \leq i \leq p)$ be the vertices of p pages. Then $|V(B_4^p)| = 2p + 2$ and $|E(B_4^p)| = 3p + 1$. Label the vertices u and v with 1 and u_i and v_i with $i + 1$ $(1 \leq i \leq p)$ respectively. This induces the lucky edge labeling of the graph. Therefore, Lucky number of rectangular book is $\eta(B_4^p) = 2p + 2$. □

Illustration 3.6. *Lucky edge labeling of B_4^3 is shown in the Figure 8 and $\eta(B_4^3) = 8$.*

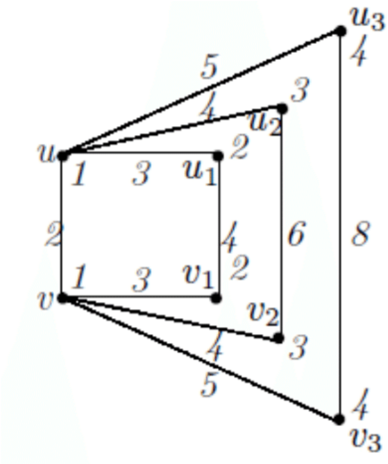

Figure 8.

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