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Adjacent vertex sum polynomial for the splitting graph of Factographs

Research Article

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Abstract: Let $G=(V,E)$ be a graph. The vertex polynomial of the graph $G=(V,E)$ is defined as $V(G,x)=\sum\limits_{k=0}^{\Delta(G)} v_k x^k$, where $\Delta(G)=\max\{d(v)/v\in V\}$ and v_k is the number of vertices of degree k. The adjacent vertex sum polynomial is defined as $S(G,x)=\sum\limits_{i=o}^{\Delta(G)} n_{\Delta(G)-i}x^{\alpha_{\Delta(G)-i}}$, where $n_{\Delta(G)-i}$ is the sum of the number of adjacent vertices of all the vertices of degree $\Delta(G)-i$ and $\alpha_{\Delta(G)-i}$ is the sum of the degree of adjacent vertices of all the vertices of degree $\Delta(G)-i$. In this paper we seek to find the vertex polynomial and the adjacent vertex sum polynomial for the splitting graph of Perfect factograph and the splitting graph of Integral Perfect factograph.

Keywords: Perfect factograph, Integral Perfect factograph, Vertex polynomial, Adjacent vertex sum polynomial, Splitting graph.

1. Introduction

In a graph $G=(V,E)$, we mean a finite undirected, non-trivial graph without loops and multiple edges. The vertex set is denoted by V and the edge set by$E$. For $v\in V$, $d(v)$ is the number of edges incident with v. The maximum degree of G is defined as $\Delta(G)=\max\{d(v)/v\in V\}$. For terms not defined here, we refer to Frank Harary [8]. For each vertex v of a graph G, take a new vertex v', join v' to all the vertices of G adjacent to v. The graph $S(G)$ thus obtained is called splitting graph of G [6]. Let $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ be two graphs, the union $G_1\cup G_2$ is defined to be (V,E) where $V=V_1\cup V_2$ and $E=E_1\cup E_2$, the sum G_1+G_2 is defined as $G_1\cup G_2$ together with all the lines joining points of V_1 to V_2. The graph $G=(V,E)$ is simply denoted by G.

2. Vertex Polynomial for the Splitting Graph of Perfect Factograph

Definition 2.1 ([3]). *A factograph G with $z=p_1^{\alpha_1}$, where p_1 is a prime and α_1 is any positive integer is called perfect factograph.*

Theorem 2.2. *The vertex polynomial for the spitting graph of the Perfect factograph G is given by* $V(S(G),x)=$

$$\begin{cases} x^{2\alpha_1}+x^{2(\alpha_1-1)}+\ldots+2x^{\alpha_1}+\ldots+x^{2(\alpha_1-(\alpha_1-1))}+x^{\alpha_1}+x^{\alpha_1-1}+\ldots+2x^{\frac{\alpha_1}{2}}+\ldots+x^{(\alpha_1-(\alpha_1-1))}, & \text{when } \alpha_1 \text{ is even;}\\ x^{2\alpha_1}+x^{2(\alpha_1-1)}+\ldots+2x^{2\lceil\frac{\alpha_1}{2}\rceil}+\ldots+x^{2(\alpha_1-(\alpha_1-1))}+x^{\alpha_1}+x^{\alpha_1-1}+\ldots 2x^{\lceil\frac{\alpha_1}{2}\rceil}+\ldots+x^{(\alpha_1-(\alpha_1-1))}, & \text{when } \alpha_1 \text{ is odd.}\end{cases}$$

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Proof. Let G be a perfect factograph. Let the vertex set be $V = \{p_1^0, p_1^1, p_1^2, \ldots, p_1^{\alpha_1}\}$. In G, we observe that for $i \neq j$, the vertex p_1^i is adjacent to p_1^j if and only if $i + j \leq \alpha_1$.

Case (i): when α_1 is even,

The degree sequence of G is $d(p_1^0) = \alpha_1$, $d(p_1^1) = \alpha_1 - 1, \ldots, d(p_1^{\frac{\alpha_1}{2}}) = \frac{\alpha_1}{2}$, $d(p_1^{\frac{\alpha_1}{2}+1}) = \frac{\alpha_1}{2}, \ldots, d(p_1^{\alpha_1}) = 1$. In $S(G)$, each new vertex corresponding to each vertex of v has same degree as in v of G and rest of vertices becomes twice the degree. Similar manor we can prove for the odd case. □

Example 2.3. *Consider the Perfect factograph $G = p_1^3$, then the corresponding graph $S(G)$ is depicted as follows;*

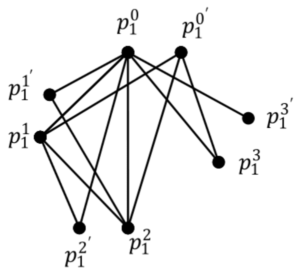

Figure 1.

Here,

$$V(S(G), x) = x^6 + 2x^4 + x^2 + x^3 + 2x^2 + x.$$
$$= x^6 + 2x^4 + x^3 + 3x^2 + x.$$

Theorem 2.4. *Let $G = p_1^{\alpha_1}$ be the perfect factograph. Then the vertex polynomial of $\zeta = S(G) \cup S(G) \cup \ldots \cup S(G)$ (n times) is given by*

$$V(\zeta, x) = \begin{cases} nx^{2\alpha_1} + nx^{2(\alpha_1-1)} + \ldots + 2nx^{\alpha_1} + \ldots + nx^{2(\alpha_1-(\alpha_1-1))} + nx^{\alpha_1} + nx^{\alpha_1-1} + \ldots 2nx^{\frac{\alpha_1}{2}} + \ldots + nx^{(\alpha_1-(\alpha_1-1))}, \\ \qquad \text{when } \alpha_1 \text{ is even.} \\ nx^{2\alpha_1} + nx^{2(\alpha_1-1)} + \ldots + 2nx^{2\lceil\frac{\alpha_1}{2}\rceil} + \ldots + nx^{2(\alpha_1-(\alpha_1-1))} + nx^{\alpha_1} + nx^{\alpha_1-1} + \ldots 2nx^{\lceil\frac{\alpha_1}{2}\rceil} + \ldots + nx^{(\alpha_1-(\alpha_1-1))}, \\ \qquad \text{when } \alpha_1 \text{ is odd.} \end{cases}$$

Theorem 2.5. *Let $G = {p_1}^{\alpha_1}$ be the perfect factograph. Then the vertex polynomial of $nS(G)$ is given by $V(nS(G), x) =$*

$$\begin{cases} nx^{2[\alpha_1+(n-1)(\alpha_1+1)]} + nx^{2[(\alpha_1-1)+(n-1)(\alpha_1+1)]} + \ldots + 2nx^{\alpha_1+2[+(n-1)(\alpha_1+1)]} + \ldots + nx^{2+[2(\alpha_1-(\alpha_1-1))+(n-1)(\alpha_1+1)]} \\ \quad + nx^{\alpha_1+2(n-1)(\alpha_1+1)} + nx^{(\alpha_1-1)+2(n-1)(\alpha_1+1)} + \ldots 2nx^{\frac{\alpha_1}{2}+2(n-1)(\alpha_1+1)} + \ldots + nx^{2[(\alpha_1-(\alpha_1-1))](n-1)(\alpha_1+1)}, \\ \qquad \text{when } \alpha_1 \text{ is even.} \\ nx^{2[\alpha_1+(n-1)(\alpha_1+1)]} + nx^{2[(\alpha_1-1)+(n-1)(\alpha_1+1)]} + \ldots + 2nx^{2[\lceil\frac{\alpha_1}{2}\rceil+(\alpha_1+1)]} + \ldots + nx^{2+[2(\alpha_1-(\alpha_1-1))+(n-1)(\alpha_1+1)]} \\ \quad + nx^{\alpha_1+2(n-1)(\alpha_1+1)} + nx^{(\alpha_1-1)+2(n-1)(\alpha_1+1)} + \ldots 2nx^{\lceil\frac{\alpha_1}{2}\rceil+2(n-1)(\alpha_1+1)} + \ldots + nx^{2[(\alpha_1-(\alpha_1-1))](n-1)(\alpha_1+1)}, \\ \qquad \text{when } \alpha_1 \text{ is odd.} \end{cases}$$

Proof. Using definition of sum of graphs, degree of each vertex in $S(G) \cup S(G) \cup \ldots \cup S(G)$ (*n times*) is increased by $2(n-1)(\alpha_1+1)$ in $nS(G)$ gives the required result. □

3. Vertex Polynomial for the Splitting Graph of Integral Perfect Factograph

Definition 3.1 ([2]). *An Integral Factograph G with $z = {p_1}^{\alpha_1}$, where p_1 is a prime and α_1 is any positive integer is called Integral perfect factograph.*

Theorem 3.2. *The vertex polynomial for the spitting graph of the Integral Perfect factograph G is given by $V(S(G), x) =$*

$$\begin{cases} 2x^{2(2\alpha_1+1)}+\ldots+2x^{2(\alpha_1+1)}+2x^{2\alpha_1}+\ldots+2x^{2(2\alpha_1+2-2\alpha_1)}+2x^{2\alpha_1+1}+\ldots+2x^{\alpha_1+1}+2x^{\alpha_1}+\ldots+2x^{(2\alpha_1+2-2\alpha_1)}, \\ \qquad when \ \ \alpha_1 \ is\ even. \\ 2x^{2(2\alpha_1+1)}+\ldots+2x^{2\left[2\left\lceil\frac{\alpha_1}{2}\right\rceil+1\right]}+2x^{4\left\lceil\frac{\alpha_1}{2}\right\rceil}+\ldots+2x^{2(2\alpha_1+2-2\alpha_1)}+2x^{2\alpha_1+1}+\ldots+2x^{2\left\lceil\frac{\alpha_1}{2}\right\rceil+1}+2x^{2\left\lceil\frac{\alpha_1}{2}\right\rceil} \\ \qquad +\ldots+2x^{(2\alpha_1+2-2\alpha_1)}, \quad when \ \ \alpha_1 \ is\ odd. \end{cases}$$

Proof. Let $G={p_1}^{\alpha_1}$be the Integral perfect factograph. Let V= $\{p_1^0$,- p_1^0, p_1^1, -p_1^1, p_1^2,-p_1^2,$\cdots$, ${p_1}^{\alpha_1}$, -${p_1}^{\alpha_1}\}$ be the vertex set of G. In G, We observe that for i?j, the vertex p_1^i (or -p_1^i) is adjacent to p_1^j (or -p_1^j) if and only if i+j $= \alpha_1$. When $\alpha_1$is odd, The degree sequence of G *is* $d\left(p_1^0\right) = 2\alpha_1+1$, $d\left(-p_1^0\right) = 2\alpha_1+1$, $\ldots$, $d\left(p_1^{\left\lfloor\frac{1}{2}\right\rfloor}\right) = 2\lceil\frac{\alpha_1}{2}\rceil+1$,

$$d\left(-p_1^{\left\lfloor\frac{1}{2}\right\rfloor}\right) = 2\left\lceil\frac{\alpha_1}{2}\right\rceil+1, d\left(p_1^{\left\lceil\frac{\alpha_1}{2}\right\rceil}\right) = 2\left\lceil\frac{\alpha_1}{2}\right\rceil, d\left(-p_1^{\left\lceil\frac{\alpha_1}{2}\right\rceil}\right) = 2\left\lceil\frac{\alpha_1}{2}\right\rceil, \ldots, d\left(p_1^{\frac{1}{2}}\right) = 2, d\left(-p_1^{\frac{1}{2}}\right) = 2.$$

In $S(G)$, each new vertex corresponding to each vertex of v has same degree as in v of G and rest of vertices becomes twice the degree. Similarly, we can prove for the even case. □

Example 3.3. *Consider the Integral Perfect factograph $G=p_1^2$, then the corresponding graph $S(G)$ is depicted as follows;*

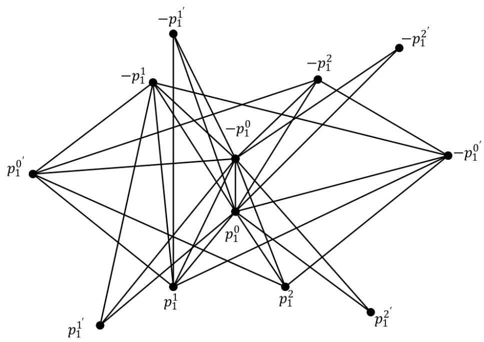

Figure 2.

Here, $V(S(G), x) = 2x^{10}+2x^6+2x^4+2x^5+2x^3+2x^2$.

Theorem 3.4. *Let $G={p_1}^{\alpha_1}$ be the Integral perfect factograph. Then the vertex polynomial of $\zeta= S(G) \cup S(G) \cup \ldots \cup S(G)$ (n times) is given by $V(\zeta, x) =$*

$$\begin{cases} 2nx^{2(2\alpha_1+1)}+\cdots+2nx^{2(\alpha_1+1)}+2nx^{2\alpha_1}+\cdots+2nx^{2(2\alpha_1+2-2\alpha_1)}+2nx^{2\alpha_1+1} \\ \quad +2nx^{2\alpha_1-1}+\cdots+2nx^{\alpha_1+1}+2nx^{\alpha_1}+\cdots+2nx^{(2\alpha_1+2-2\alpha_1)}, \ when \ \ \alpha_1 \ is\ even. \\ 2nx^{2(2\alpha_1+1)}+\cdots+2nx^{2\left[2\left\lceil\frac{\alpha_1}{2}\right\rceil+1\right]}+2nx^{4\left\lceil\frac{\alpha_1}{2}\right\rceil}+\cdots+2nx^{2(2\alpha_1+2-2\alpha_1)}+2nx^{2\alpha_1+1} \\ \quad +\cdots+2nx^{2\left\lceil\frac{\alpha_1}{2}\right\rceil+1}+2nx^{2\left\lceil\frac{\alpha_1}{2}\right\rceil}+\cdots+2nx^{(2\alpha_1+2-2\alpha_1)}, \quad when \ \ \alpha_1 \ is\ odd. \end{cases}$$

Theorem 3.5. *The vertex polynomial for the spitting graph of the Integral Perfect factograph $nS(G)$ is given by* $V(nS(G), x) =$

$$\begin{cases} 2nx^{2[(2\alpha_1+1)+(n-1)(2\alpha_1+2)]}+\ldots+2nx^{2[(\alpha_1+1)+(n-1)(2\alpha_1+2)])}+2nx^{2[\alpha_1+(n-1)(2\alpha_1+2)]}+\ldots+2nx^{2[2+(n-1)(2\alpha_1+2)]} \\ +2nx^{(2\alpha_1+1)+(n-1)(2\alpha_1+2)}+\ldots+2nx^{(\alpha_1+1)+(n-1)(2\alpha_1+2)}+2nx^{\alpha_1+(n-1)(2\alpha_1+2)}+\ldots+2nx^{2+(n-1)(2\alpha_1+2)}, \\ \text{when } \alpha_1 \text{ is even.} \\ 2nx^{2[(2\alpha_1+1)+(n-1)(2\alpha_1+2)]}+\ldots+2nx^{2\left[\left(2\left\lceil\frac{\alpha_1}{2}\right\rceil+1\right)+(n-1)(2\alpha_1+2)\right]}+2nx^{2\left[2\left\lceil\frac{\alpha_1}{2}\right\rceil+(n-1)(2\alpha_1+2)\right]}+\ldots+2nx^{2[2+(n-1)(2\alpha_1+2)]} \\ +2x^{(2\alpha_1+1)+2(n-1)(2\alpha_1+2)}+\ldots+2x^{(\alpha_1+1)+2(n-1)(2\alpha_1+2)}+2x^{\alpha_1+2(n-1)(2\alpha_1+2)}+\ldots+2x^{2[1+(n-1)(2\alpha_1+2)]}, \\ \text{when } \alpha_1 \text{ is odd.} \end{cases}$$

Proof. Using definition of sum of graphs, degree of each vertex in $S(G) \cup S(G) \cup \ldots \cup S(G)$ (n *times*) is increased by $2(n-1)(2\alpha_1+2)$ in $nS(G)$ gives the required result. □

4. Adjacent Vertex Sum Polynomial for the Splitting Graph of Perfect Factograph and Integral Perfect Factograph

Theorem 4.1. *Let $G = {p_1}^{\alpha_1}$ be the perfect factograph. The adjacent vertex sum polynomial of $S(G)$ is given by* $S(S(G), x) =$

$$\begin{cases} 2\alpha_1 x^{3[(\alpha_1-1)+\ldots+\alpha_1+\ldots+(\alpha_1-(\alpha_1-1))]}+\ldots+2\alpha_1 x^{3[\alpha_1+\ldots+(\alpha_1-(\frac{\alpha_1}{2}-1))]}+\ldots+2x^{3\alpha_1}+\alpha_1 x^{2[(\alpha_1-1)+\ldots+\alpha_1+\ldots+(\alpha_1-(\alpha_1-1))]} \\ +\cdots+\alpha_1 x^{2[\alpha_1+\ldots+(\alpha_1-(\frac{\alpha_1}{2}-1))]}+\ldots+x^{2\alpha_1}, \text{ when } \alpha_1 \text{ is even.} \\ 2\alpha_1 x^{3\left[(\alpha_1-1)+\ldots+2\left\lceil\frac{\alpha_1}{2}\right\rceil+\ldots+(\alpha_1-(\alpha_1-1))\right]}+\ldots+4\left\lceil\frac{\alpha_1}{2}\right\rceil x^{3\left[\alpha_1+\ldots+\left(\alpha_1-\left\lfloor\frac{\alpha_1}{2}\right\rfloor\right)\right]}+\ldots+2x^{3\alpha_1}+\alpha_1 x^{2\left[(\alpha_1-1)+\ldots+2\left\lceil\frac{\alpha_1}{2}\right\rceil+\ldots+(\alpha_1-(\alpha_1-1))\right]} \\ +\cdots+2\left\lceil\frac{\alpha_1}{2}\right\rceil x^{2\left[\alpha_1+\ldots+\left(\alpha_1-\left\lfloor\frac{\alpha_1}{2}\right\rfloor\right)\right]}+\ldots+x^{2\alpha_1}, \text{ when } \alpha_1 \text{ is odd.} \end{cases}$$

Proof. Let G be a perfect factograph with α_1 is even, $V = \{p_1^0, p_1^1, p_1^2, \ldots, {p_1}^{\alpha_1}\}$ a vertex set. We know that the degree sequence G is $d(p_1^0) = \alpha_1$, $d(p_1^1) = \alpha_1 - 1, \ldots, d\left(p_1^{\frac{\alpha_1}{2}}\right) = \frac{\alpha_1}{2}$, $d\left(p_1^{\frac{\alpha_1}{2}+1}\right) = \frac{\alpha_1}{2}, \ldots, d(p_1^{\alpha_1}) = 1$. Note that all the vertices except $p_1^{\frac{\alpha_1}{2}}$ and $p_1^{\frac{\alpha_1}{2}+1}$ have different degrees. In $S(G)$, each new vertex corresponding to each vertex of v has same degree as in v of G and rest of vertices becomes twice the degree. Here, it is obvious to find the degree and sum of the degrees of the adjacent vertices of each vertex. □

Example 4.2. *The adjacent vertex sum polynomial of the graph $S(G)$ in Figure: 1 is*

$$\begin{aligned} S(S(G), x) &= 6x^{15}+8x^{15}+2x^9+3x^{10}+4x^{10}+x^6 \\ &= 14x^{15}+2x^9+7x^{10}+x^6. \end{aligned}$$

Theorem 4.3. *Let $G = {p_1}^{\alpha_1}$ be an Integral perfect factograph. The adjacent vertex sum polynomial of $S(G)$ is given by* $S(S(G), x) =$

$$\begin{cases} 4(2\alpha_1+1)x^{3(2\alpha_1+1)+6\left[(2\alpha_1-1)+\ldots+((2\alpha_1+1)-(2\alpha_1-1))\right]}+\cdots+4(\alpha_1+1)x^{3((2\alpha_1+1)-\alpha_1))+6\left[(2\alpha_1+1)+\ldots+\left((2\alpha_1+1)-(\alpha_1-2)\right)\right]} \\ +\cdots+4x^{6(2\alpha_1+1)}+2(2\alpha_1+1)x^{2(2\alpha_1+1)+4\left[(2\alpha_1-1)+\ldots+((2\alpha_1+1)-(2\alpha_1-1))\right]}+\cdots+2(\alpha_1+1)x^{2(\;(2\alpha_1+1)-\alpha_1)} \\ +4\left[(2\alpha_1+1)+\ldots+\left(\,(2\alpha_1+1)-(\alpha_1-2)\right)\right]+\cdots+2x^{4(2\alpha_1+1)}, \text{when } \alpha_1 \text{ is even.} \\ 4(2\alpha_1+1)x^{3(2\alpha_1+1)+6\left[(2\alpha_1+1-2)+\ldots+((2\alpha_1+1)-(2\alpha_1-1))\right]}+\cdots+4\left\lceil\frac{\alpha_1}{2}\right\rceil x^{6\left[(2\alpha_1+1)+(2\alpha_1-1)+\ldots+\left((2\alpha_1+1)-2\left\lfloor\frac{\alpha_1}{2}\right\rfloor\right)\right]}+\ldots \\ +4x^{6(2\alpha_1+1)}+2(2\alpha_1+1)x^{2(2\alpha_1+1)+4\left[(2\alpha_1+1-2)+\ldots+((2\alpha_1+1)-(2\alpha_1-1))\right]}+\cdots+2\left\lceil\frac{\alpha_1}{2}\right\rceil x^{4\left[(2\alpha_1+1)+(2\alpha_1-1)+\ldots+\left((2\alpha_1+1)-2\left\lfloor\frac{\alpha_1}{2}\right\rfloor\right)\right]} \\ +\cdots+2x^{4(2\alpha_1+1)}, \text{when } \alpha_1 \text{ is odd.} \end{cases}$$

Proof. Let $G{=}{p_1}^{\alpha_1}$ be the Integral perfect factograph with α_1 is even. Let V= $\{p_1^0,\, -p_1^0,\, p_1^1,\, -p_1^1,\, p_1^2,\, -p_1^2, \cdots,\, {p_1}^{\alpha_1},\, -{p_1}^{\alpha_1}\}$ be the vertex set of G. The degree sequence of G, $d\left(p_1^0\right) = 2\alpha_1{+}1$, $d\left(-p_1^0\right) = 2\alpha_1{-}1$, $\ldots$, $d\left({p_1}^{\frac{\alpha_1}{2}}\right){=}\alpha_1{+}1$, $d\left(-{p_1}^{\frac{\alpha_1}{2}}\right){=}\alpha_1{+}1$, $d\left({p_1}^{\frac{\alpha_1}{2}+1}\right){=}\alpha_1$, $d\left(-{p_1}^{\frac{\alpha_1}{2}+1}\right){=}\alpha_1, \ldots$, $d\left(p_1^{\frac{1}{2}}\right) = 2$, $d\left(-p_1^{\frac{1}{2}}\right) = 2$. In $S\left(G\right)$, each new vertex corresponding to each vertex of v has same degree as in v of G and rest of vertices becomes twice the degree. Here, it is obvious to find the degree and sum of the degrees of the adjacent vertices of each vertex. □

Example 4.4. *The adjacent vertex sum polynomial of the graph $S\left(G\right)$ in Figure: 2 is*

$$\begin{aligned} S\left(S\left(G\right),\ x\right) &= 20x^{45}{+}12x^{39}{+}4x^{30}{+}10x^{30}{+}6x^{26}{+}2x^{10} \\ &= 20x^{45}{+}12x^{39}{+}14x^{30}{+}6x^{26}{+}2x^{10}. \end{aligned}$$

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