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Intuitionistic Fuzzy $\ddot{g}$-Closed Sets

Research Article

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Abstract: In this paper, we introduce the concepts of intuitionistic fuzzy $\ddot{g}$-closed sets and intuitionistic fuzzy $\ddot{g}$-open sets. Further, we study some of their properties.

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1. Introduction

The concept of fuzzy sets was introduced by Zadeh [17] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. The concept of generalized closed sets in topological spaces was introduced by Levine [8]. In this paper we introduce intuitionistic fuzzy $\ddot{g}$-closed sets and intuitionistic fuzzy $\ddot{g}$-open sets. The relations between intuitionistic fuzzy $\ddot{g}$-closed sets and other generalizations of intuitionistic fuzzy closed sets are given.

2. Preliminaries

Definition 2.1 ([1])**.** *Let X be a non empty set. An intuitionistic fuzzy set (IFS in short) A in X can be described in the form*

$$A = \{\langle, \mu_A(x), \nu_A(x)\rangle | x \in X\}$$

where the function $\mu_A : X \to [0, 1]$ is called the membership function and $\mu_A(x)$ denotes the degree to which $x \in A$ and the function $\nu_A : X \to [0, 1]$ is called the non-membership function and $\nu_A(x)$ denotes the degree to which $x \notin A$ and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote IFS(X), the set of all intuitionistic fuzzy sets in X. Throughout the paper, X denotes a non empty set.

Definition 2.2 ([1])**.** *Let A and B be any two IFSs of the form $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ and $B=\{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}$. Then*

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(1). $A \subseteq B$ *if and only if* $\mu_A(x) \leq \mu_B(x)$ *and* $\nu_A(x) \geq \nu_B(x)$ *for all* $x \in X$,

(2). $A = B$ *if and only if* $A \subseteq B$ *and* $B \subseteq A$,

(3). $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in X\}$,

(4). $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\}$,

(5). $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\}$.

Definition 2.3 ([1]). *The intuitionistic fuzzy sets* $0_\sim = \{\langle x, 0, 1\rangle \mid x \in X\}$ *and* $1_\sim = \{\langle x, 1, 0\rangle \mid x \in X\}$ *are called the empty set and the whole set of X respectively.*

Definition 2.4 ([1]). *Let A and B be any two IFSs of the form* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ *and* $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}$. *Then*

(1). $A \subseteq B$ *and* $A \subseteq C \Rightarrow A \subseteq B \cap C$,

(2). $A \subseteq C$ *and* $B \subseteq C \Rightarrow A \cup B \subseteq C$,

(3). $A \subseteq B$ *and* $B \subseteq C \Rightarrow A \subseteq C$,

(4). $(A \cup B)^c = A^c \cap B^c$ *and* $(A \cap B)^c = A^c \cup B^c$,

(5). $((A)^c)^c = A$,

(6). $(1_\sim)^c = 0_\sim$ *and* $(0_\sim)^c = 1_\sim$.

Definition 2.5 ([3]). *An intuitionistic fuzzy topology (IFT in short) on X is a family* τ *of IFSs in X satisfying the following axioms :*

(1). $0_\sim, 1_\sim \in \tau$,

(2). $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$,

(3). $\cup G_i \in \tau$ *for any family* $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair (X, τ) *is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in* τ *is known as an intuitionistic fuzzy open set(IFOS in short) in X. The complement* A^c *of an IFOS A in an IFTS* (X, τ) *is called an intuitionistic fuzzy closed set (IFCS in short) in X.*

Definition 2.6 ([3]). *Let* (X, τ) *be an IFTS and* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ *be an IFS in X. Then the intuitionistic fuzzy interior and the intuitionistic fuzzy closure are defined as follows:*

$$int(A) = \cup \{G \mid G \text{ is an IFOS in } X \text{ and } G \subseteq A\},$$
$$cl(A) = \cap \{K \mid K \text{ is an IFCS in } X \text{ and } A \subseteq K\}.$$

Proposition 2.7 ([3]). *For any IFSs A and B in* (X, τ), *we have*

(1). $int(A) \subseteq A$,

(2). $A \subseteq cl(A)$,

(3). A *is an IFCS in X* $\Leftrightarrow cl(A) = A$,

(4). A is an IFOS in X $\Leftrightarrow$ *int(A) = A,*

(5). $A \subseteq B \Rightarrow int(A) \subseteq int(B)$ *and* $cl(A) \subseteq cl(B)$*,*

(6). $int(int(A)) = int(A)$*,*

(7). $cl(cl(A)) = cl(A)$*,*

(8). $cl(A \cup B) = cl(A) \cup cl(B)$*,*

(9). $int(A \cap B) = int(A) \cap int(B)$*.*

Proposition 2.8 ([3]). *For any IFS A in* (X, τ)*, we have*

(1). $int(0_\sim) = 0_\sim$ *and* $cl(0_\sim) = 0_\sim$*,*

(2). $int(1_\sim) = 1_\sim$ *and* $cl(1_\sim) = 1_\sim$*,*

(3). $(int(A))^c = cl(A^c)$*,*

(4). $(cl(A))^c = int(A^c)$*.*

Proposition 2.9 ([3]). *If A is an IFCS in* (X, τ) *then* $cl(A) = A$ *and if A is an IFOS in* (X, τ) *then* $int(A) = A$*. The arbitrary union of IFCSs is an IFCS in* (X, τ)*.*

Definition 2.10. *An IFS A in an IFTS* (X, τ) *is said to be an*

(1). intuitionistic fuzzy α*-closed set (IF*α*CS in short) if* $cl(int(cl(A))) \subseteq A$*, [5]*

(2). intuitionistic fuzzy semi closed set (IFSCS in short) if $int(cl(A)) \subseteq A$*, [2]*

(3). intuitionistic fuzzy semi pre closed set (IFSPCS in short) if $int(cl(int(A))) \subseteq A$*. [16]*

Definition 2.11. *An IFS A in an IFTS* (X, τ) *is said to be an*

(1). intuitionistic fuzzy α*-open set (IF*α*OS in short) if* $A \subseteq int(cl(int(A)))$*, [5]*

(2). intuitionistic fuzzy semi open set (IFSOS in short) if $A \subseteq cl(int(A))$*, [4]*

(3). intuitionistic fuzzy semi pre open set (IFSPOS in short) if $A \subseteq cl(int(cl(A)))$*. [16]*

Remark 2.12 ([7]). *We have the following implications.*

$$IFCS \rightarrow IF\alpha CS \rightarrow IFSCS \rightarrow IFSPCS$$

None of the above implications are reversible.

Definition 2.13 ([12]). *Let A be an IFS in an IFTS* (X, τ)*. Then the* α*-interior of A (*$\alpha int(A)$ *in short) and the* α*-closure of A (*$\alpha cl(A)$ *in short) are defined as*

$$\alpha int(A) = \cup \{G \mid G \text{ is an IF}\alpha\text{OS in } (X, \tau) \text{ and } G \subseteq A\},$$
$$\alpha cl(A) = \cap \{K \mid K \text{ is an IF}\alpha\text{CS in } (X, \tau) \text{ and } A \subseteq K\}.$$

sint(A), scl(A), spint(A) and spcl(A) are similarly defined. For any IFS A in (X, τ)*, we have* $\alpha cl(A^c) = (\alpha int(A))^c$ *and* $\alpha int(A^c) = (\alpha cl(A))^c$*.*

Remark 2.14 ([12]). *Let A be an IFS in an IFTS (X, τ). Then*

(1). $\alpha cl(A) = A \cup cl(int(cl(A)))$,

(2). $\alpha int(A) = A \cap int(cl(int(A)))$.

Definition 2.15. *An IFS A in (X, τ) is said to be an*

(1). intuitionistic fuzzy generalized closed set (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ), [14]

(2). intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ), [11]

(3). intuitionistic fuzzy semi generalized closed set (IFSGCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in (X, τ), [13]

(4). intuitionistic fuzzy α generalized closed set (IFαGCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ), [12]

(5). intuitionistic fuzzy α generalized semi closed set (IFαGSCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in (X, τ), [6]

(6). intuitionistic fuzzy ω closed set (IFωCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in (X, τ), [13]

(7). intuitionistic fuzzy generalized semi pre closed set (IFGSPCS in short) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ). [10]

The complements of the above mentioned intuitionistic fuzzy closed sets are called their respective intuitionistic fuzzy open sets.

Remark 2.16 ([13]).

(1). Every IFOS is an IFSGOS,

(2). Every IFSOS is an IFSGOS.

Definition 2.17 ([15]). *Two IFSs A and B are said to be q-coincident (AqB in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$. For any two IFS A and B of (X, τ), $A\bar{q}B$ if and only if $A \subseteq B^c$.*

3. Intuitionistic Fuzzy $\ddot{g}$-closed Sets

In this section we introduce intuitionistic fuzzy $\ddot{g}$-closed sets and study some of their properties.

Definition 3.1. *An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy $\ddot{g}$-closed set (IF$\ddot{G}$CS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSGOS in (X, τ). The collection of all intuituionistic fuzzy $\ddot{g}$-closed sets in X is denoted by IF$\ddot{G}C(X)$.*

Example 3.2. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. We have $\mu_G(a) = 0.3$, $\mu_G(b)$=0.4, $\nu_G(a) = 0.6$ and $\nu_G(b) = 0.5$. Consider an IFS $A = \langle x, (0.6, 0.5), (0.3, 0.4)\rangle$. Then A is an IF$\ddot{G}$CS.*

Example 3.3. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. We have $\mu_G(a) = 0.3$, $\mu_G(b)=0.4$, $\nu_G(a) = 0.6$ and $\nu_G(b) = 0.5$. Consider an IFS $A = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. Then A is not an IF$\ddot{G}$CS.*

Theorem 3.4. *Every IFCS in an IFTS (X, τ) is an IF$\ddot{G}$CS, but not conversely.*

Proof. Let A be an IFCS in (X, τ). Let U be an IFSGOS such that $A \subseteq U$. Since A is IFCS, cl(A) = A. Thus we have cl(A) $\subseteq$ U, whenever $A \subseteq U$ and U is an IFSGOS in (X, τ). Therefore A is an IF$\ddot{G}$CS in (X, τ). Hence every IFCS is an IF$\ddot{G}$CS. □

Example 3.5. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.6, 0.7), (0.3, 0.2)\rangle$. Consider an IFS $A = \langle x, (0.2, 0.1), (0.7, 0.8)\rangle$. Here the IFSGOS $= \langle x, (l_1, m_1), (l_2, m_2)\rangle$ where $l_1 \in [0.6, 1]$, $m_1 \in [0.7, 1]$, $l_2 \in [0, 0.3]$ and $m_2 \in [0, 0.2]$. Then the IFS A is an IF$\ddot{G}$CS but not an IFCS in (X, τ).*

Theorem 3.6. *Every IF$\ddot{G}$CS in an IFTS (X, τ) is an IFGSPCS, but not conversely.*

Proof. Let $A \subseteq U$ where U is an IFOS in (X, τ). Since A is an IF$\ddot{G}$CS in (X, τ), we have cl(A) $\subseteq$ U, whenever $A \subseteq U$ and U is an IFSGOS in (X, τ). Since spcl(A) $\subseteq$ cl(A), we have spcl(A) $\subseteq$ U. Therefore A is an IFGSPCS in (X, τ). Hence every IF$\ddot{G}$CS is an IFGSPCS. □

Example 3.7. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. Consider an IFS $A = \langle x, (0.4, 0.5), (0.5, 0.4)\rangle$. Then A is an IFGSPCS but not an IF$\ddot{G}$CS in (X, τ).*

Theorem 3.8. *Every IF$\ddot{G}$CS in an IFTS (X, τ) is an IFωCS, but not conversely.*

Proof. Let $A \subseteq U$ where U is an IFSOS in (X, τ). Since A is an IF$\ddot{G}$CS in (X, τ), we have cl(A) $\subseteq$ U, whenever $A \subseteq U$ and U is an IFSGOS in (X, τ). Therefore A is an IFωCS in (X, τ). Hence every IF$\ddot{G}$CS is an IFωCS. □

Example 3.9. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.4, 0.3), (0.5, 0.6)\rangle$. Consider an IFS $A = \langle x, (0.6, 0.7), (0.3, 0.2)\rangle$. Then A is an IF$\omega$CS but not an IF$\ddot{G}$CS in (X, τ).*

Theorem 3.10. *Every IF$\ddot{G}$CS in an IFTS (X, τ) is an IFGCS, but not conversely.*

Proof. Let $A \subseteq U$ where U is an IFOS in (X, τ). Since A is an IF$\ddot{G}$CS in (X, τ), we have cl(A) $\subseteq$ U, whenever $A \subseteq U$ and U is an IFSGOS in (X, τ). Therefore A is an IFGCS in (X, τ). Hence every IF$\ddot{G}$CS is an IFGCS. □

Example 3.11. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.6, 0.5), (0.3, 0.4)\rangle$. Consider an IFS $A = \langle x, (0.7, 0.6), (0.2, 0.3)\rangle$. Then A is an IFGCS but not an IF$\ddot{G}$CS in (X, τ).*

Theorem 3.12. *Every IF$\ddot{G}$CS in an IFTS (X, τ) is an IFαGCS, but not conversely.*

Proof. Let $A \subseteq U$ where U is an IFOS in (X, τ). Since A is an IF$\ddot{G}$CS in (X, τ), we have cl(A) $\subseteq$ U, whenever $A \subseteq U$ and U is an IFSGOS in (X, τ). Since αcl(A) $\subseteq$ cl(A), we have αcl(A) $\subseteq$ U. Therefore A is an IFαGCS in (X, τ). Hence every IF$\ddot{G}$CS is an IFαGCS. □

Example 3.13. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.5, 0.6), (0.4, 0.3)\rangle$. Consider an IFS $A = \langle x, (0.6, 0.7), (0.3, 0.2)\rangle$. Then A is an IF$\alpha$GCS but not an IF$\ddot{G}$CS in (X, τ).*

Theorem 3.14. *Every IF$\ddot{G}$CS in an IFTS (X, τ) is an IFGSCS, but not conversely.*

Proof. Let A $\subseteq$ U where U is an IFOS in (X, τ). Since A is an IF$\ddot{G}$CS in (X, τ), we have cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFSGOS in (X, τ). Since scl(A) $\subseteq$ cl(A), we have scl(A) $\subseteq$ U. Therefore A is an IFGSCS in (X, τ). Hence every IF$\ddot{G}$CS is an IFGSCS. □

Example 3.15. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.4), (0.7, 0.5)\rangle$. Consider an IFS $A = \langle x, (0.6, 0.7), (0.3, 0.2)\rangle$. Then A is an IFGSCS but not an IF$\ddot{G}$CS in (X, τ).*

Definition 3.16. *An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy ψ-closed set (IFψCS in short) if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFSGOS in (X, τ). The complement of intuitionistic fuzzy ψ-closed set is called intuitionistic fuzzy ψ-open set (IFψOS in short).*

Theorem 3.17. *Every IF$\ddot{G}$CS in an IFTS (X, τ) is an IFψCS, but not conversely.*

Proof. Let A $\subseteq$ U where U is an IFSGOS in (X, τ). Since A is an IF$\ddot{G}$CS in (X, τ), we have cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFSGOS in (X, τ). Since scl(A) $\subseteq$ cl(A), we have scl(A) $\subseteq$ U. Therefore A is an IFψCS in (X, τ). Hence every IF$\ddot{G}$CS is an IFψCS. □

Example 3.18. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.2), (0.6, 0.7)\rangle$. Consider an IFS $A = \langle x, (0.4, 0.3), (0.5, 0.6)\rangle$. Then A is an IF$\psi$CS but not an IF$\ddot{G}$CS in (X, τ).*

Theorem 3.19. *Every IF$\ddot{G}$CS in an IFTS (X, τ) is an IFαGSCS, but not conversely.*

Proof. Let A $\subseteq$ U where U is an IFSOS in (X, τ). Since A is an IF$\ddot{G}$CS in (X, τ), we have cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFSGOS in (X, τ). Since αcl(A) $\subseteq$ cl(A), we have αcl(A) $\subseteq$ U. Therefore A is an IFαGSCS in (X, τ). Hence every IF$\ddot{G}$CS is an IFαGSCS. □

Example 3.20. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. Consider an IFS $A = \langle x, (0.7, 0.6), (0.2, 0.3)\rangle$. Then A is an IF$\alpha$GSCS but not an IF$\ddot{G}$CS in (X, τ).*

Definition 3.21. *An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy $\ddot{g}_\alpha$-closed set (IF$\ddot{G}_\alpha$CS in short) if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFSGOS in (X, τ). The complement of intuitionistic fuzzy $\ddot{g}_\alpha$-closed set is called an intuitionistic fuzzy $\ddot{g}_\alpha$-open set (IF$\ddot{G}_\alpha$OS in short).*

Theorem 3.22. *Every IF$\ddot{G}$CS in an IFTS (X, τ) is an IF$\ddot{G}_\alpha$CS, but not conversely.*

Proof. Let A be an IF$\ddot{G}$CS in (X, τ). Then we have cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFSGOS in (X, τ). Since αcl(A) $\subseteq$ cl(A), we have αcl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFSGOS in (X, τ). Therefore A is an IF$\ddot{G}_\alpha$CS in (X, τ). Hence every IF$\ddot{G}$CS is an IF$\ddot{G}_\alpha$CS. □

Example 3.23. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.7), (0.6, 0.2)\rangle$. Consider an IFS $A = \langle x, (0.3, 0.2), (0.6, 0.7)\rangle$. Then A is an IF$\ddot{G}_\alpha$CS but not an IF$\ddot{G}$CS in (X, τ).*

Remark 3.24. *IFαCS and IF$\ddot{G}$CS are independent.*

Example 3.25. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.2), (0.7, 0.8)\rangle$. Consider an IFS $A = \langle x, (0.2, 0.4), (0.8, 0.6)\rangle$. Then A is an IF$\alpha$CS but not an IF$\ddot{G}$CS in (X, τ).*

Example 3.26. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on X, where $G_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$ and $G_2 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. Consider an IFS $A = \langle x, (0.75, 0.65), (0.15, 0.25)\rangle$. Then A is an IF$\ddot{G}$CS but not an IF$\alpha$CS in (X, τ).*

Remark 3.27. *IFSCS and IF$\ddot{G}$CS are independent.*

Example 3.28. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$. Consider an IFS $A = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. Then A is an IFSCS but not an IF$\ddot{G}$CS in (X, τ).*

Example 3.29. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ be an IFT on X, where $G_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$ and $G_2 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. Consider an IFS $A = \langle x, (0.8, 0.7), (0.1, 0.2)\rangle$. Then A is an IF$\ddot{G}$CS but not an IFSCS in (X, τ).*

Theorem 3.30. *If A and B are IF$\ddot{G}$CSs in an IFTS (X, τ), then $A \cup B$ is also an IF$\ddot{G}$CS in (X, τ).*

Proof. If $A \cup B \subseteq G$ and G is IFSGOS, then $A \subseteq G$ and $B \subseteq G$. Since A and B are IF$\ddot{G}$CSs, $cl(A) \subseteq G$ and $cl(B) \subseteq G$ and hence $cl(A) \cup cl(B) = cl(A \cup B) \subseteq G$. Thus $A \cup B$ is IF$\ddot{G}$CS in (X, τ). □

Remark 3.31. *The intersection of two IF$\ddot{G}$CSs in an IFTS (X, τ) need not be an IF$\ddot{G}$CS in (X, τ).*

Example 3.32. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ be an IFT on X, where $G_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$ and $G_2 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. Consider the two IFSs $A = \langle x, (0.1, 0.7), (0.8, 0.2)\rangle$ and $B = \langle x, (0.8, 0.2), (0.1, 0.7)\rangle$. Then A and B are IF$\ddot{G}$CSs. But $A \cap B = \langle x, (0.1, 0.2), (0.8, 0.7)\rangle$ is not an IF$\ddot{G}$CS in (X, τ).*

Theorem 3.33. *If A is an IF$\ddot{G}$CS in an IFTS (X, τ) and $A \subseteq B \subseteq cl(A)$, then B is an IF$\ddot{G}$CS in (X, τ).*

Proof. Let $B \subseteq U$ where U is an IFSGOS in (X, τ). Since $A \subseteq B$, $A \subseteq U$. Since A is an IF$\ddot{G}$CS in (X, τ), $cl(A) \subseteq U$. Since $B \subseteq cl(A)$, $cl(B) \subseteq cl(A) \subseteq U$. Therefore B is an IF$\ddot{G}$CS in (X, τ). □

Theorem 3.34. *Let A be an IFS in an IFTS (X, τ). Then A is an IF$\ddot{G}$CS if and only if $A\bar{q}F$ implies $cl(A)\bar{q}F$ for every IFSGCS F in (X, τ).*

Proof. Necessary Part: Let F be an IFSGCS in (X, τ) and let $A\bar{q}F$. Then $A \subseteq F^c$, where F^c is an IFSGOS in (X, τ). Therefore by hypothesis $cl(A) \subseteq F^c$. Hence $cl(A)\bar{q}F$.

Sufficient Part: Let F be an IFSGCS in (X, τ) and let A be an IFS in (X, τ). By hypothesis, $A\bar{q}F$ implies $cl(A)\bar{q}F$. Then $cl(A) \subseteq F^c$ whenever $A \subseteq F^c$ and F^c is an IFSGOS in (X, τ). Hence A is an IF$\ddot{G}$CS in (X, τ). □

Theorem 3.35. *Let (X, τ) be an IFTS. Then $IFC(X) = IF\ddot{G}C(X)$ if every IFS in (X, τ) is an IFSGOS in X, where IFC(X) denotes the collection of IFCSs of an IFTS (X, τ).*

Proof. Suppose that every IFS in (X, τ) is an IFSGOS in X. Let $A \in IF\ddot{G}C(X)$. Then $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSGOS in X. Since every IFS is an IFSGOS, A is also an IFSGOS and $A \subseteq A$. Therefore $cl(A) \subseteq A$. Hence $cl(A) = A$. Therefore $A \in IFC(X)$. Hence $IF\ddot{G}C(X) \subseteq IFC(X) \rightarrow$ (i). Let $A \in IFC(X)$. Then by Theorem 3.4, $A \in IF\ddot{G}C(X)$. Hence $IFC(X) \subseteq IF\ddot{G}C(X) \rightarrow$ (ii). From (i) and (ii), we have $IFC(X) = IF\ddot{G}C(X)$. □

Proposition 3.36. *If A is an IFSGOS and IF$\ddot{G}$CS in an IFTS (X, τ), then A is an IFCS in (X, τ).*

Proof. Since A is an IFSGOS and IF$\ddot{G}$CS, $cl(A) \subseteq A$. Hence A is an IFCS in (X, τ). □

4. Intuitionistic Fuzzy $\ddot{g}$-open Sets

In this section we introduce intuitionistic fuzzy $\ddot{g}$-open sets and study some of their properties.

Definition 4.1. *An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy $\ddot{g}$-open set (IF$\ddot{G}$OS in short) if A^c is an intuitionistic fuzzy $\ddot{g}$-closed set in (X, τ). The collection of all intuituionistic fuzzy $\ddot{g}$-open sets in X is denoted by IF$\ddot{G}$O(X).*

Example 4.2. *Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.3, 0.4), (0.6, 0.5)$\rangle$. We have $\mu_G(a)$ = 0.3, $\mu_G(b)$=0.4, $\nu_G(a)$ = 0.6 and $\nu_G(b)$ = 0.5. Consider an IFS A = $\langle$x, (0.3, 0.4), (0.6, 0.5)$\rangle$. Then A is an IF$\ddot{G}$OS.*

Example 4.3. *Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.3, 0.4), (0.6, 0.5)$\rangle$. We have $\mu_G(a)$ = 0.3, $\mu_G(b)$=0.4, $\nu_G(a)$ = 0.6 and $\nu_G(b)$ = 0.5. Consider an IFS A = $\langle$x, (0.6, 0.5), (0.3, 0.4)$\rangle$. Then A is not an IF$\ddot{G}$CS.*

Theorem 4.4. *An IFS A in an IFTS (X, τ) is IF$\ddot{G}$OS if and only if F $\subseteq$ int(A) whenever F $\subseteq$ A and F^c is an IFSGOS.*

Proof. Necessary Part: Let A be an IF$\ddot{G}$OS in (X, τ). Let F^c be an IFSGOS such that F $\subseteq$ A. Then $A^c \subseteq F^c$ where A^c is an IF$\ddot{G}$CS. Hence cl(A^c) $\subseteq F^c$. This implies $(\text{int(A)})^c \subseteq F^c$. Thus we have F $\subseteq$ int(A) whenever F $\subseteq$ A and F^c is an IFSGOS. Sufficient Part: Let F $\subseteq$ int(A) whenever F $\subseteq$ A and F^c is an IFSGOS in (X, τ). This implies $(\text{int(A)})^c \subseteq F^c$ whenever $A^c \subseteq F^c$ and F^c is an IFSGOS. That is cl(A^c) $\subseteq F^c$ whenever $A^c \subseteq F^c$ and F^c is an IFSGOS. Therefore A^c is an IF$\ddot{G}$CS. Hence A is an IF$\ddot{G}$OS in (X, τ). □

Theorem 4.5. *Every IFOS in an IFTS (X, τ) is an IF$\ddot{G}$OS, but not conversely.*

Proof. Let A be an IFOS in (X, τ). Therefore A^c is an IFCS in (X, τ). Then by Theorem 3.4, A^c is an IF$\ddot{G}$CS in (X, τ). Therefore A is an IF$\ddot{G}$OS in (X, τ). □

Example 4.6. *Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.6, 0.7), (0.3, 0.2)$\rangle$. Consider an IFS A = $\langle$x, (0.7, 0.8), (0.2, 0.1)$\rangle$. Then A is an IF$\ddot{G}$OS but not an IFOS in (X, τ).*

Theorem 4.7. *Every IF$\ddot{G}$OS in an IFTS (X, τ) is an IFGSPOS, but not conversely.*

Proof. Let A be an IF$\ddot{G}$OS in (X, τ). Therefore A^c is an IF$\ddot{G}$CS in (X, τ). Then by Theorem 3.6, A^c is an IFGSPCS in (X, τ). Therefore A is an IFGSPOS in (X, τ). □

Example 4.8. *Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.3, 0.4), (0.6, 0.5)$\rangle$. Consider an IFS A = $\langle$x, (0.5, 0.4), (0.4, 0.5)$\rangle$. Then A is an IFGSPOS but not an IF$\ddot{G}$OS in (X, τ).*

Theorem 4.9. *Every IF$\ddot{G}$OS in an IFTS (X, τ) is an IFωOS, but not conversely.*

Proof. Let A be an IF$\ddot{G}$OS in (X, τ). Therefore A^c is an IF$\ddot{G}$CS in (X, τ). Then by Theorem 3.8, A^c is an IFωCS in (X, τ). Therefore A is an IFωOS in (X, τ). □

Example 4.10. *Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.4, 0.3), (0.5, 0.6)$\rangle$. Consider an IFS A = $\langle$x, (0.3, 0.2), (0.6, 0.7)$\rangle$. Then A is an IFωOS but not an IF$\ddot{G}$OS in (X, τ).*

Theorem 4.11. *Every IF$\ddot{G}$OS in an IFTS (X, τ) is an IFGOS, but not conversely.*

Proof. Let A be an IF$\ddot{G}$OS in (X, τ). Therefore A^c is an IF$\ddot{G}$CS in (X, τ). Then by Theorem 3.10, A^c is an IFGCS in (X, τ). Therefore A is an IFGOS in (X, τ). □

Example 4.12. *Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.6, 0.5), (0.3, 0.4)$\rangle$. Consider an IFS A = $\langle$x, (0.2, 0.3), (0.7, 0.6)$\rangle$. Then A is an IFGOS but not an IF$\ddot{G}$OS in (X, τ).*

Theorem 4.13. *Every IF$\ddot{G}$OS in an IFTS (X, τ) is an IFαGOS, but not conversely.*

Proof. Let A be an IF$\ddot{G}$OS in (X, τ). Therefore A^c is an IF$\ddot{G}$CS in (X, τ). Then by Theorem 3.12, A^c is an IFαGCS in (X, τ). Therefore A is an IFαGOS in (X, τ). □

Example 4.14. *Let* $X = \{a, b\}$. *Let* $\tau = \{0_\sim, G, 1_\sim\}$ *be an IFT on X, where* $G = \langle x, (0.5, 0.6), (0.4, 0.3)\rangle$. *Consider an IFS* $A = \langle x, (0.3, 0.2), (0.6, 0.7)\rangle$. *Then A is an IF$\alpha$GOS but not an IF$\ddot{G}$OS in (X, τ).*

Theorem 4.15. *Every IF$\ddot{G}$OS in an IFTS (X, τ) is an IFGSOS, but not conversely.*

Proof. Let A be an IF$\ddot{G}$OS in (X, τ). Therefore A^c is an IF$\ddot{G}$CS in (X, τ). Then by Theorem 3.14, A^c is an IFGSCS in (X, τ). Therefore A is an IFGSOS in (X, τ). □

Example 4.16. *Let* $X = \{a, b\}$. *Let* $\tau = \{0_\sim, G, 1_\sim\}$ *be an IFT on X, where* $G = \langle x, (0.2, 0.4), (0.7, 0.5)\rangle$. *Consider an IFS* $A = \langle x, (0.3, 0.2), (0.6, 0.7)\rangle$. *Then A is an IFGSOS but not an IF$\ddot{G}$OS in (X, τ).*

Theorem 4.17. *Every IF$\ddot{G}$OS in an IFTS (X, τ) is an IFψOS, but not conversely.*

Proof. Let A be an IF$\ddot{G}$OS in (X, τ). Therefore A^c is an IF$\ddot{G}$CS in (X, τ). Then by Theorem 3.17, A^c is an IFψCS in (X, τ). Therefore A is an IFψOS in (X, τ). □

Example 4.18. *Let* $X = \{a, b\}$. *Let* $\tau = \{0_\sim, G, 1_\sim\}$ *be an IFT on X, where* $G = \langle x, (0.3, 0.2), (0.6, 0.7)\rangle$. *Consider an IFS* $A = \langle x, (0.5, 0.6), (0.4, 0.3)\rangle$. *Then A is an IF$\psi$OS but not an IF$\ddot{G}$OS in (X, τ).*

Theorem 4.19. *Every IF$\ddot{G}$OS in an IFTS (X, τ) is an IFαGSOS, but not conversely.*

Proof. Let A be an IF$\ddot{G}$OS in (X, τ). Therefore A^c is an IF$\ddot{G}$CS in (X, τ). Then by Theorem 3.19, A^c is an IFαGSCS in (X, τ). Therefore A is an IFαGSOS in (X, τ). □

Example 4.20. *Let* $X = \{a, b\}$. *Let* $\tau = \{0_\sim, G, 1_\sim\}$ *be an IFT on X, where* $G = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. *Consider an IFS* $A = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$. *Then A is an IF$\alpha$GSOS but not an IF$\ddot{G}$OS in (X, τ).*

Theorem 4.21. *Every IF$\ddot{G}$OS in an IFTS (X, τ) is an IF$\ddot{G}_\alpha$OS, but not conversely.*

Proof. Let A be an IF$\ddot{G}$OS in (X, τ). Therefore A^c is an IF$\ddot{G}$CS in (X, τ). Then by Theorem 3.22, A^c is an IF$\ddot{G}_\alpha$CS in (X, τ). Therefore A is an IF$\ddot{G}_\alpha$OS in (X, τ). □

Example 4.22. *Let* $X = \{a, b\}$. *Let* $\tau = \{0_\sim, G, 1_\sim\}$ *be an IFT on X, where* $G = \langle x, (0.3, 0.7), (0.6, 0.2)\rangle$. *Consider an IFS* $A = \langle x, (0.6, 0.7), (0.3, 0.2)\rangle$. *Then A is an IF$\ddot{G}_\alpha$OS but not an IF$\ddot{G}$OS in (X, τ).*

Remark 4.23. *IFαOS and IF$\ddot{G}$OS are independent.*

Example 4.24. *Let* $X = \{a, b\}$. *Let* $\tau = \{0_\sim, G, 1_\sim\}$ *be an IFT on X, where* $G = \langle x, (0.3, 0.2), (0.7, 0.8)\rangle$. *Consider an IFS* $A = \langle x, (0.8, 0.6), (0.2, 0.4)\rangle$. *Then A is an IF$\alpha$OS but not an IF$\ddot{G}$OS in (X, τ).*

Example 4.25. *Let* $X = \{a, b\}$. *Let* $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ *be an IFT on X, where* $G_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$ *and* $G_2 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. *Consider an IFS* $A = \langle x, (0.15, 0.25), (0.75, 0.65)\rangle$. *Then A is an IF$\ddot{G}$OS but not an IF$\alpha$OS in (X, τ).*

Remark 4.26. *IFSOS and IF$\ddot{G}$OS are independent.*

Example 4.27. *Let* $X = \{a, b\}$. *Let* $\tau = \{0_\sim, G, 1_\sim\}$ *be an IFT on X, where* $G = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$. *Consider an IFS* $A = \langle x, (0.6, 0.5), (0.3, 0.4)\rangle$. *Then A is an IFSOS but not an IF$\ddot{G}$OS in (X, τ).*

Example 4.28. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on X, where $G_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$ and $G_2 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. Consider an IFS $A = \langle x, (0.1, 0.2), (0.8, 0.7)\rangle$. Then A is an IF$\ddot{G}$OS but not an IFSOS in (X, τ).*

Theorem 4.29. *If A and B are IF$\ddot{G}$OSs in an IFTS (X, τ), then $A \cap B$ is also an IF$\ddot{G}$OS in (X, τ).*

Proof. Let A and B be IF$\ddot{G}$OSs in (X, τ). Therefore A^c and B^c are IF$\ddot{G}$CSs in (X, τ). By Theorem 3.30, $(A^c \cup B^c)$ is an IF$\ddot{G}$CS in (X, τ). Since $(A^c \cup B^c) = (A \cap B)^c$, $A \cap B$ is an IF$\ddot{G}$OS in (X, τ). □

Remark 4.30. *The union of two IF$\ddot{G}$OSs in an IFTS (X, τ) need not be an IF$\ddot{G}$OS in (X, τ).*

Example 4.31. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on X, where $G_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$ and $G_2 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. Consider the two IFSs $A = \langle x, (0.1, 0.7), (0.8, 0.2)\rangle$ and $B = \langle x, (0.8, 0.2), (0.1, 0.7)\rangle$. Then A and B are IF$\ddot{G}$OSs. But $A \cup B = \langle x, (0.8, 0.7), (0.1, 0.2)\rangle$ is not an IF$\ddot{G}$OS in (X, τ).*

Theorem 4.32. *If A is an IF$\ddot{G}$OS in an IFTS (X, τ) such that $int(A) \subseteq B \subseteq A$, then B is IF$\ddot{G}$OS in (X, τ).*

Proof. Let A be an IF$\ddot{G}$OS in (X, τ) such that $\mathrm{int}(A) \subseteq B \subseteq A$. It implies $A^c \subseteq B^c \subseteq \mathrm{cl}(A^c)$ where A^c is an IF$\ddot{G}$CS in (X, τ). By Theorem 3.33, B^c is an IF$\ddot{G}$CS in (X, τ). Therefore B is an IF$\ddot{G}$OS in (X, τ). □

Theorem 4.33. *Let (X, τ) be an IFTS. Then $IFO(X) = IF\ddot{G}O(X)$ if every IFS in (X, τ) is an IFSGOS in X, where IFO(X) denotes the collection of IFOSs of an IFTS (X, τ).*

Proof. Suppose that every IFS in (X, τ) is an IFSGOS in X. Then by Theorem 3.35, we have $\mathrm{IFC}(X) = \mathrm{IF\ddot{G}C}(X)$. Therefore $\mathrm{IFO}(X) = \mathrm{IF\ddot{G}O}(X)$. □

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