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Cordial Labeling for Star Graphs

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Abstract: Cordial labeling is used to label the vertices and edges of a graph with $\{0, 1\}$ under constraint, such that the number of vertices with label 0 and 1 differ by atmost 1 and the number of edges with label 1 and 0 differ by atmost 1. In this paper we prove that the two star graph $K_{1,m} \wedge K_{1,n}$ with a wedge in common is a cordial graph for all $m \geq 1$ and $n \geq 1$.

MSC: 05C78.

Keywords: Cordial graph, Star graph, Cordial labeling.

1. Introduction

In [7], it is proved that a two star graph with an edge in common is a mean graph. In [2], it is proved that product of shell are cordial graph. In [1], it is proved that wheel graphs are cordial graph. In [5], cordial graphs for smaller graphs are given. In this paper we prove that the two star graph with wedge is a cordial graph.

Definition 1.1. *A graph G is called a bigraph or bipartite graph if V can be a partitioned into two disjoint subsets V_1 and V_2 such that every line of G joins a point of V_1 to a point of V_2. is called a bipartition of G. If further G contains every line joining the points of V_1 to the points of V_2 then G is called a complete bigraph. If V_1 contains m points and V_2 contains n points then the complete bigraph G is denoted by $K_{m,n}$. $K_{1,m}$ is called a Star for $m \geq 1$.*

Definition 1.2. *Let f be a function from the vertices(nodes) of G to $\{0, 1\}$ and for each edge xy assign the label $|f(x) - f(y)|$, call f a cordial labeling of G if the number of vertices(nodes) labeled 0 and the number vertices(nodes) labeled 1 differs by at most 1 and the number of edges(links) labeled 0 and the number of edges(links) labeled 1 differs by at most 1.*

Theorem 1.3. *Any two star graph $K_{1,m} \wedge K_{1,n}$ with a wedge in common is a cordial graph for all $m \geq 1, n \geq 1$.*

Proof. Let $G = K_{1,m} \wedge K_{1,n}$. The vertex and edge set of G is given by

$$V(G) = \{\alpha, \beta\} \cup \{\alpha_i \ ; 1 \leq i \leq m\} \cup \{\beta_i \ ; 1 \leq i \leq n\}$$

$$E(G) = \{\alpha\alpha_i \ ; 1 \leq i \leq m\} \cup \{\beta\beta_i \ ; 1 \leq i \leq n\} \cup \{\alpha_i\beta_i\}$$

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Then G has $m+n+2$ vertices and $m+n+1$ edges.

To prove that G is a cordial graph for all $m \geq 1$ $n \geq 1$, we shall consider the following cases. The node and link labeling of G is given by,

$$f : V(G) \to \{0,1\}$$
$$f^* : E(G) \to \{0,1\}$$

Case 1: When m and n are even. The vertex labeling of G is given by,

$$\begin{aligned} f(\alpha) &= 1 \\ f(\alpha_{2i-1}) &= 1 \quad \text{for } 1 \leq i \leq \frac{m}{2} \\ f(\alpha_{2i}) &= 0 \quad \text{for } 1 \leq i \leq \frac{m}{2} \\ f(\beta) &= 0 \\ f(\beta_{2i-1}) &= 1 \quad \text{for } 1 \leq i \leq \frac{n}{2} \\ f(\beta_{2i}) &= 0 \quad \text{for } 1 \leq i \leq \frac{n}{2} \end{aligned}$$

Then the link labeling is given by, $\alpha\alpha_{2i-1}$ is 0 for $1 \leq i \leq \frac{m}{2}$; $\alpha\alpha_{2i}$ is 1 for $1 \leq i \leq \frac{m}{2}$; $\beta\beta_{2i-1}$ is 1 for $1 \leq i \leq \frac{n}{2}$; $\beta\beta_{2i}$ is 0 for $1 \leq i \leq \frac{n}{2}$. The labeling of the wedge $\alpha_1\beta_2$ is 1. Then the total number of vertices with label 1 is $\left(\frac{m}{2}+1+\frac{n}{2}\right)$. The total number of vertices with label 0 is $\left(\frac{m}{2}+\frac{n}{2}+1\right)$. And also the total number of edges with label 1 is $\left(\frac{m}{2}+\frac{n}{2}+1\right)$. Then the total number of edges with label 0 is $\left(\frac{m}{2}+\frac{n}{2}\right)$. Hence the difference in the the number of vertices with label 0 and 1 is zero, and the difference in the number of edges with label 0 and 1 is one. Hence G is a cordial graph if m and n are even.

Case 2: When m and n are odd. The vertex labeling of G is given by,

$$\begin{aligned} f(\alpha) &= 1 \\ f(\alpha_{2i-1}) &= 0 \quad \text{for } 1 \leq i \leq \left\lceil \frac{m}{2} \right\rceil \\ f(\alpha_{2i}) &= 1 \quad \text{for } 1 \leq i \leq \left\lfloor \frac{m}{2} \right\rfloor \\ f(\beta) &= 1 \\ f(\beta_{2i-1}) &= 0 \quad \text{for } 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\ f(\beta_{2i}) &= 1 \quad \text{for } 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \end{aligned}$$

Then the edge labeling is given by, $\alpha\alpha_{2i-1}$ is 1 for $1 \leq i \leq \left\lceil \frac{m}{2} \right\rceil$; $\alpha\alpha_{2i}$ is 0 for $1 \leq i \leq \left\lfloor \frac{m}{2} \right\rfloor$; $\beta\beta_{2i-1}$ is 1 for $1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$; $\beta\beta_{2i}$ is 0 for $1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$. The labeling of the wedge $\alpha_2\beta_2$ is 0. Then the cumulative number of nodes with label 1 is $\left(\left\lfloor \frac{m}{2} \right\rfloor + 1 + \left\lfloor \frac{n}{2} \right\rfloor + 1\right)$ and the cumulative number of nodes with label 0 is $\left(\left\lceil \frac{m}{2} \right\rceil + \left\lceil \frac{n}{2} \right\rceil\right)$. And also the cumulative number of links with label 1 is $\left(\left\lceil \frac{m}{2} \right\rceil + \left\lceil \frac{n}{2} \right\rceil\right)$. Then the cumulative number of links with label 0 is $\left(\left\lfloor \frac{m}{2} \right\rfloor + \left\lfloor \frac{n}{2} \right\rfloor + 1\right)$. Hence the difference in the the number of vertices with label 0 and 1 is zero, and the difference in the number of edges with label 0 and 1 is one. Hence G is a cordial graph if m and n are odd.

Case 3: When $m \neq n$

Sub Case (a): m is odd, n is even. The node labeling of G is given by,

$$f(\alpha) = 1$$

$$\begin{aligned} f(\alpha_{2i-1}) &= 0 \quad \text{for } 1 \le i \le \left\lceil \frac{m}{2} \right\rceil \\ f(\alpha_{2i}) &= 1 \quad \text{for } 1 \le i \le \left\lfloor \frac{m}{2} \right\rfloor \\ f(\beta) &= 1 \\ f(\beta_{2i-1}) &= 0 \quad \text{for } 1 \le i \le \frac{n}{2} \\ f(\beta_{2i}) &= 1 \quad \text{for } 1 \le i \le \frac{n}{2} \end{aligned}$$

Then the link labeling is given by, $\alpha\alpha_{2i-1}$ is 1 for $1 \le i \le \lceil \frac{m}{2} \rceil$; $\alpha\alpha_{2i}$ is 0 for $1 \le i \le \lfloor \frac{m}{2} \rfloor$; $\beta\beta_{2i-1}$ is 1 for $1 \le i \le \frac{n}{2}$; $\beta\beta_{2i}$ is 0 for $1 \le i \le \frac{n}{2}$. The labeling of the wedge $\alpha_2\beta_2$ is 0. Then the overall vertices with label 1 is $\lfloor \frac{m}{2} \rfloor + 1 + \frac{n}{2} + 1$ and the overall vertices with label 0 is $\lceil \frac{m}{2} \rceil + \frac{n}{2}$. And also the overall edges with label 1 is $\lceil \frac{m}{2} \rceil + \frac{n}{2}$. Then the overall edges with label 0 is $\lfloor \frac{m}{2} \rfloor + 1 + \frac{n}{2}$. Hence the difference in the the number of vertices with label 0 and 1 is one, and the difference in the number of edges with label 0 and 1 is zero. Hence G is a cordial graph for m is odd, n is even and not equal.

Sub Case (b): m is even, n is odd. The node labeling of G is given by,

$$\begin{aligned} f(\alpha) &= 1 \\ f(\alpha_{2i-1}) &= 0 \quad \text{for } 1 \le i \le \frac{m}{2} \\ f(\alpha_{2i}) &= 1 \quad \text{for } 1 \le i \le \frac{m}{2} \\ f(\beta) &= 1 \\ f(\beta_{2i-1}) &= 0 \quad \text{for } 1 \le i \le \left\lceil \frac{n}{2} \right\rceil \\ f(\beta_{2i}) &= 1 \quad \text{for } 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \end{aligned}$$

Then the link labeling is given by, $\alpha\alpha_{2i-1}$ is 1 for $1 \le i \le \frac{m}{2}$; $\alpha\alpha_{2i}$ is 0 for $1 \le i \le \frac{m}{2}$; $\beta\beta_{2i-1}$ is 1 for $1 \le i \le \lceil \frac{n}{2} \rceil$; $\beta\beta_{2i}$ is 0 for $1 \le i \le \lfloor \frac{n}{2} \rfloor$. The labeling of the wedge $\alpha_2\beta_2$ is 0. Then the all nodes with label 1 is $\left(\frac{m}{2} + 1 + \lfloor \frac{n}{2} \rfloor + 1\right)$ and the all nodes with label 0 is $\left(\frac{m}{2} + \lceil \frac{n}{2} \rceil\right)$. And also the all links with label 1 is $\left(\frac{m}{2} + \lceil \frac{n}{2} \rceil\right)$. Then the all links with label 0 is $\left(\frac{m}{2} + \lfloor \frac{n}{2} \rfloor + 1\right)$. Hence the difference in the number of vertices with label 0 and 1 is one, and the difference in the number of edges with label 0 and 1 is zero. Hence G is a cordial graph for m is even, n is odd and not equal. □

2. Application [3, 7, 8]

The communications network addressing; A communication network is composed of nodes, each of which has computing power and can transmit and receive messages over communication links, wireless or cabled. The basic network topologies include fully connected, mesh, star, ring, tree, bus. A single network may consist of several interconnected subnets of different topologies. Networks are further classified as Local Area Networks (LAN), e.g. inside one building, or Wide Area Networks (WAN), e.g. between buildings. It might be useful to assign to each user terminal a node label, subject to the constraint that all connecting edges (communication links) receive distinct labels. In this way, the number of any two communicating terminals automatically specify (by simple subtraction) the link label of the connecting path; and conversely, the path label uniquely specifies the pair of user terminals which it interconnects. Researches may get some information related to graph labeling and its applications in communication field.

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