Int. J. Math. And Appl., 6(1–A)(2018), 115–120

ISSN: 2347-1557

Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Geometric Mean Labeling of Union and Product of Some Standard Graphs with Zero Divisor Graphs

J. Periaswamy[1,*] and N. Selvi[2]

1 Department of Mathematics, A.V.C.College(Autonomous), Mayiladuthurai, Tamil Nadu, India.

2 Department of Mathematics, A.D.M.College For Women(Autonomous), Nagappattinam, Tamil Nadu.

Abstract: In this paper, Geometric mean labeling of $C_m \cup \Gamma(Z_n)$, $mC_n \cup \Gamma(Z_n)$, $nC_3 \cup \Gamma(Z_n)$, $n\Gamma(Z_{25})$, $n\Gamma(Z_{25}) \cup P_m$, $n\Gamma(Z_{25}) \cup C_m$, $n\Gamma(Z_{25}) \cup mC_k$, $\Gamma(Z_{2p}) \cup \Gamma(Z_{2p})$, $P_m \times \Gamma(Z_8)$ and $P_n \times \Gamma(Z_9)$ are investigated.

MSC: 05C25, 05C70.

Keywords: Graph Lebeling, Geometric Mean Labeling.

1. Introduction

In 2012, Somasundaram and Vidhyarani [6] introduced the concept of Geometric mean labelings. Let a graph G be simple with p vertices and q edges. G is said to be a Geometric mean graph if it possible to label vertices $x \in V$ with distinct labels $f(x)$ from $1, 2, ..., q+1$ in such a way that when edge e=uv is labeled with $\left\lceil \sqrt{f(u)f(v)} \right\rceil$ or $\left\lfloor \sqrt{f(u)f(v)} \right\rfloor$ then the resulting edge labels are distinct. In this case f is called Geometric mean labeling of G. In this paper, we evaluate the geometric mean labelings of zero divisor graphs. Let R be a commutative ring and let $Z(R)$ be its set of zero-divisors. We associate a graph $\Gamma(R)$ to R with vertices $Z(R)^* = Z(R) - \{0\}$, the set of non-zero divisors of R and for distinct $u, v \in Z(R)^*$, the vertices u and v are adjacent if and only if $uv = 0$. The zero divisor graph is very useful to find the algebraic structures and properties of rings. The idea of a zero divisor graph of a commutative ring was introduced by I. Beck in [2]. The first simplification of Beck's zero divisor graph was introduced by D. F. Anderson and P. S. Livingston [1]. Their motivation was to give a better illustration of the zero divisor structure of the ring. D. F. Anderson and P. S. Livingston, and others, e.g., [3–5], investigate the interplay between the graph theoretic properties of $\Gamma(R)$ and the ring theoretic properties of R. Throughout this paper, we consider the commutative ring R by Z_n and zero divisor graph $\Gamma(R)$ by $\Gamma(Z_n)$.

Definition 1.1. *Let a graph G be simple with p vertices and q edges. G is said to be a Geometric mean graph if it possible to label vertices $x \in V$ with distinct labels $f(x)$ from $1, 2, ..., q+1$ in such a way that when edge $e = uv$ is labeled with $\left\lceil \sqrt{f(u)f(v)} \right\rceil$ or $\left\lfloor \sqrt{f(u)f(v)} \right\rfloor$ then the resulting edge labels are distinct. In this case, f is called Geometric mean labeling of G.*

* E-mail: jperiaswamy@gmail.com

2. Geometric Mean Labeling of Union and Product of Some Standard Graphs with $\Gamma(Z_n)$

Theorem 2.1. *For $m \geq 3$ and $\Gamma(Z_n)$ is a path graph, then $C_m \cup \Gamma(Z_n)$ is a geometeic mean graph.*

Proof. Let C_m be the cycle $u_1u_2....u_mu_1$. Let $\Gamma(Z_n)$ be the path $v_1v_2....v_n$. The graph $C_m \cup \Gamma(Z_n)$ has $m+n$ vertices and $m+n-1$ edges. Define a function $f : V(C_m \cup \Gamma(Z_n)) \rightarrow \{1, 2, ..., q+1\}$ as $f(u_i) = i$, $1 \leq i \leq m$ and $f(v_i) = m+i$, $1 \leq i \leq n$. Then the set of labels of the edges of cycle C_m is $\{1, 2, ..., m\}$. The set of labels of the edges of $\Gamma(Z_n)$ is $\{m+1, m+2, ..., m+n-1\}$. Hence, f is a geometric mean labeling of $C_m \cup \Gamma(Z_n)$. That is, $C_m \cup \Gamma(Z_n)$ is a geometric mean graph for $m \geq 3$ and $n \geq 1$. We know that the only non-trivial paths in $\Gamma(Z_n)$ are $\Gamma(Z_6), \Gamma(Z_8)$ and $\Gamma(Z_9)$. Therefore using above theorem, $C_m \cup \Gamma(Z_6)$, $C_m \cup \Gamma(Z_8)$, $C_m \cup \Gamma(Z_9)$ are geometric mean graph. In general, $k_3 \cup \Gamma(Z_n)$ for $n \geq 1$ is geometric mean graphs, where $\Gamma(Z_n)$ is a path graph. □

Theorem 2.2. *For any path $\Gamma(Z_n)$, $mC_n \cup \Gamma(Z_n)$ is a geometric mean graph for $m \geq 1$ and the length of cycle is at least 3.*

Proof. Let mC_n be the m copies of C_n and $\Gamma(Z_n)$ be a path of length k. The vertex set of mC_n be $V = V_1 \cup V_2 \cup ... \cup V_m$, where $V_i = \{v_i^1, v_i^2, ..., v_i^n\}$ and the edge set of mC_n is $E = E_1 \cup E_2 \cup ... \cup E_m$, where $E_i = \{e_i^1, e_i^2, ..., e_i^n\}$. Let $w_1, w_2, ..., w_k$ be the vertex of $\Gamma(Z_n)$. A function $f : V(mC_n \cup \Gamma(Z_n)) \rightarrow \{1, 2, ..., q+1\}$ is defined as $f(v_i^j) = n(i-1)+j$, $1 \leq i \leq m$, $1 \leq j \leq n$ and $f(w_i) = mn+i$, $1 \leq i \leq k$. Then the set of labels of the edges of mC_n is $\{1, 2, ..., mn\}$. The set of labels of the edges of $\Gamma(Z_n)$ is $\{mn+1, mn+2, ..., mn+k-1\}$. Hence, $mC_n \cup \Gamma(Z_n)$ is a geometric mean graph. □

Theorem 2.3. *$nk_3 \cup \Gamma(Z_n)$ is a geometric mean graph for $n \geq 1$.*

Proof. Let the vertex set of nk_3 be $V = V_1 \cup V_2 \cup .... \cup V_n$, where $V_i = \{u_i^1, u_i^2, u_i^3\}$ and the edge set $E = E_1 \cup E_2 \cup .... \cup E_n$, where $E_i = \{e_i^1, e_i^2, e_i^3\}$. $\Gamma(Z_n)$ be the path $v_1v_2...v_m...nk_3 \cup \Gamma(Z_n)$ has $3n+m$ vertices and $3n+m-1$ edges. A function $f : V(nk_3 \cup \Gamma(Z_n)) \rightarrow \{1, 2, ..., q+1\}$ is defined as $f(u_i) = 3(i-1)+j$, $1 \leq i \leq m$. The label of the edges $u_i^1u_i^2 = f(u_i^1)$, $1 \leq i \leq n$. The label of the edges $u_i^2u_i^3 = f(u_i^3)$, $1 \leq i \leq n$. The label of the edges $u_i^1u_i^3 = \frac{f(u_i^1)+f(u_i^3)}{2}$, $1 \leq i \leq n$. Then the set of labels of the edges of nk_3 is $\{1, 2, ..., 3n\}$. The set of labels of the edges of $\Gamma(Z_n)$ is $\{3n+1, 3n+2, ..., 3n+m-1\}$. Hence, $nk_3 \cup \Gamma(Z_n)$ is a geometric mean graph for $n \geq 1$. □

Theorem 2.4. *$n\Gamma(Z_{25})$ is a geometric mean graph for $n \geq 1$.*

Proof. We know that $\Gamma(Z_{25})$ is isomorphic to k_4. Let $\Gamma(Z_{25})$ be the complete graph with 4 vertices $n\Gamma(Z_{25})$ be the n copies of $\Gamma(Z_{25})$. Let the vertex set of $n\Gamma(Z_{25})$. Let the vertex set of $n\Gamma(Z_{25})$ be $V = V_1 \cup V_2 \cup ... \cup V_n$, where $V_i = \{v_i^1, v_i^2, v_i^3, v_i^4\}$ and the edge set $E = \{E_1 \cup E_2 \cup ... \cup E_n\}$, where $E_i = \{e_i^1, e_i^2, e_i^3, e_i^4, e_i^5, e_i^6\}$. Define a function $f : V(n(\Gamma(Z_{25}))) \rightarrow \{1, 2, ..., q+1\}$ as $f(v_i^j) = f(v_i^{j-1})+2$, $1 \leq i \leq 4$, $j = 2, 3$, $f(v_i^4) = f(v_i^3)+1$, $1 \leq i \leq n$ and $f(v_i^1) = f(v_{i-1}^4)+1$, $2 \leq i \leq n$. It is known that the geometric mean of two consecutive integers b and $b+1$ lies between b and $b+1$. It may assign the edge label $6i-5$ for the edge joining the vertices $6i-1$ and $6i-3$ and the edge joining the vertices $6i$ and $6i-1$ as $6i$. The label of the edge joining the vertices $6i-5$ and $6i-1$ is $6i-4$, Since $6i-5 < \sqrt{(6i-5)(6i-1)} < 6i-1$. By similar argument the edge labels $6i-1$, $6i-2$ and $6i-3$ can be found. Then the set of labels of the edges of $\Gamma(Z_{25})$ is $\{1, 2, ..., 6n\}$. In view of the above labeling pattern $n\Gamma(Z_{25})$ is a geometric mean graph. □

Theorem 2.5.

(1). $n\Gamma(Z_{25}) \cup \Gamma(Z_6)$ is a geometric mean graph.

(2). $n\Gamma(Z_{25}) \cup \Gamma(Z_8)$ is a geometric mean graph.

(3). $n\Gamma(Z_{25}) \cup \Gamma(Z_9)$ is a geometric mean graph.

Proof.

(1). Let $n\Gamma(Z_{25})$ be the n copies of $\Gamma(Z_{25})$ with the vertex set $V = V_1 \cup V_2 \cup .... \cup V_n$, where $V_i = \{v_i^1, v_i^2, v_i^3, v_i^4\}$ and the edge set $E = E_1 \cup E_2 \cup ... \cup E_n$, where $E_i = \{e_i^1, e_i^2, e_i^3, e_i^4, e_i^5, e_i^6\}$. Let $\Gamma(Z_6)$ be the path $u_1u_2u_3$. The graph $n\Gamma(Z_{25}) \cup \Gamma(Z_6)$ has $4n+3$ vertices and $6n+2$ edges. Define a function $f : V(n\Gamma(Z_{25}) \cup \Gamma(Z_6)) \to \{1, 2, ..., q+1\}$ as $f(v_i^1) = 1$, $f(v_i^3) = f(v_i^{j-1}) + 2$, $1 \le i \le n$, $j = 2, 3$ and $f(v_i^4) = f(v_i^3) + 1$, $1 \le i \le n$, $f(v_i^1) = f(v_{i-1}^4) + 1$, $2 \le i \le n$ and $f(u_i) = f(v_n^4) + i$, $1 \le i \le 3$. Then the set of labels of the edges of $n\Gamma(Z_{25})$ is $\{1, 2, ..., 6n\}$. the set of labels of the edges of $\Gamma(Z_6)$ is $\{6n+1, 6n+2, 6n+3\}$. Thus the edge labels of $n\Gamma(Z_{25}) \cup \Gamma(Z_6)$ is a distinct. Hence, $n\Gamma(Z_{25}) \cup \Gamma(Z_6)$ is a geometric mean graph.

(2). We know that $n\Gamma(Z_{25}) \cup \Gamma(Z_6)$ is isomorphic to $n\Gamma(Z_{25}) \cup \Gamma(Z_8)$. Hence, using the above result, $n\Gamma(Z_{25}) \cup \Gamma(Z_8)$ is a geometric mean graph.

(3). Let $\Gamma(Z_9)$ be a path u_1u_2. The graph $n\Gamma(Z_{25}) \cup \Gamma(Z_9)$ has $4n+2$ vertices and $6n+1$ edges. Define a function $f : V(n\Gamma(Z_{25}) \cup \Gamma(Z_9)) \to \{1, 2, ..., q+1\}$ as

$$
\begin{aligned}
f(v_1^1) &= f(v_i^{j-1}) + 2, && 1 \le i \le n, j = 2, 3 \\
f(v_i^4) &= f(v_i^3) + 1, && 1 \le i \le n \\
f(v_i^1) &= f(v_{i-1}^4) + 1, && 2 \le i \le n \quad \text{and} \\
f(u_i) &= f(v_4^n) + i, && 1 \le i \le 2
\end{aligned}
$$

Then the set of labels of the edge of $n\Gamma(Z_{25})$ is $\{1, 2, ..., 6n\}$. The set of labels of the edges of $\Gamma(Z_9)$ is $\{6n+1, 6n+2\}$. Thus, the edge labels of $n\Gamma(Z_{25}) \cup \Gamma(Z_9)$ is distinct. Hence, $n\Gamma(Z_{25}) \cup \Gamma(Z_9)$ is a geometric mean graph. □

Theorem 2.6. *$n\Gamma(Z_{25}) \cup C_m$ is a geometric mean graph for $n \ge 1$ and $m \ge 3$.*

Proof. Let $n\Gamma(Z_{25})$ be the n copies of $\Gamma(Z_{25})$ with the vertex set $V = V_1 \cup V_2 \cup ... \cup V_n$, where $V_i = \{v_i^1, v_i^2, v_i^3, v_i^4\}$ and the edge set $E = E_1 \cup E_2 \cup ... \cup E_n$ where $E_i = \{e_i^1, e_i^2, e_i^3, e_i^4, e_i^5, e_i^6\}$. let C_m be the cycle $u_1u_2...u_mu_1$. The graph $n\Gamma(Z_{25}) \cup C_m$ has $4n+m$ vertices and $6n+m$ edges. Define a function $f : V(n\Gamma(Z_{25}) \cup C_m) \to \{1, 2, ..., q+1\}$ as

$$
\begin{aligned}
f(v_i^1) &= 1 \\
f(v_i^j) &= f(v_i^{j-1}) + 2, && 1 \le i \le n, j = 2, 3 \\
f(v_i^4) &= f(v_i^3) + 1, && 1 \le i \le n \\
f(v_i^1) &= f(v_{i-1}^4) + 1, && 2 \le i \le n \quad \text{and} \\
f(u_i) &= f(v_n^4) + i, && 1 \le i \le m
\end{aligned}
$$

In the above pattern f is a geometric mean labeling of $n\Gamma(Z_{25}) \cup C_m$. Hence, $n\Gamma(Z_{25}) \cup C_m$ is a geometric mean graph, for $n \ge 1$ and $m \ge 3$. □

Remark 2.7.

(1). $n\Gamma(Z_{25}) \cup mC_k$ is the geometric mean graph for $n, m \ge 1$ and $k \ge 3$.

(2). $n\Gamma(Z_6)$, $n\Gamma(Z_8)$, $n\Gamma(Z_9)$, $n\Gamma(Z_8) \cup C_m$, $n\Gamma(Z_9) \cup C_m$, $n\Gamma(Z_6) \cup C_m$, $n\Gamma(Z_6) \cup m\Gamma(Z_{25})$, $n\Gamma(Z_8) \cup m\Gamma(Z_{25})$, $n\Gamma(Z_9) \cup m\Gamma(Z_{25})$, *and* $n\Gamma(Z_6) \cup mC_k$, $n\Gamma(Z_8) \cup mC_k$, $n\Gamma(Z_9) \cup mC_k$ *are not geometric mean graphs. Because all the above have number of vertices is greater than the number of edges.*

Theorem 2.8. *If* $\Gamma(Z_{2p})$ *and* $\Gamma(Z_8)$ *are trees then* $n\Gamma(Z_{2p}) \cup \Gamma(Z_8)$ *is not a geometric mean graph.*

Proof. Let $\Gamma(Z_{2p}) = (p_1, q_1)$ and $\Gamma(Z_8) = (p_2, q_2)$ be the given trees and let $\Gamma(Z_{2p}) \cup \Gamma(Z_8)$ be a (p, q) graph therefore, $p = p_1 + p_2$ and $q = q_1 + q_2$. Since, $\Gamma(Z_{2p})$ and $\Gamma(Z_8)$ are trees, $q_1 = p_1 - 1$, $q_2 = p_2 - 1$ then $q = q_1 + q_2 = p_1 - 1 + p_2 - 1 = p_1 + p_2 - 2 = p - 2$. Clearly, the number of vertices is greater then the number of edges. Hence, $\Gamma(Z_{2p}) \cup \Gamma(Z_8)$ is not a geometric mean graph. □

Remark 2.9.

(1). $\Gamma(Z_{2p}) \cup \Gamma(Z_6)$ *is not a geometric mean graph.*

(2). $\Gamma(Z_{2p}) \cup \Gamma(Z_9)$ *is not a geometric mean graph.*

(3). $\Gamma(Z_{2p}) \cup \Gamma(Z_{2p})$ *is not a geometric mean graph.*

(4). $\Gamma(Z_6) \cup \Gamma(Z_6)$ *is not a geometric mean graph.*

(5). $\Gamma(Z_6) \cup \Gamma(Z_8)$ *is not a geometric mean graph.*

(6). $\Gamma(Z_6) \cup \Gamma(Z_9)$ *is not a geometric mean graph.*

(7). $\Gamma(Z_8) \cup \Gamma(Z_8)$ *is not a geometric mean graph.*

(8). $\Gamma(Z_8) \cup \Gamma(Z_9)$ *is not a geometric mean graph.*

(9). $\Gamma(Z_9) \cup \Gamma(Z_9)$ *is not a geometric mean graph.*

Theorem 2.10. *The planer graph* $\Gamma(Z_9) \times \Gamma(Z_6)$ *is a geometric mean graph.*

Proof. Let the vertex set of $\Gamma(Z_9) \times \Gamma(Z_6)$ be $\{a_{ij}, 1 \le i \le 2, 1 \le j \le 3\}$ and the edge set be $E(\Gamma(Z_9) \times \Gamma(Z_6)) = \{a_{i(j-1)}a_{ij}, 1 \le i \le 2, 2 \le j \le 3\} \cup \{a_{(i-1)j}a_{ij}, 2 \le i \le 2, 1 \le j \le 3\}$. The Graph $\Gamma(Z_9) \times \Gamma(Z_6)$ has 6 vertices and 7 edges. Define a function $f : V(\Gamma(Z_9) \times \Gamma(Z_6)) \to [1, 2, ..., q+1]$ as $f(a_{ij}) = j$, $i = 1$, $1 \le j \le 3$ and $f(a_{ij}) = f(a_{(i-1)3}) + 2 + j$, $2 \le i \le 2$, $1 \le j \le 3$. The label of the edge $a_{ij}a_{i(j+1)}$ is $5(i-1) + j$, $1 \le i \le 2$, $1 \le j \le 2$. The label of the edge $a_{ij}a_{(i+1)j}$ is $5(i-1) + 2 + j$, $1 \le i \le 1$, $1 \le j \le 3$. Hence, $\Gamma(Z_9) \times \Gamma(Z_6)$ is a geometric mean graph. □

Remark 2.11.

(1). $\Gamma(Z_6) \times \Gamma(Z_6)$ *is a geometric mean graph.*

(2). $\Gamma(Z_6) \times \Gamma(Z_8)$ *is a geometric mean graph.*

(3). $\Gamma(Z_8) \times \Gamma(Z_8)$ *is a geometric mean graph.*

(4). $\Gamma(Z_8) \times \Gamma(Z_9)$ *is a geometric mean graph.*

(5). $\Gamma(Z_9) \times \Gamma(Z_9)$ *is a geometric mean graph.*

Theorem 2.12. *Let* G *be a graph obtained from* $P_n \times \Gamma(Z_9)$ *by detecting an egde that joins to end points of the* p_n *paths.*

Proof. Let the graph G defined above has $2n$ vertices and $3n-3$ edges. Because, we know that $\Gamma(Z_9)$ is isomorphic to p_2. Define a function $f : V(G) \to \{1, 2, ..., q+1\}$ as

$$f(u_1) = 1$$
$$f(u_i) = 3i - 3 \text{ if } 2 \leq i \leq n$$
$$f(v_i) = 3i - 1 \text{ if } 1 \leq i \leq n-1 \text{ and}$$
$$f(v_n) = 3n - 2.$$

Then the label of the edge u_iv_i is $3i-2$, $1 \leq i \leq n-1$. The label of the edge u_iu_{i+1} is $3i-1$, $1 \leq i \leq n-1$. the label of the edge v_iv_{i+1} $3i$, $1 \leq i \leq n-1$. Hence, the graph $P_n \times \Gamma(Z_9)$ is a geometric mean graph. □

3. Conclusion

In this paper we mention several open construction and theorems related to Geometric mean labeling of zero divisor graphs. The flavour of finding the labeling of $\Gamma(Z_n)$, particularly in a geometric mean labeling is similar to that of determining properties of vertices arrangements in the plane. The main difficulty is finding the geometric mean is that a graph has so many essentially different drawings that the computation of any of the labelings for a graph of only 35 vertices, appears to be a hopelessly difficult task, even for a very fast computer. Finally, we conclude the following results,

(1). For $m \geq 3$ and $\Gamma(Z_n)$ is a path graph; then $C_m \cup \Gamma(Z_n)$ is a geometric mean graph.

(2). For any path $\Gamma(Z_n)$, $mCn \cup \Gamma(Z_n)$ is a geometric mean graph for $m \geq 1$ and the length of cycle is at least 3.

(3). $nK_3 \cup \Gamma(Z_n)$ is a geometric mean graph for $n \geq 1$.

(4). $n\Gamma(Z_{25})$ is a geometric mean graph for $n \geq 1$.

(5). (a). $n\Gamma(Z_{25}) \cup \Gamma(Z_6)$ is a geometric mean graph.

(b). $n\Gamma(Z_{25}) \cup \Gamma(Z_8)$ is a geometric mean graph.

(c). $n\Gamma(Z_{25}) \cup \Gamma(Z_9)$ is a geometric mean graph.

(6). $n\Gamma(Z_{25}) \cup C_m$ is a geometric mean graph for $n \geq 1$ and $m \geq 3$.

(7). If $\Gamma(Z_{2p})$ and $\Gamma(Z_8)$ are trees then $\Gamma(Z_{2p} \cup \Gamma(Z_8)$ is not a geometric mean graph.

(a). $\Gamma(Z_{2p}) \cup \Gamma(Z_6)$ is not a geometric mean graph.

(b). $\Gamma(Z_{2p}) \cup \Gamma(Z_9)$ is not a geometric mean graph.

(c). $\Gamma(Z_{2p}) \cup \Gamma(Z_{2p})$ is not a geometric mean graph.

(d). $\Gamma(Z_6) \cup \Gamma(Z_6)$ is not a geometric mean graph.

(e). $\Gamma(Z_6) \cup \Gamma(Z_8)$ is not a geometric mean graph.

(f). $\Gamma(Z_6) \cup \Gamma(Z_9)$ is not a geometric mean graph.

(g). $\Gamma(Z_8) \cup \Gamma(Z_8)$ is not a geometric mean graph.

(h). $\Gamma(Z_8) \cup \Gamma(Z_9)$ is not a geometric mean graph.

(8). The planer graph $\Gamma(Z_9) \times \Gamma(Z_6)$ is a geometric mean graph.

(9). Let G be a graph obtained from $P_m \times \Gamma(Z_9)$ by detecting an edge that joins to end points of the p_n paths.

Finally, we conclude that the finding geometric mean labeling problems are easy to state but notoriously and profoundly difficult to solve.

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