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Edge Sum Index of a Graph in a Commutative Ring

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Abstract: Let $\Gamma(Z_n)$ be a graph. A bijection $f : E(\Gamma(Z_n)) \to Z^+$, where Z^+ is a set of positive integers is called an edge mapping of the graph $\Gamma(Z_n)$. Now, we define, $F(v) = \Sigma\{f(e);$ e is incident on $v\}$ on $V(\Gamma(Z_n))$. Then, F is called the edge sum mapping of the edge mapping f. $\Gamma(Z_n)$ is said to be an edge sum graph if there exists an edge mapping $f : E(\Gamma(Z_n)) \to N^+$ such that f and its corresponding edge sum mapping. F on $V(\Gamma(Z_n))$ satisfy the following conditions: (i) F is into mapping toZ^+. That is, $F(v) \in Z^+$, for every $v \in E(\Gamma(Z_n))$. (ii) If $e_1, e_2, \ldots, e_n \in E(\Gamma(Z_n))$ such that $f(e_1) + f(e_2) + \ldots f(e_n) \in Z^+$, then $e_1, e_2, \ldots, e_n$ are incident on a vertex in $\Gamma(Z_n)$. In this paper, we evaluated the edge sum index of some standard graphs in zero divisor graph.

MSC: 05C25, 05C70.

Keywords: Graph Lebeling, Edge Sum Index of a Graph, Commutative Ring.

1. Introduction

Let R be a commutative ring and let Z(R) be its set of zero-divisors. We associate a graph $\Gamma(R)$ to R with vertices $Z(R)^* = Z(R) - \{0\}$, the set of non-zero divisors of R and for distinct $u, v \in Z(R)^*$, the vertices u and v are adjacent if and only if $uv = 0$. The zero divisor graph is very useful to find the algebraic structures and properties of rings. The idea of a zero divisor graph of a commutative ring was introduced by I. Beck in [2]. The first simplication of Beck's zero divisor graph was introduced by D. F. Anderson and P. S. Livingston [1]. Their motivation was to give a better illustration of the zero divisor structure of the ring. D. F. Anderson and P. S. Livingston, and others, e.g., [5, 6, 7], investigate the interplay between the graph theoretic properties of $\Gamma(R)$ and the ring theoretic properties of R. Throughout this paper, we consider the commutative ring R by Z_n and zero divisor graph $\Gamma(R)$ by $\Gamma(Z_n)$. The egde sum labelings was introduced by Paulraj Joseph et al.,[3, 4]. In this paper, we discuss the concepts of edge sum lebeling of some standard graphs in zero divisor graphs. Let us consider a graph, $V(\Gamma(Z_n)) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ be the vertex set and $E(\Gamma(Z_n)) = \{v_1v_4, v_2v_3, v_2v_5, v_3v_4, v_3v_5, v_3v_6, v_4v_5, v_4v_7, v_5v_6\}$ be the edge set of the graph $\Gamma(Z_n)$. The edge mapping $f : E(\Gamma(Z_n)) \to Z^+$ is defined by $f(v_1v_4) = 3, f(v_2v_3) = 5, f(v_2v_5) = 2, f(v_3v_4) = 9, f(v_3v_5) = 10, f(v_3v_6) = 6, f(v_4v_5) = 8f(v_4v_7) = 11, f(v_5v_6) = 12$. The corresponding edge sum mapping F is given by, $F(v_1) = 3, F(v_2) = 7, F(v_3) = 20, F(v_4) = 31, F(v_5) = 32, F(v_6) = 18, F(v_7) = 11$. Clearly $\Gamma(Z_R)$ is an edge sum graph.

Theorem 1.1. *Let $\Gamma(Z_n)$ be an edge sum graph. Then $\Gamma(Z_9)$ is a component of $\Gamma(Z_n)$.*

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Proof. Let $\Gamma(Z_n)$ be an edge sum graph with edge mapping $f : E(\Gamma(Z_n)) \to Z^+$ and edge sum mapping F. Let z be the largest number in Z^+. Since, $f : E(\Gamma(Z_n)) \to Z^+$ is a bijection, there exist an edge e=uv joining the vertices u and v such that f(e)=z. Our aim is, to prove that both u and v are pendent vertices in $\Gamma(Z_n)$.

Case (1): Let u is adjacent to a vertex other than v, say us clearly, $F(u) \geq f(uv) + f(uw) > f(uv) = z$. This is a contradiction to our assumption z is the largest number in Z^+. Clearly, u is a pendent vertex in $\Gamma(Z_n)$.

Case (2): Let v is adjacent to a vertex other than u, say r. Clearly, $F(v) \geq f(vu) + f(vr) \geq f(vu) = z$. This is a contradiction to our assumption z is the largest number in Z^+. Clearly, v is a pendent vertex in $\Gamma(Z_n)$.

Case (3): Let u and v are adjacent with a common vertex, say t. Using case(i) and case(ii), we got a contradiction for Z is a largest number in Z^+.

Similarly, let u and v are adjacent with different vertices, say a and b. Once again using case(i) and case(ii), we get a contradiction. Therefore, the vertices u and v form a $\Gamma(Z_9)$ component in $\Gamma(Z_n)$. □

Theorem 1.2. *Let $\Gamma(Z_n)$ be an edge sum graph with edge mapping $f : E(\Gamma(Z_n)) \to Z^+$ and edge sum mapping F. Let $e_1, e_2, \ldots, e_n$ where $n > 1$ be a collection of edges incident on a vertex $u \in V(\Gamma(Z_n))$. Let $\acute{e_1}, \acute{e_2}, \ldots \acute{e_m}$ be another collection of edges incident on a verrtex v such that $f(e_1) + f(e_2) + \cdots + f(e_n) = f(\acute{e_1}) + f(\acute{e_2}) + \cdots + f(\acute{e_m})$. Then, the degree of u and degree of v belongs to $\{n, (n+1)\} \times \{m, (m+1)\}$ and one of the following statements holds:*

(1). u and v are adjacent and (deg u, deg v) $\neq (n, m)$.

(2). u and v are non adjacent and (deg u, deg v)=(n,m).

Proof. We divide into two cases with respect to deg u.

Case (1): *deg* $u = n$. That is, $e_1, e_2, \ldots, e_n$ are the only edges incident on u. Then, $f = F(u) \in Z^+$. Hence, $f(e_1) + f(e_2) + \cdots + f(e_n) = f(\acute{e_1}) + f(\acute{e_2}) + \cdots + f(\acute{e_m}) = F(u) \in Z^+$. Hence, $\acute{e_1}, \acute{e_2}, \ldots \acute{e_m}$ are all incident on a vertex v. Since, we know that none of the edges $\acute{e_i}$ is incident on the vertex u. Clearly, $u \neq v$. Let deg v=m+k and $\acute{e_1}, \acute{e_2}, \ldots \acute{e_m}, \acute{e_{m+1}}, \ldots \acute{e_{m+k}}$ be the edges incident on v. Then, $f(v) = f(\acute{e_1}) + f(\acute{e_2}) + \cdots + f(\acute{e_m}) + f(\acute{e_{m+1}}), \cdots + f(\acute{e_{m+k}}) = f(e_1) + f(e_2) + \cdots + f(e_n) + f(\acute{e_{m+1}}) + \cdots + f(\acute{e_{m+k}}) \in Z^+$. Hence, $e_1, e_2, \ldots, e_n, \acute{e_{m+1}}, \ldots \acute{e_{m+k}}$ are all incident on a vertex v. But we know that $e_1, e_2, \ldots .e_n$ are incident on u and the edges $\acute{e_{m+1}}, \acute{e_{m+2}}, \ldots, \acute{e_{m+k}}$ are incident on v and there can be atmost one edge incident or both the vertices. Therefore $k = 0$ or $k = 1$. When, $k = 0$, either u and v are not adjacent and *deg* $u = n$, *deg* $v = m$ (or) u and v are adjacent with one edge $e_i = uv$ for $1 \leq i \leq n$ and *deg* $u = n$, *deg* $v = m + 1$. When $k = 1$, u and v are adjacent with $uv = \acute{e_{m+1}}$ and *deg* $u = n$, *deg* $v = m + 1$.

Case (2): *deg* $u > n$. Let *deg* $u = (n + s)$ with $s > 0$. Let $e_1, e_2, \ldots, e_n, e_{n+1}, \ldots, e_{n+s}$ be the edges incident on u. Then, $F(u) = f(e_1) + f(e_2) + \cdots + f(e_n) + f(e_{n+1}) + \cdots + f(e_{n+s}) \in Z^+ = f(\acute{e_1}) + f(\acute{e_2}) + \cdots + f(\acute{e_m}) + f(e_{n+1}) + \cdots + f(e_{n+s}) \in Z^+$.

Hence $\acute{e_1}, \acute{e_2}, \ldots \acute{e_m}, e_{n+1}, e_{n+2}, \ldots, e_{n+s}$ are all incident on a vertex. Let us consider that vertex as v. But $\acute{e_1}, \acute{e_2}, \ldots \acute{e_m}$ are all not incident on u and $e_{n+1}, \ldots, e_{n+s}$ are incident on u and therefore $v \neq u$. As those can be atmost one edge incident on both the vertices, s=0 or s=1. Since, therefore u and v are adjacent with $uv = e_i$ for same $i, 1 \leq i \leq m+1$, and *deg* $u = n+1$ and *deg* $v = m + 1$. □

Theorem 1.3. *Let $\Gamma(Z_n)$ be a edge mapping $f : E(\Gamma(Z_n)) \to Z^+$ and edge sum function F. If v is the only vertex such that $Fv \notin Z^+$, then v is adjacent to a pendent vertex.*

Proof. Let $deg\ v = m$ and $v_1, v_2, \dots, v_m$ be the vertices adjacent to v. None, we define a new graph $\Gamma(\acute{Z_n})$ with respect to the vertex v, namely that graph is called bloom the vertex v. The following conditions are holds for bloom graph of the vertex v.

(1). Number of vertices in $\Gamma(\acute{Z_n})$ is greater than number of vertices in $\Gamma(Z_n)$ by $m - 1$.

(2). Number of edges in $E(\Gamma(Z_n))$ and $E(\Gamma(\acute{Z_n}))$ are equal.

(3). If we define $\acute{f} : E(\Gamma(\acute{Z_n})) \to Z^+$ as $\acute{f}(e) = f(e)$ for all $e \in E(\Gamma(Z_n)) \cap E(\Gamma(\acute{Z_n}))$ and $\acute{f}(v_i\acute{v_i}) = f(vv_i)$ for $1 \leq i \leq m$, it is easy to see that $\acute{f} : E(\Gamma(\acute{Z_n})) \to Z^+$ is an edge mapping and its edge sum mapping $\acute{F}$ or $V\Gamma(\acute{Z_n}))$ is $\acute{F}(u) = F(u)$ for all $u \in V(\Gamma(Z_n)) \cap V(\Gamma(\acute{Z_n}))$ and $\acute{f}(\acute{v_i}) \in Z^+ for 1 \leq i \leq m$ where as $F(v) \notin Z+$.

Bloom the vertex v_1 we get a new graph $\Gamma(\acute{Z_n})$ and let $\acute{V}$ be the vertex set of $\Gamma(\acute{Z_n})$ and $\acute{E}$ be the edge set of $\Gamma(\acute{Z_n})$. By definition of bloom graph of vertex v, $\acute{V} = \{V - \{v\}\} \cup \{\acute{v_1}, \acute{v_2}, \dots, \acute{v_m}\}$ and $\acute{E} = [E - \{vv_1, vv_2, \dots, vvm\}] \cup \{v_1\acute{v_1}, v_2\acute{v_2} \dots, v_m\acute{v_m}\}$, where E be the edge set of $\Gamma(Z_n)$.
The edge mapping $f : E(\Gamma(Z_n)) \to Z^+$ gives rise to an edge mapping $\acute{f} : E(\Gamma(\acute{Z_n})) \to Z^+$ of the graph $\Gamma(\acute{Z_n})$ such that the edge sum of mapping $\acute{F}$ of $\acute{f}$ has the following conditions: that is $\acute{F}(u) = F(u)$ for all $u \in V \cap \acute{V}$ and hence, $\acute{F}(u) \in Z^+$ for all $u \in V \cap \acute{V}$ and $\acute{F}(\acute{V_i}) \in Z^+$ for $1 \leq i \leq m$. Hence, $\Gamma(\acute{Z_n})$ is an edge sum graph. Using theorem (1.1), $\Gamma(\acute{Z_n})$=$\Gamma(Z_9)$ or $\Gamma(Z_9)$ is a component of $\Gamma(\acute{Z_n})$. Since, $\Gamma(Z_n)$ is connected graph, one of $v_i\acute{v_i}$ is a $\Gamma(Z_9)$ is a component of $\Gamma(\acute{Z_n})$ which implies that v_i is a pendent vertex in $\Gamma(Z_n)$ adjacent to v. Hence, v is the only vertex such that $F(v) \notin Z^+$, then v is adjacent to a pendent vertex. □

Theorem 1.4. *Let $\Gamma(Z_n)$ be a non pendent vertices graph. Let $f : E(\Gamma(Z_n)) \to Z^+$ be an edge mapping of $\Gamma(Z_n)$ and F be the edge sum mapping of f. Let $v_1, v_2, \dots, v_m$ be the vertices of $\Gamma(Z_n)$ such that $F(v_i) \notin Z^+$ for $1 \leq i \leq m$. Then, the induced subgraph of G with the vertex set $\{v_1, v_2, \dots, v_m\}$ is not $\overline{\Gamma(Z_{p^2})}$, where $p \geq 5$ is any prime number.*

Proof. Using above Theorem 1.3, bloom the vertices $v_1, v_2, \dots, v_m$ in $\Gamma(Z_n)$, we get a new graph $\Gamma(Z_n)^*$ which is an edge sum graph. Therefore, $\Gamma(Z_n)^* = \Gamma(Z_9)$ or $\Gamma(Z_9)$ is a component of $\Gamma(Z_n)^*$. since, $\Gamma(Z_n)$ has no pendent vertex only an edge between v_i and v_j will be a $\Gamma(Z_9)$ component of $\Gamma(Z_n)^*$. Then, the induced subgraph of $\Gamma(Z_n)$ with the vertex set $\{v_1, v_2, \dots, v_m\}$ has the edge v_iv_j and is not $\overline{\Gamma(Z_{p^2})}$. Hence, proved. □

Theorem 1.5. *Let $\Gamma(Z_n)$ be a non pendent vertices graph. Let $f : E(\Gamma(Z_n)) \to Z^+$ be an edge mapping of $\Gamma(Z_n)$ and F be the edge sum mapping of f. If u and v are the only two vertices such that $F(u), F(v) \notin Z^+$, then u and v are adjacent.*

Proof. Let $deg\ v = m$ and $v_1, v_2, \dots, v_m$ be the vertices adjacent to v. Blooming the vertex v, we get a new graph $\Gamma(Z_n)^* = (\acute{V}, \acute{E})$ where, $\acute{V} = \{V - (v)\} \cup \{\acute{v_1}, \acute{v_2}, \dots, \acute{v_m}\}$ and $\acute{E} = \{E - \{vv_1, vv_2, ..., vv_m\}\} \cup \{v_1\acute{v_1}, v_2\acute{v_2}, \dots, v_m\acute{v_m}\}$. The edge mapping $f : E(\Gamma(Z_n)) \to Z^+$ of the graph $\Gamma(Z_n)$ given rise to an edge mapping $\acute{f} : \acute{E}(\Gamma(Z_n)^*) \to Z^+$ of the graph $\Gamma(Z_n)^*$ such that the edge sum mapping $\acute{F}$ of $\acute{f}$ has the following property: $\acute{F}(u) = F(u)$ for all $u \in V \cap \acute{V}$ and hence $\acute{F}(u) \in Z^+$ for all $u \in V \cap \acute{V}$ and $F(\acute{v}) \in Z^+$ for $1 \leq i \leq m$. Hence, $\Gamma(Z_n)^*$ is an edge sum graph. therefore, $\Gamma(Z_n)^*$=$\Gamma(Z_9)$ or $\Gamma(Z_9)$ is a component of $\Gamma(Z_n)^*$. Using Theorem 1.2, let $e_1, e_2, \dots, e_n$, where, $n > 1$ be a collection of edges incident on a vertex $u \in V(\Gamma(Z_n))$, such that u and v are adjacent and $(deg\ u, deg\ v) \neq (n, m)$. Using Theorem 1.3, if v is the only verteex such that $F(v) \notin Z^+$, then v is adjacent to a pendent vertex. But, an assumption, $\Gamma(Z_n)$ contains no pendent vertices, which implies v is adjacent to same non-pendent vertices. Using Theorem 1.4, let v_1, v_2, v_m be the vertices of $\Gamma(Z_n)$

such that $F(v_i) \notin Z^+$ for $1 \leq i \leq m$.Then the induced subgraph of $\Gamma(Z_n)$ with the vertex set $\{v_1, v_2, \ldots, v_n\}$ is not $\overline{\Gamma(Z_{p^2})}$. Clearly, let $m = 2$, in Theorem 1.4, we get $v_1 = v$ and $v_2 = u$ such that u and v are adjacent vertices and $F(u), F(v) \notin Z^+$. Hence proved theorem. □

Let $\Gamma(Z_n)$ be an edge sum mapping graph. Let v_1, v_2, v_m be the vertices of $\Gamma(Z_n)$ such that $F(v_i) \notin Z^+$ for $1 \leq i \leq m$.Then, any vertex v_i is adjacent with non pendent vertices such that, the induced subgraph of $\Gamma(Z_n)$ with the vertex set $\{v_1, v_2, \ldots, v_n\}$ is not $\overline{\Gamma(Z_{p^2})}$.

Theorem 1.6. *Let $\Gamma(Z_n)$ be an edge sum graph with edge mapping $f : E(\Gamma(Z_n)) \to Z^+$ and edge sum mapping F of f . Let w be a non pendent vertex and $e = uv \in E(\Gamma(Z_n))$ be such that $F(w) = F(uv) = F(e)$. Then one of the following Holds:*

(1). $\{u, v\}$ forms a $\Gamma(Z_9)$ component in $\Gamma(Z_n)$.

(2). There is no induced subgraph $< \{u, v, w\} >$ in $\Gamma(Z_n)$.

(3). Otherwise $< \{u, v, w\} >$ is a P_2 graph with one of u,v as a pendent vertex in $\Gamma(Z_n)$. That is p_2 is isomorphic with $\Gamma(Z_9)$.

Proof. Let $w_1, w_2, \ldots, w_n$ be the vertices adjacent to w. Then, $F(w) = f(e_1) + f(e_2) + \cdots + f(e_n) \in Z^+$, where $e_i = ww_i$ for $1 \leq i \leq n$. Let us consider the case when u is not adjacent to w. let u be adjacent to $u_1, u_2, \ldots, u_m$ apart from v. Then,

$$\begin{aligned} F(u) &= f(\acute{e_1}) + f(\acute{e_2}) + \cdots + f(\acute{e_m}) + f(uv) \\ &= f(\acute{e_1}) + f(\acute{e_2}) + \cdots + f(\acute{e_m}) + f(e) \in Z^+, \text{ where } \acute{e_i} = uu_i \text{ for } 1 \leq i \leq m. \\ &= f(\acute{e_1}) + f(\acute{e_2}) + \cdots + f(\acute{e_m}) + f(e_1) + f(e_2) + \cdots + f(e_n) \in Z^+. \end{aligned}$$

Hence, $\acute{e_1}, \acute{e_2}, \ldots, \acute{e_m}, e_1, e_2, \ldots, e_n$ are incident on a vertex. But, we know that $\acute{e_1}, \acute{e_2}, \ldots, \acute{e_m}$ are incident on u and $e_1, e_2, \ldots, e_n$ are incident on w. Since, u and w are non adjacent vertices and m=0, implies that u is a pendent vertex. Clearly, $\{u, v\}$ forms a $\Gamma(Z_9)$ component in $\Gamma(Z_n)$. Using Theorem 1. 1, $\Gamma(Z_n)$ contains a component of $\Gamma(Z_9)$. But, we know that $\Gamma(Z_9)$ contains only two vertices 3 and 6. So, there is impossible to find one more vertex in $\Gamma(Z_9)$. Therefore, there is no induced subgraph $< \{u, v, w\} >$in $\Gamma(Z_n)$. Suppose both u and v are not adjacent vertices to w then both are pendent vertices forming a $\Gamma(Z_9)$ component in $\Gamma(Z_n)$. Clearly, any of the vertex $\{u, v\}$ is adjacent to w and other is non adjacent vertex. Clearly, the second vertex is a pendent vertex which gives a path length two. that is $\Gamma(Z_n)$ contains a component in $\Gamma(Z_9)$, which implies that $\Gamma(Z_9)$ is isomorphic to P_2. Hence, proved. □

Theorem 1.7. *Let $\Gamma(Z_n)$ be an edge sum graph with edge mapping $f : E(\Gamma(Z_n)) \to Z^+$ and edge sum mapping F of f. Let $\acute{e_1}, \acute{e_2}, \ldots, \acute{e_n}$, where $n > 1$ be a collection of edges incident on a vertex w. Let $ww_i = \acute{e_i}$, for $1 \leq i \leq n$. If there exists an edge e=uv such that $f(\acute{e_1}) + f(\acute{e_2}) + \cdots + f(\acute{e_n}) = f(uv) = f(e)$, then one of the following holds:*

(1). $\{u, v\}$ forms a $\Gamma(Z_9)$ component in G.

(2). $< \{u, v, w\} >$ is P_2 or P_1 with one of u,v as a pendent vertex in $\Gamma(Z_n)$.

Proof.

Case (1): u is not adjacent to w. Let u be adjacent to $u_1, u_2, \ldots, u_m$ apart from v. Then,

$$F(u) = f(e_1) + f(e_2) + \cdots + f(e_m) + f(uv)$$

$$= f(e_1) + f(e_2) + \cdots + f(e_m) + f(e) \in Z^+, \quad \text{where } e_i = uu_i \text{ for } 1 \leq i \leq n.$$
$$= f(e_1) + f(e_2) + \cdots + f(e_m) + f(\acute{e_1}) + \cdots + f(\acute{e_n}) \in Z^+.$$

Hence, $e_1, e_2, \ldots, e_m, \acute{e_1}, \acute{e_2}, \ldots, \acute{e_n}$ are incident on a vertex . But $e_1, e_2, \ldots, e_m$ are incident on u and $\acute{e_1}, \acute{e_2}, \ldots, \acute{e_n}$ are incident on w. Since, u and w are not adjacent vertices with $m = 0$ and u is a pendent vertex.

Case (2): u is adjacent to w and $uw \neq \acute{e_i}$, for $1 \leq i \leq n$. Let u be adjacent to $u_1, u_2, \ldots, u_m$ other than v and w. Then,

$$F(u) = f(e_1) + \cdots + f(e_m) + f(uv) + f(uw) \in Z^+, \quad \text{where, } e_i = uu_i \text{ for } 1 \leq i \leq m.$$
$$= f(e_1) + f(e_2) + \cdots + f(e_m) + f(\acute{e_1}) + f(\acute{e_2}) + \cdots + f(\acute{e_n}) + f(uw) \in Z^+.$$

hence, $e_1, e_2, \ldots, e_m, \acute{e_1}, \acute{e_2}, ..., \acute{e_n}$ and uw are incident on a vertex. But $e_1, e_2, \ldots, e_m$ are incident on u , $\acute{e_1}, \acute{e_2}, \ldots, \acute{e_n}$ are incident on w and uw is the only edge incident on both u and w. hence, $m = 0$ and u is adjacent only to v and w. The other two possible cases are u is adjacent to w with $uw = \acute{e_i}$, for some i, $1 \leq i \leq n$ and that u coincides with w. hence, if both u and v are not adjacent to w, they form a $\Gamma(Z_9)$ component in $\Gamma(Z_n)$; if one of u,v say u, is adjacent to w with $uw \neq \acute{e_i}$ for $1 \leq i \leq n$, thenn deg u=2 and v is a pendent vertex, so that $< \{u, v, w\} >= P_2 \cong \Gamma(Z_9)$; if u is adjacent to w with $uw = \acute{e_i}$ for some i and v is not adjacent to w, then $< \{u, v, w\} is P_2$ with v is a pendent vertex in $\Gamma(Z_n)$. Hence, proved. □

Theorem 1.8. *Let $\Gamma(Z_n)$ be an edge sum graph with edge mapping $f : E(\Gamma(Z_n)) \to Z^+$ and edge sum mapping F of f. Let $e_1, e_2, \ldots, e_n$ be the edges incident on u and $\acute{e_1}, \acute{e_2}, \ldots, \acute{e_m}$ be on v. If there exists proper edge subset $e_1, e_2, \ldots, e_r$ of $e_1, e_2, \ldots, e_n$ and $\acute{e_1}, \acute{e_2}, \ldots, \acute{e_s}$ of $\acute{e_1}, \acute{e_2}, \ldots, \acute{e_m}$ such that $f(e_1) + f(e_2) + \cdots + f(e_r) + f(\acute{e_1}) + f(\acute{e_2}) + \cdots + f(\acute{e_s})$, then u and v are adjacent and $r = n - 1$ and $s = m - 1$.*

Proof. We know that,

$$F(u) = f(e_1) + f(e_2) + \cdots + f(e_r) + f(e_{r+1}) + \cdots + f(e_n) \in Z^+, \quad \text{where } r < n.$$
$$= f(\acute{e_1}) + f(\acute{e_2}) + \cdots + f(\acute{e_s}) + f(e_{r+1}) + \cdots + f(e_n) \in Z^+,$$

Hence, $\acute{e_1}, \acute{e_2}, \ldots, \acute{e_s}, e_{r+1}, \ldots, e_n$ are all incident on a vertex in $\Gamma(Z_n)$. But $\acute{e_1}, \acute{e_2}, \ldots, \acute{e_s}$ are incident on u. Therefore, $n = r + 1$ and $e_{r+1} = uv$. Similarly, $s = m - 1$ and $\acute{e_{s+1}} = uv$. that is, $r = n - 1$, $s = m - 1$ and $\acute{e_m} = e_n = uv$. Hence, proved. □

References

[1] D.F.Anderson and P.S.Livingston, *The zero-divisor graph of a commutative ring*, J. Algebra, 217(2)(1999), 434-447.

[2] I.Beck, *Colouring of Commutative Rings*, J. Algebra, 116(1988), 208-226.

[3] F.Harary, *Sum Graphs and Difference Graphs*, Congress. Number, 72(1990), 101-108.

[4] J.Paulraj Joseph, D.S.T.Ramesh and S.Somasundaram, *Edge sum graphs*, IJOMAS, 18(1)(2002), 71-78.

[5] J.Ravi Sankar and S.Meena, *Changing and Unchanging the Domination Number of a Commutative ring*, International Journal of Algebra, 6(27)(2012), 1343-1352.

[6] J.Ravi Sankar and S.Meena, *Connected Domination number of a commutative ring*, International Journal of Mathematical Research, 5(1)(2012), 5-11.

[7] J.Ravi Sankar and S.Meena, *On Weak Domination in a Zero Divisor Graph*, International Journal of Applied Mathematics, 26(1)(2013), 83-91.