Int. J. Math. And Appl., 6(1–A)(2018), 139–145
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Cordiality in the Context of Duplication in Helm and Closed Helm

U. M. Prajapati[1,*] and R. M. Gajjar[2]

1 Department of Mathematics, St. Xavier's College, Ahmedabad, Gujarat, India.

2 Research Scholar, Department of Mathematics, Gujarat University, Ahmedabad, Gujarat, India.

Abstract: Let $G = (V(G), E(G))$ be a graph and let $f : V(G) \to \{0, 1\}$ be a mapping from the set of vertices to {0,1} and for each edge $uv \in E$ assign the label $|f(u) - f(v)|$. If the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labled with 0 and the number of edges labeled with 1 differ by at most 1, then f is called a cordial labeling. We discuss cordial labeling of graphs obtained from duplication of certain graph elements in helm and closed helm.

MSC: 05C78.

Keywords: Graph Labeling, Cordial Labeling, Cordial Graph.

1. Introduction

We begin with simple, finite, undirected graph $G = (V(G), E(G))$ where $V(G)$ and $E(G)$ denotes the vertex set and the edge set respectively. For a finite set A, $|A|$ denotes the number of elements of A. For all other terminology we follow Gross [2]. We provide some useful definitions for the present work.

Definition 1.1. *The graph labeling is an assignment of numbers to the vertices or edges or both subject to certain condition(s).*

A detailed survey of various graph labeling is explained in Gallian [1].

Definition 1.2. *For a graph $G = (V(G), E(G))$, a mapping $f : V(G) \to \{0, 1\}$ is called a binary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f. For an edge $e = uv$, the induced edge labeling $f^* : E(G) \to \{0, 1\}$ defined as $f^*(uv) = |f(u) - f(v)|$. Let $v_f(0), v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and let $e_f(0), e_f(1)$ be the number of edges having labels 0 and 1 respectively under f^*.*

Definition 1.3. *Duplication of a vertex v of a graph G produces a new graph G' by adding a new vertex v' such that $N(v') = N(v)$. In other words a vertex v' is said to be duplication of v if all the vertices which are adjacent to v in G are also adjacent to v' in G'.*

* E-mail: udayan64@yahoo.com

Definition 1.4. *Duplication of an edge $e = uv$ of a graph G produces a new graph G' by adding an edge $e' = u'v'$ such that $N(u') = N(u) \cup \{v'\} - \{v\}$ and $N(v') = N(v) \cup \{u'\} - \{u\}$.*

Definition 1.5. *The wheel W_n, is join of the graphs C_n and K_1. i.e $W_n = C_n + K_1$. Here vertices corresponding to C_n are called rim vertices and C_n is called rim of W_n while, the vertex corresponding to K_1 is called the apex vertex, edges joining the apex vertex and a rim vertex is called spoke.*

Definition 1.6 ([1]). *The helm H_n, is the graph obtained from the wheel W_n by adding a pendant edge at each rim vertex. Each pendent edges are called outer spoke.*

Definition 1.7 ([1]). *The closed helm CH_n, is the graph obtained from a helm by joining each pendent vertex to form a cycle, here vertices corresponding to this cycle are called outer rim vertices and vertices corresponding to wheel except the apex vertex are called inner rim vertiecs.*

Definition 1.8. *A binary vertex labeling f of a graph G is called a cordial labeling if $|v_f(1)-v_f(0)| \leq 1$ and $|e_f(1)-e_f(0)| \leq 1$. A graph G is said to be cordial if it admits cordial labeling.*

The concept of cordial labeling was introduced by Cahit [3] in which he proved that the wheel W_n is cordial if and only if $n \not\equiv 3 \pmod 4$. Vaidya and Dani [4] proved that the graphs obtained by duplication of an arbitrary edge of a cycle and a wheel admit a cordial labeling. Prajapati and Gajjar [5] proved that complement of wheel graph and complement of cycle graph are cordial if $n \not\equiv 4 \pmod 8$ or $n \not\equiv 7 \pmod 8$. Prajapati and Gajjar [6] proved that cordial labeling in the context of duplication of cycle graph and path graph. In this paper, for every natural number n the set $\{1, 2, ..., n\}$ will be denoted by $[n]$.

2. Main Results

Theorem 2.1. *The graph obtained by duplicating all the vertices except the apex vertex of the helm H_n is cordial.*

Proof. Let $V(H_n) = \{w\} \cup \{u_i, v_i/1 \leq i \leq n\}$ and $E(H_n) = \{wu_i, u_iv_i/1 \leq i \leq n\} \cup \{u_nu_1\} \cup \{u_iu_{i+1}/1 \leq i \leq n-1\}$. Let G be the graph obtained by duplicating all the vertices except the apex vertex in H_n. Let $u'_1, u'_2, ..., u'_n, v'_1, v'_2, ..., v'_n$ be the new vertices of G by duplicating $u_1, u_2, ..., u_n, v_1, v_2, ..., v_n$ respectively. Then $V(G) = \{w\} \cup \{u_i, v_i, u'_i, v'_i/1 \leq i \leq n\}$ and $E(G) = \{u_nu_1, u'_nu_1, u_nu'_1\} \cup \{u_iu_{i+1}, u'_iu_{i+1}, u_iu'_{i+1}/1 \leq i \leq n-1\} \cup \{wu_i, wu'_i, u_iv_i, u'_iv_i, u_iv'_i/1 \leq i \leq n\}$. Therefore $|V(G)| = 4n + 1$ and $|E(G)| = 8n$. Define a vertex labeling $f : V(G) \to \{0, 1\}$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = w; \\ 1 & \text{if } x \in \{u_i, {v'}_i\},\ i \in [n]; \\ 0 & \text{if } x \in \{v_i, {u'}_i\},\ i \in [n]. \end{cases}$$

Thus $v_f(1) = 2n + 1$ and $v_f(0) = 2n$. The induced edge labeling $f^* : E(G) \to \{0, 1\}$ is $f^*(uv) = |f(u) - f(v)|$, for every edge $e = uv \in E$. Therefore

$$f^*(e) = \begin{cases} 1 & \text{if } e \in \{u_iv_i, w{u'}_i\},\ i \in [n]; \\ 1 & \text{if } e \in \{{u'}_iu_{i+1}, u_i{u'}_{i+1}\},\ i \in [n-1]; \\ 0 & \text{if } e \in \{wu_i, v_i{u'}_i, u_i{v'}_i\},\ i \in [n]; \\ 0 & \text{if } e = u_iu_{i+1},\ i \in [n-1]; \\ 0 & \text{if } e = u_nu_1; \\ 1 & \text{if } e \in \{u'_nu_1, u_nu'_1\}. \end{cases}$$

Thus $e_f(1) = 4n$ and $e_f(0) = 4n$. Therefore f satisfies the conditions $|v_f(1) - v_f(0)| \leq 1$ and $|e_f(1) - e_f(0)| \leq 1$. So, f admits cordial labeling on G. Hence G is cordial. $\square$

Theorem 2.2. *The graph obtained by duplicating all the vertices of the helm H_n is cordial.*

Proof. Let $V(H_n) = \{w\} \cup \{u_i, v_i/1 \leq i \leq n\}$ and $E(H_n) = \{wu_i, u_iv_i/1 \leq i \leq n\} \cup \{u_nu_1\} \cup \{u_iu_{i+1}/1 \leq i \leq n-1\}$. Let G be the graph obtained by duplicating all the vertices in H_n. Let $w', u'_1, u'_2, ..., u'_n, v'_1, v'_2, ..., v'_n$ be the new vertices of G by duplicating $w, u_1, u_2, ..., u_n, v_1, v_2, ..., v_n$ respectively. Then $V(G) = \{w, w'\} \cup \{u_i, v_i, u'_i, v'_i/1 \leq i \leq n\}$ and $E(G) = \{u_nu_1, u'_nu_1, u_nu'_1\} \cup \{u_iu_{i+1}, u'_iu_{i+1}, u_iu'_{i+1}/1 \leq i \leq n-1\} \cup \{wu_i, wu'_i, u_iv_i, u'_iv_i, u_iv'_i, w'u_i/1 \leq i \leq n\}$. Therefore $|V(G)| = 4n+2$ and $|E(G)| = 9n$. Define a vertex labeling $f : V(G) \to \{0, 1\}$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = w'; \\ 1 & \text{if } x = u_i, \ i \in [n]; \\ \frac{1+(-1)^{i+1}}{2} & \text{if } x = v'_i, \ i \in [n]; \\ 0 & \text{if } x = w; \\ \frac{1+(-1)^{i}}{2} & \text{if } x = v_i, \ i \in [n]; \\ 0 & \text{if } x = u'_i, \ i \in [n]. \end{cases}$$

Thus even $v_f(1) = 2n+1$ and $v_f(0) = 2n+1$. The induced edge labeling $f^* : E(G) \to \{0, 1\}$ is $f^*(uv) = |f(u) - f(v)|$, for every edge $e = uv \in E$. Therefore

$$f^*(e) = \begin{cases} 1 & \text{if } e = wu_i, \ i \in [n]; \\ \frac{1+(-1)^{i+1}}{2} & \text{if } e = u_iv_i, \ i \in [n]; \\ 1 & \text{if } e \in \{u'_iu_{i+1}, u_iu'_{i+1}\}, \ i \in [n-1]; \\ 0 & \text{if } e = u_iu_{i+1}, \ i \in [n-1]; \\ 0 & \text{if } e \in \{w'u_i, wu'_i\}, \ i \in [n]; \\ \frac{1+(-1)^{i}}{2} & \text{if } e \in \{u_iv'_i, v_iu'_i\}, \ i \in [n]; \\ 0 & \text{if } e = u_nu_1; \\ 1 & \text{if } e \in \{u'_nu_1, u_nu'_1\}. \end{cases}$$

Thus $e_f(1) = \dfrac{9n - \left(\frac{1+(-1)^{n+1}}{2}\right)}{2}$ and $e_f(0) = \dfrac{9n + \left(\frac{1+(-1)^{n+1}}{2}\right)}{2}$. Therefore f satisfies the conditions $|v_f(1) - v_f(0)| \leq 1$ and $|e_f(1) - e_f(0)| \leq 1$. So, f admits cordial labeling on G. Hence G is cordial. $\square$

Theorem 2.3. *The graph obtained by duplicating all the edges other than spoke edges of the helm H_n is cordial.*

Proof. Let $V(H_n) = \{w\} \cup \{u_i, v_i/1 \leq i \leq n\}$ and $E(H_n) = \{k_i = wu_i, m_i = u_iv_i/1 \leq i \leq n\} \cup \{l_i = u_iu_{i+1}/1 \leq i \leq n-1\} \cup \{l_n = u_nu_1\}$. Let G be the graph obtained by duplicating all the edges other than spoke edges in H_n. For each $i \in 1, 2, ...n$, let $l'_i = a_ib_i$ and $m'_i = c_id_i$ be the new edges of G by duplicating l_i and m_i respectively. Then $V(G) = \{w\} \cup \{u_i, v_i, a_i, b_i, c_i, d_i/1 \leq i \leq n\}$ and $E(G) = \{wu_i, u_iv_i, wc_i, a_ib_i, c_id_i, a_iv_i, wa_i, wb_i/1 \leq i \leq n\} \cup \{u_nu_1, c_nu_1, u_nc_1, b_nv_1, u_na_1, b_{n-1}u_1, b_nu_2\} \cup \{u_iu_{i+1}, c_iu_{i+1}, u_ic_{i+1}, b_iv_{i+1}, u_ia_{i+1}/1 \leq i \leq n-1\} \cup \{b_iu_{i+2}/1 \leq i \leq n-2\}$. Therefore $|V(G)| = 6n+1$ and $|E(G)| = 14n$. Define a vertex labeling $f : V(G) \to \{0, 1\}$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = w; \\ 1 & \text{if } x \in \{u_i, v_i, c_i\}, \ i \in [n]; \\ 0 & \text{if } x \in \{a_i, b_i, d_i\}, \ i \in [n]. \end{cases}$$

Thus $v_f(1) = 3n + 1$ and $v_f(0) = 3n$. The induced edge labeling $f^* : E(G) \to \{0, 1\}$ is $f^*(uv) = |f(u) - f(v)|$, for every edge $e = uv \in E$. Therefore

$$f^*(e) = \begin{cases} 1 & \text{if } e \in \{c_i d_i, a_i v_i, w a_i, w b_i\},\ i \in [n]\,; \\ 0 & \text{if } e \in \{w u_i, u_i v_i, w c_i, a_i b_i\},\ i \in [n]\,; \\ 0 & \text{if } e \in \{u_i u_{i+1}, c_i u_{i+1}, u_i c_{i+1}\},\ i \in [n-1]\,; \\ 1 & \text{if } e \in \{b_i v_{i+1}, u_i a_{i+1}\},\ i \in [n-1]\,; \\ 1 & \text{if } e = b_i u_{i+2},\ i \in [n-2]\,; \\ 1 & \text{if } e \in \{b_{n-1} u_1, b_n u_2, b_n v_1, u_n a_1\}; \\ 0 & \text{if } e \in \{u_n u_1, c_n u_1, u_n c_1\}. \end{cases}$$

Thus $e_f(1) = 7n$ and $e_f(0) = 7n$. Therefore f satisfies the conditions $|v_f(1) - v_f(0)| \leq 1$ and $|e_f(1) - e_f(0)| \leq 1$. So, f admits cordial labeling on G. Hence G is cordial. $\square$

Theorem 2.4. *The graph obtained by duplicating all the vertices of the closed helm CH_n is cordial.*

Proof. Let $V(CH_n) = \{w\} \cup \{u_i, v_i / 1 \leq i \leq n\}$ and $E(CH_n) = \{w u_i, u_i v_i / 1 \leq i \leq n\} \cup \{u_n u_1, v_n v_1\} \cup \{u_i u_{i+1}, v_i v_{i+1} / 1 \leq i \leq n-1\}$. Let G be the graph obtained by duplicating all vertices of CH_n. Let $w', u'_1, u'_2, ..., u'_n, v'_1, v'_2, ..., v'_n$ be the new vertices of G by duplicating $w, u_1, u_2, ..., u_n, v_1, v_2, ..., v_n$ respectively. Then $V(G) = \{w, w'\} \cup \{u_i, v_i, u'_i, v'_i / 1 \leq i \leq n\}$ and $E(G) = \{u_n u_1, v_n v_1, v'_n v_1, v_n v'_1, u'_n u_1, u_n u'_1\} \cup \{w u_i, u_i v_i, w_i u'_i, u'_i v_i, w'_i u_i, u_i v'_i / 1 \leq i \leq n\} \cup \{u_i u_{i+1}, v_i v_{i+1}, v'_i v_{i+1}, v_i v'_{i+1}, u'_i u_{i+1}, u_i u'_{i+1} / 1 \leq i \leq n-1\}$. Therefore $|V(G)| = 4n + 2$ and $|E(G)| = 12n$. Define a vertex labeling $f : V(G) \to \{0, 1\}$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = w'; \\ 1 & \text{if } x \in \{u_i, v'{}_i\},\ i \in [n]\,; \\ 0 & \text{if } x = w; \\ 0 & \text{if } x \in \{v_i, u'{}_i\},\ i \in [n]\,. \end{cases}$$

Thus $v_f(1) = 2n + 1$ and $v_f(0) = 2n + 1$. The induced edge labeling $f^* : E(G) \to \{0, 1\}$ is $f^*(uv) = |f(u) - f(v)|$, for every edge $e = uv \in E$. Therefore

$$f^*(e) = \begin{cases} 1 & \text{if } e \in \{u_i v_i, w u_i\},\ i \in [n]\,; \\ 1 & \text{if } e \in \{v'{}_i v_{i+1}, u_i u'{}_{i+1}, u'{}_i u_{i+1}, v_i v'{}_{i+1}\},\ i \in [n-1]\,; \\ 0 & \text{if } e \in \{u_i u_{i+1}, v_i v_{i+1}\},\ i \in [n-1]\,; \\ 0 & \text{if } e \in \{w'{}_i u_i, w_i u'{}_i, u_i v'{}_i, u'{}_i v_i\},\ i \in [n]\,; \\ 0 & \text{if } e \in \{u_n u_1, v_n v_1\}; \\ 1 & \text{if } e \in \{v'_n v_1, v_n v'_1, u'_n u_1, u_n u'_1\}. \end{cases}$$

Thus $e_f(1) = 6n$ and $e_f(0) = 6n$. Therefore f satisfies the conditions $|v_f(1) - v_f(0)| \leq 1$ and $|e_f(1) - e_f(0)| \leq 1$. So, f admits cordial labeling on G. Hence G is cordial. $\square$

Theorem 2.5. *The graph obtained by duplicating all the outer rim vertices of the closed helm CH_n is cordial.*

Proof. Let $V(CH_n) = \{w\} \cup \{u_i, v_i / 1 \leq i \leq n\}$ and $E(CH_n) = \{w u_i, u_i v_i / 1 \leq i \leq n\} \cup \{u_n u_1, v_n v_1\} \cup \{u_i u_{i+1}, v_i v_{i+1} / 1 \leq i \leq n-1\}$. Let G be the graph obtained by duplicating all the outer rim vertices in CH_n. Let $v'_1, v'_2, ..., v'_n$ be the new vertices of G by duplicating $v_1, v_2, ..., v_n$ respectively. Then $V(G) = \{w\} \cup \{u_i, v_i, v'_i / 1 \leq i \leq n\}$ and $E(G) = \{w u_i, u_i v_i, u_i v'_i / 1 \leq i \leq$

$n\} \cup \{u_nu_1, v_nv_1, v'_nv_1, v_nv'_1\} \cup \{u_iu_{i+1}, v_iv_{i+1}, v'_iv_{i+1}, v_iv'_{i+1}/1 \le i \le n-1\}$. Therefore $|V(G)| = 3n+1$ and $|E(G)| = 7n$. Define a vertex labeling $f : V(G) \to \{0,1\}$ as follows:

$$f(x) = \begin{cases} 0 & \text{if } x = w; \\ 1 & \text{if } x = u_i,\ i \in [n]\,; \\ 0 & \text{if } x = v_i,\ i \in [n]\,; \\ \frac{1+(-1)^{i+1}}{2} & \text{if } x = v'_i,\ i \in [n]\,. \end{cases}$$

Thus $v_f(1) = \dfrac{3n + \left(\frac{1+(-1)^n}{2}\right)}{2}$ and $v_f(0) = \dfrac{3n+1+\left(\frac{1+(-1)^{n+1}}{2}\right)}{2}$. The induced edge labeling $f^* : E(G) \to \{0,1\}$ is $f^*(uv) = |f(u) - f(v)|$, for every edge $e = uv \in E$. Therefore

$$f^*(e) = \begin{cases} 1 & \text{if } e \in \{wu_i, u_iv_i\},\ i \in [n]\,; \\ \frac{1+(-1)^{i+1}}{2} & \text{if } e = v'_iv_{i+1},\ i \in [n-1]\,; \\ \frac{1+(-1)^{i+1}}{2} & \text{if } e = u_iv'_i,\ i \in [n]\,; \\ \frac{1+(-1)^{i}}{2} & \text{if } e = v_iv'_{i+1},\ i \in [n-1]\,; \\ 0 & \text{if } e \in \{v_iv_{i+1}, u_iu_{i+1}\},\ i \in [n-1]\,; \\ 0 & \text{if } e \in \{u_nu_1, v_nv_1\}; \\ \frac{1+(-1)^{n}}{2} & \text{if } e = v_nv'_1; \\ \frac{1+(-1)^{n+1}}{2} & \text{if } e = v'_nv_1. \end{cases}$$

Thus $e_f(1) = \dfrac{7n + \left(\frac{1+(-1)^{n+1}}{2}\right)}{2}$ and $e_f(0) = \dfrac{7n - \left(\frac{1+(-1)^{n+1}}{2}\right)}{2}$. Therefore f satisfies the conditions $|v_f(1) - v_f(0)| \le 1$ and $|e_f(1) - e_f(0)| \le 1$. So, f admits cordial labeling on G. Hence G is cordial. □

Theorem 2.6. *The graph obtained by duplicating all the vertices except the apex vertex of the closed helm CH_n is cordial.*

Proof. Let $V(CH_n) = \{w\} \cup \{u_i, v_i/1 \le i \le n\}$ and $E(CH_n) = \{wu_i, u_iv_i/1 \le i \le n\} \cup \{u_nu_1, v_nv_1\} \cup \{u_iu_{i+1}, v_iv_{i+1}/1 \le i \le n-1\}$. Let G be the graph obtained by duplicating all the vertices except the apex vertex in CH_n. Let $u'_1, u'_2, ..., u'_n, v'_1, v'_2, ..., v'_n$ be the new vertices of G by duplicating $u_1, u_2, ..., u_n, v_1, v_2, ..., v_n$ respectively. Then $V(G) = \{w\} \cup \{u_i, v_i, u'_i, v'_i/1 \le i \le n\}$ and $E(G) = \{wu_i, u_iv_i, w_iu'_i, u_iv'_i, u'_iv_i/1 \le i \le n\} \cup \{u_iu_{i+1}, v_iv_{i+1}, v'_iv_{i+1}, v_iv'_{i+1}, u'_iu_{i+1}, u_iu'_{i+1}/1 \le i \le n-1\} \cup \{u_nu_1, v_nv_1, v'_nv_1, v_nv'_1, u'_nu_1, u_nu'_1\}$. Therefore $|V(G)| = 4n+1$ and $|E(G)| = 11n$. Define a vertex labeling $f : V(G) \to \{0,1\}$ as follows:

$$f(x) = \begin{cases} 0 & \text{if } x = w; \\ 1 & \text{if } x = u_i,\ i \in [n]\,; \\ 0 & \text{if } x = v_i,\ i \in [n]\,; \\ \frac{1+(-1)^{i+1}}{2} & \text{if } x \in \{u'_i, v'_i\},\ i \in [n]\,. \end{cases}$$

Thus $v_f(1) = 2n + \dfrac{1+(-1)^{n+1}}{2}$ and $v_f(0) = 2n + \dfrac{1+(-1)^{n}}{2}$. The induced edge labeling $f^* : E(G) \to \{0,1\}$ is $f^*(uv) =$

$|f(u) - f(v)|$, for every edge $e = uv \in E$. Therefore

$$f^*(e) = \begin{cases} 1 & \text{if } e \in \{wu_i, u_iv_i\},\ i \in [n]\,; \\ 0 & \text{if } e \in \{v_iv_{i+1}, u_iu_{i+1}\},\ i \in [n-1]\,; \\ \frac{1+(-1)^i}{2} & \text{if } e = u_iv'_i,\ i \in [n]\,; \\ \frac{1+(-1)^{i+1}}{2} & \text{if } e \in \{w_iu'_i, u'_iv_i\},\ i \in [n]\,; \\ \frac{1+(-1)^{i+1}}{2} & \text{if } e \in \{v'_iv_{i+1}, u_iu'_{i+1}\},\ i \in [n-1]\,; \\ \frac{1+(-1)^i}{2} & \text{if } e \in \{u'_iu_{i+1}, v_iv'_{i+1}\},\ i \in [n-1]\,; \\ \frac{1+(-1)^n}{2} & \text{if } e = u'_nu_1; \\ \frac{1+(-1)^{n+1}}{2} & \text{if } e = v'_nv_1; \\ 0 & \text{if } e \in \{u_nu_1, v_nv_1, u_nu'_1\}; \\ 1 & \text{if } e = v_nv'_1. \end{cases}$$

Thus $e_f(1) = \dfrac{11n + \left(\frac{1+(-1)^{n+1}}{2}\right)}{2}$ and $e_f(0) = \dfrac{11n - \left(\frac{1+(-1)^{n+1}}{2}\right)}{2}$. Therefore f satisfies the conditions $|v_f(1) - v_f(0)| \leq 1$ and $|e_f(1) - e_f(0)| \leq 1$. So, f admits cordial labeling on G. Hence G is cordial. □

Theorem 2.7. *The graph obtained by duplicating all the edges other than spoke edges of the closed helm CH_n is cordial.*

Proof. Let $V(CH_n) = \{w\} \cup \{u_i, v_i/1 \leq i \leq n\}$ and $E(CH_n) = \{j_i = wu_i, l_i = u_iv_i/1 \leq i \leq n\} \cup \{k_i = u_iu_{i+1}, m_i = v_iv_{i+1}/1 \leq i \leq n-1\} \cup \{k_n = u_nu_1, m_n = v_nv_1\}$. Let G be the graph obtained by duplicating all the edges other than spoke edges in CH_n. For each $i \in 1, 2, ...n$ let $k'_i = a_ib_i, l'_i = c_id_i$ and $m'_i = e_if_i$ be the new edges of G by duplicating $k_i, l_i$and m_i respectively. Then $V(G) = \{w\} \cup \{u_i, v_i, a_i, b_i, c_i, d_i, e_i, f_i/1 \leq i \leq n\}$ and $E(G) = \{wu_i, u_iv_i, c_id_i, a_ib_i, a_iv_i, wa_i, wb_i, wc_i, e_if_i, e_iu_i/1 \leq i \leq n\} \cup \{b_iu_{i+2}, v_if_{i+2}/1 \leq i \leq n-2\} \cup \{u_iu_{i+1}, v_iv_{i+1}, b_iv_{i+1}, f_iu_{i+1}, v_ie_{i+1}, u_ia_{i+1}, d_iv_{i+1}, v_id_{i+1}, c_iu_{i+1}, u_ic_{i+1}/1 \leq i \leq n-1\} \cup \{u_nu_1, v_nv_1, b_nv_1, f_nu_1, v_ne_1, u_na_1, d_nv_1, v_nd_1, c_nu_1, u_nc_1, b_{n-1}u_1, b_nu_2, v_{n-1}f_1, v_nf_2\}$. Therefore $|V(G)| = 10n + 1$ and $|E(G)| = 22n$. Define a vertex labeling $f : V(G) \to \{0, 1\}$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = w; \\ 0 & \text{if } x \in \{u_i, d_i, e_i, f_i\},\ i \in [n]; \\ 1 & \text{if } x \in \{v_i, a_i, b_i, c_i\},\ i \in [n]\,. \end{cases}$$

Thus $v_f(1) = 5n + 1$ and $v_f(0) = 5n$. The induced edge labeling $f^* : E(G) \to \{0, 1\}$ is $f^*(uv) = |f(u) - f(v)|$, for every edge $e = uv \in E$. Therefore

$$f^*(e) = \begin{cases} 1 & \text{if } e \in \{wu_i, u_iv_i, c_id_i\}, i \in [n]; \\ 0 & \text{if } e \in \{u_iu_{i+1}, v_iv_{i+1}, b_iv_{i+1}, f_iu_{i+1}\}, i \in [n-1]; \\ 1 & \text{if } e \in \{b_iu_{i+2}, v_if_{i+2}\}, i \in [n-2]; \\ 0 & \text{if } e \in \{a_ib_i, a_iv_i, wa_i, wb_i, wc_i, e_if_i, e_iu_i\}, i \in [n]; \\ 1 & \text{if } e \in \{v_ie_{i+1}, u_ia_{i+1}, d_iv_{i+1}, v_id_{i+1}, c_iu_{i+1}, u_ic_{i+1}\}, i \in [n-1]; \\ 0 & \text{if } e \in \{u_nu_1, v_nv_1, b_nv_1, f_nu_1\}; \\ 1 & \text{if } e \in \{v_ne_1, u_na_1, d_nv_1, v_nd_1, c_nu_1, u_nc_1, b_{n-1}u_1, b_nu_2, v_{n-1}f_1, v_nf_2\}. \end{cases}$$

Thus $e_f(1) = 11n$ and $e_f(0) = 11n$. Therefore f satisfies the conditions $|v_f(1) - v_f(0)| \leq 1$ and $|e_f(1) - e_f(0)| \leq 1$. So, f admits cordial labeling on G. Hence G is cordial. □

3. Conclusion

we have derived seven new results by investigating cordial labeling in the context of duplication in helm and closed helm. More exploration is possible for other graph families and in the context of different graph labeling problems.

References

[1] J.A.Gallian, *A Dynamic Survey of Graph Labeling*, The Electronic Journal of Combinatorics, 19(2016), #DS6.

[2] J.Gross and J.Yellen, *Graph Theory and Its Applications*, CRC Press, Boca Raton, (1999).

[3] I.Cahit, *On cordial and 3-equitable labellings of graphs*, Util. Math., 37(1990), 189-198.

[4] S.K.Vaidya and N.A.Dani, *Cordial and 3-Equitable Graphs Induced by Duplication of Edge*, Mathematics Today, 27(2011), 71-82.

[5] U.M.Prajapati and R.M.Gajjar, *Cordial Labeling of Complement of Some Graph*, Mathematics Today, 30(2015), 99-118.

[6] U.M.Prajapati and R.M.Gajjar, *Cordial Labeling in the Context of Duplication of some Graph Elements*, International Journal of Mathematics and Soft Computing, 6(2016), 65-73.