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Solution of Fractional Telegraph Equation with Fuzzy Initial Condition

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Abstract: In this work, we analyze the method of finding the solution of fractional telegraph equation of the form $\frac{\partial^{2\alpha} u}{\partial x^{2\alpha}} + a \odot \frac{\partial^{\alpha} u}{\partial t^{\alpha}} + b \odot u = \frac{\partial^2 u}{\partial x^2}$, with fuzzy initial condition, where ${}^{C}D$ is a Caputo fractional derivative. Comparison has done between integer order and fractional order model. Some numerical illustrations are provided to explain the proposed theory.

MSC: 92D25, 70K50, 35B36

Keywords: Fractional Telegraph equations, fuzzy differential equations, Laplace Transform.

1. Introduction

Fractional differential equations have been quickly developed in recent years. The theory of derivative of fractional order is very useful in demonstrating many phenomena in including fractals theory, control theory and other sciences. Fuzzy partial differential equations plays a major role in the fields of science and engineering. The concepts of Fuzzy were introduced by Zadeh [18] and it is enriched by several authors. One of the most significant field of the Mathematics is the fuzzy differential equation. The Fuzzy Fractional Partial differential Equations are obtained when a physical system is modeled under differential sense and also used to model uncertainty of dynamical systems. Therefore the solution of the fuzzy fractional Partial differential equations are very important in various Fields.

The idea of fuzzy partial differential equations (FPDEs) was first introduced by Buckley in [6]. In [5] Balachandran investigated the existence of solutions of nonlinear fuzzy integral equations. Classical solution of PIDEs were discussed by using classical Laplace transform in [17]. In [6], Buckley and Feuring proposed a method to solutions of elementary fuzzy partial differential equations. In [16] a numerical method for solving fuzzy partial differential equation (FPDE) is analyzed and the conditions for the stability of this method are given. Thorwe and Bhalekar [17] solved the partial integro-differential equations by Laplace transform method. Also, Salahshour and Allahviranloo [4] investigated the method of solving fractional differential equations by fuzzy laplace transform method. Mazandarani [9] solved fuzzy fractional differential equation numerically by using modified fractional Euler method. Many contributions have been done by several authors, for further details see [1, 15] for solving fractional differential equations Solving Fuzzy Partial Differential Equations (FPDE) can be used for finding the particular structure behavior, e.g., by fixing unknown parameters to some acceptable values.

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In the process of optimizing guided communication systems, it is necessary to determine the signal losses. This is the main reason to study the fractional telegraph equation. To determine these losses with uncertainty, it is formulated as the Fractional Telegraph Equations with Fuzzy initial condition in which we can calculate these losses. This motivates us to study the analysis of fractional telegraph equation. The solutions of fuzzy fractional Partial differential equations are constructed by using the fuzzy Laplace transform is given in [16]. In this paper, the same method is used for analyze the solution of fractional Partial Telegraph equations with fuzzy initial conditions, but with fractional derivative considered in the sense of the Caputo derivative because it provides us the advantage of requiring initial conditions given in terms of integer order derivatives. In order to determine the lower and upper functions of the solution we convert the given FPIDEs to two crisp ordinary differential equations by using FLT.

This work is organized as follows. In section 2, we recall some basic definitions, notations and notions. In section 3, we propose the Fuzzy Laplace Transform method to solve the fractional partial Telegraph equation with fuzzy initial conditions. It is shown that FLTM is a simple and reliable approach for solving such equations analytically. Then by using the method appeared in section 3, some examples are illustrated in section 4 to show the ability of the proposed method and conclusions are drawn in section 5.

1.1. Preliminaries

Basic definitions and results regarding the fractional calculus are given in this section.

Definition 1.1 (Riemann - Liouville Fractional Integral [12]). *The Riemann-Liouville fractional integral operator of order $\alpha > 0$ of a function $f \in L^1(\mathcal{R}^+)$ is defined by*

$$I^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s)\mathrm{d}s, \tag{1}$$

where $\Gamma(.)$ is the Euler gamma function.

Definition 1.2 (Riemann - Liouville Fractional Derivative [12]). *The Riemann-Liouville fractional derivative of order $\alpha > 0, n-1 < \alpha < n, n \in N$, is defined as*

$$D^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_0^t (t-s)^{n-\alpha-1} f(s)\mathrm{d}s, \tag{2}$$

where D^n is the ordinary differential operator and the function $f(t)$ has absolutely continuous derivative up to order $(n-1)$.

Definition 1.3 (Caputo Fractional Derivative [12]). *The Caputo fractional derivative of order $\alpha > 0, n-1 < \alpha < n, n \in N$, is defined as*

$$^{C}D^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-s)^{n-\alpha-1} f^n(s)\mathrm{d}s, \tag{3}$$

where the function $f(t)$ has absolutely continuous derivative up to order $(n-1)$.

Definition 1.4 (Fuzzy Number [18]). *A fuzzy number r in parametric form is a pair $(\underline{r}, \overline{r})$ of functions $\underline{r}(t), \overline{r}(t), 0 <\leq t \leq 1$, which satisfy the following requirements:*

(1). $\underline{r}(t)$ is a bounded non-decreasing left continuous function in (0,1], and right continuous at 0,

(2). $\overline{r}(t)$ is a bounded non-increasing left continuous function in (0,1], and right continuous at 0,

(3). $\underline{r}(t) \leq \overline{r}(t), 0 \leq t \leq 1$.

Addition, scalar multiplication and other operations of a fuzzy number can be seen in [3]

Definition 1.5 (Fuzzy Laplace Transform of a Fuzzy valued function [3]). *Let f(x) be a continuous fuzzy-valued function;then the fuzzy Laplace transform can be denoted as:*

$$U(x,s) = L\{u(x,t)\} = \int_0^\infty u(x,t) \odot e^{-st} \mathrm{d}t \tag{4}$$

$$U(x,s;r) = L\{u(x,t;r)\} = [l\{\underline{u}(x,t;r)\}, l\{\overline{u}(x,t;r)\}], \tag{5}$$

where $l\{\underline{u}(x,t;r)\} = \int_0^\infty \underline{u}(x,t;r) \odot e^{-st}\mathrm{d}t, l\{\overline{u}(x,t;r)\} = \int_0^\infty \overline{u}(x,t;r) \odot e^{-st}\mathrm{d}t$

Theorem 1.6 (Laplace Transform of a Partial Derivatives [3]). *If* $u : (a,b) \times (a,b) \to E$ *is a fuzzy valued function such that its derivatives up to* $(n-1)^{th}$ *order are continuous for all* $t > 0$*, then*

$$\mathcal{L}[\frac{\partial u}{\partial x}; s] = \frac{\mathrm{d}}{\mathrm{d}x}\mathcal{L}[u] \tag{6}$$

$$\mathcal{L}[\frac{\partial^2 u}{\partial x^2}; s] = \frac{\mathrm{d}^2}{\mathrm{d}x^2}\mathcal{L}[u] \tag{7}$$

$$\mathcal{L}[\frac{\partial^\alpha u}{\partial t^\alpha}; s] = s^\alpha \mathcal{L}[u] - s^{\alpha-1}u(x,0), where \quad 0 < \alpha \le 1 \tag{8}$$

$$\mathcal{L}[\frac{\partial^{2\alpha} u}{\partial t^{2\alpha}}; s] = s^{2\alpha} \mathcal{L}[u] - s^{2\alpha-1}u(x,0) - s^{2\alpha-2}u_t(x,0), where \quad 1 < 2\alpha \le 2 \tag{9}$$

The one-parameter Mittag-Leffler function is defined as

$$E_\alpha(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + 1)}, \qquad (\alpha > 0, z \in \mathbb{C}). \tag{10}$$

The two-parameter Mittag-Leffler function is defined as

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}, \qquad (\alpha, \beta > 0, z \in \mathbb{C}). \tag{11}$$

The Mittag-Leffler function in more pars defined as

$$E^{\nu}_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{(\nu)_k}{\Gamma(\alpha k + \beta)} \frac{z^k}{\Gamma(k+1)}, \tag{12}$$

$$E^{\nu,q}_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{(\nu)_{kq}}{\Gamma(\alpha k + \beta)} \frac{z^k}{\Gamma(k+1)}, \tag{13}$$

Definition 1.7 (Laplace Transform of a Mittag Leffler function [12]). *The Laplace transform of Mittag-Leffler functions are given by*

$$\mathcal{L}\{t^{\beta-1}E_{\alpha,\beta}(-\lambda t^\alpha)\} = \frac{s^{\alpha-\beta}}{s^\alpha + \lambda}, \tag{14}$$

$$\mathcal{L}\{t^{\beta-1}E^{\nu}_{\alpha,\beta}(-\lambda t^\alpha)\} = \frac{s^{\alpha\nu-\beta}}{(s^\alpha + \lambda)^\nu}, \tag{15}$$

Definition 1.8 (Inverse Laplace Transform of a Mittag Leffler function[4]).

$$\mathcal{L}^{-1}\{\frac{s^{\rho-1}}{s^\alpha + as^\beta + b}\} = t^{\alpha-\rho}\sum_{k=0}^{\infty}(-a)^k t^{(\alpha-\beta)k} E^{k+1}_{\alpha,\alpha+(\alpha-\beta)k-\rho+1}(-bt^\alpha) \tag{16}$$

Theorem 1.9 ([3]). *Let f(x),g(x) be continuous-fuzzy-valued functions; suppose that* c_1, c_2 *are constants, then:*

$$L\{(c_1 \odot f(x)) + (c_2 \odot g(x))\} = c_1 \odot L\{f(x)\} + c_2 \odot L\{g(x)\}.$$

Theorem 1.10 ([3]). *Let f(x) bea continuous-fuzzy-valued function on* $[0,\infty]$ *and* $\lambda \in R$*, then :*

$$L\{\lambda \odot f(x)\} = \lambda \odot L\{f(x)\}.$$

2. Main Work

2.1. Solution of Fuzzy Fractional Telegraph Equation

In this section, we will investigate solution of Fractional Telegraph Equation with fuzzy initial condition. Let us consider the general linear fractional telegraph equation

$$\frac{\partial^{2\alpha} u}{\partial x^{2\alpha}} + a \odot \frac{\partial^{\alpha} u}{\partial t^{\alpha}} + b \odot u = \frac{\partial^2 u}{\partial x^2}, \tag{17}$$

with the boundary conditions (BCs)

$$u(0,t) = 0, u(l,t) = 0.$$

and the initial conditions (ICs)

$$u(x,0) = 0, u_t(x,0) = (\underline{f}, \overline{f}), 0 \leq x \leq l$$

where $f = (\underline{f}, \overline{f})$ is a fuzzy function.

Fuzzy Laplace Transform Method

Laplace Transform of a function of two variables concept is used to solve the Telegraph equations. Suppose x and t are two independent variables here t is considered as the principal variable and x as the secondary variable. When the Laplace transform is applied with t as a variable, the PDE is reduced to an ordinary differential equation of the $t-$ transform $U(x,s)$, where x is the independent variable. The general solution $U(x,s)$ of the ODE is then fitted to the BCs of the original problem. Finally, the solution $u(x,t)$ is obtained by using complex inversion formula. Thus, the Laplace Transform is specially suited to solving initial boundary value problems (IBVP), when conditions are prescribed at $t = 0$. Taking the Laplace Transform of both sides of Telegraph equation (17), we have

$$s^{2\alpha}\mathcal{L}[u] - s^{2\alpha-1}u(x,0) - s^{2\alpha-2}u_t(x,0) + as^{\alpha}\mathcal{L}[u] - as^{\alpha-1}u(x,0) + \mathcal{L}[u] = \frac{\partial^2}{\partial x^2}\mathcal{L}[u], \tag{18}$$

Let $\mathcal{L}[u] = U(x,s)$, equation (18) becomes

$$\frac{\partial^2}{\partial x^2}U(x,s) - [s^{2\alpha} + as^{\alpha} + b]U(x,s) + (s^{\alpha} + a)s^{\alpha-1}u(x,0) + s^{2\alpha-2}u_t(x,0) = 0, \tag{19}$$

The classical form of (19) gives the following two ordinary differential equations as follows:

$$\frac{\partial^2}{\partial x^2}\underline{U}(x,s,r) - [s^{2\alpha} + as^{\alpha} + b]\underline{U}(x,s,r) + (s^{\alpha} + a)s^{\alpha-1}\underline{u}(x,0) + s^{2\alpha-2}\underline{u}_t(x,0) = 0, \tag{20}$$

$$\frac{\partial^2}{\partial x^2}\overline{U}(x,s,r) - [s^{2\alpha} + as^{\alpha} + b]\overline{U}(x,s,r) + (s^{\alpha} + a)s^{\alpha-1}\overline{u}(x,0) + s^{2\alpha-2}\overline{u}_t(x,0) = 0, \tag{21}$$

Using the given fuzzy initial conditions we have,

$$\frac{\partial^2}{\partial x^2}\underline{U}(x,s,r) - [s^{2\alpha} + as^{\alpha} + b]\underline{U}(x,s,r) + s^{2\alpha-2}\underline{f} = 0, \tag{22}$$

$$\frac{\partial^2}{\partial x^2}\overline{U}(x,s,r) - [s^{2\alpha} + as^{\alpha} + b]\overline{U}(x,s,r) + s^{2\alpha-2}\overline{f} = 0, \tag{23}$$

The Boundary conditions $u(0,t) = 0, u(1,t) = 1$, and their Laplace Transforms are

$$U(0,s) = U(l,s) = 0$$

Equations (22) and (23) represents a non homogeneous ordinary differential equation, by using boundary conditions one can find the value of complementary function is equal to zero and the upper and lower solutions of (17) can be find out from the particular integral of the equations (22) and (23) respectively.

2.2. Examples

In this section we will discuss the solution of fuzzy partial Telegraph differential equations using FLT to show the utility of the proposed method in Section 3. Solutions are plotted by using MATLAB.

Example 2.1. *Consider the linear fractional telegraph equation of the form*

$$\frac{\partial^{2\alpha} u}{\partial x^{2\alpha}} + 2\frac{\partial^{\alpha} u}{\partial t^{\alpha}} + u = \frac{\partial^2 u}{\partial x^2}, \tag{24}$$

with the initial condition $u(x,0) = 0, u_t(x,0) = (r-1, 1-r)sin(\pi x/l)$ *and boundary condition* $u(0,t) = 0, u(l,t) = 0$, *where* $1/2 < \alpha \leq 1$

Taking the Laplace Transform of both sides of Telegraph equation (24), we have

$$s^{2\alpha}\mathcal{L}[u] - s^{2\alpha-1}u(x,0) - s^{2\alpha-2}u_t(x,0) + 2s^{\alpha}\mathcal{L}[u] - 2s^{\alpha-1}u(x,0) + \mathcal{L}[u] = \frac{\partial^2}{\partial x^2}\mathcal{L}[u], \tag{25}$$

Now using the definition of fuzzy Laplace stated in section 3, equation (25) becomes

$$\frac{\partial^2}{\partial x^2}U(x,s) - [s^{2\alpha} + 2s^{\alpha} + 1]U(x,s) + (s^{\alpha}+2)s^{\alpha-1}u(x,0) + s^{2\alpha-2}u_t(x,0) = 0, \tag{26}$$

The classical form of (26) gives the following two ordinary differential equations as follows:

$$\frac{\partial^2}{\partial x^2}\underline{U}(x,s,r) - [s^{2\alpha} + 2s^{\alpha} + 1]\underline{U}(x,s,r) + (s^{\alpha}+2)s^{\alpha-1}\underline{u}(x,0) + s^{2\alpha-2}\underline{f} = 0, \tag{27}$$

$$\frac{\partial^2}{\partial x^2}\overline{U}(x,s,r) - [s^{2\alpha} + 2s^{\alpha} + 1]\overline{U}(x,s,r) + (s^{\alpha}+2)s^{\alpha-1}\overline{u}(x,0) + s^{2\alpha-2}\overline{f} = 0, \tag{28}$$

Using the given fuzzy initial conditions we have,

$$\frac{\partial^2}{\partial x^2}\underline{U}(x,s,r) - [s^{2\alpha} + 2s^{\alpha} + 1]\underline{U}(x,s,r) + s^{2\alpha-2}(r-1)sin(\pi x/l) = 0, \tag{29}$$

$$\frac{\partial^2}{\partial x^2}\overline{U}(x,s,r) - [s^{2\alpha} + 2s^{\alpha} + 1]\overline{U}(x,s,r) + s^{2\alpha-2}(1-r)sin(\pi x/l) = 0, \tag{30}$$

Equations (29) and (30) represents a non homogeneous ordinary differential equation, by using the ordinary method , the solution can be represented as a sum of complementary function and a particular integral as follows;

$$\underline{U} = Ae^{\beta x} + Be^{-\beta x} + \frac{s^{2\alpha-2}}{\pi^2/l^2 - \beta^2}(1-r)sin(\pi x/l) \tag{31}$$

$$\overline{U} = Ae^{\beta x} + Be^{-\beta x} + \frac{s^{2\alpha-2}}{\pi^2/l^2 - \beta^2}(r-1)sin(\pi x/l) \tag{32}$$

where $\beta = p^{\alpha} + 1$. By using boundary conditions we have $A = B = 0$ i.e the complementary function is equal to zero. Finally,

$$\underline{U} = \frac{s^{2\alpha-2}}{s^{2\alpha} + 2s^{\alpha} + 1 + \pi^2/l^2}(1-r)sin(\pi x/l) \tag{33}$$

$$\overline{U} = \frac{s^{2\alpha-2}}{s^{2\alpha} + 2s^{\alpha} + 1 + \pi^2/l^2}(r-1)sin(\pi x/l) \tag{34}$$

Consequently, taking inverse of Laplace on the both sides of equation (33) and (34) we have

$$\underline{u} = (1-r)\sum_{k=0}^{\infty}(-2)^k t^{\alpha k+1}E^{k+1}_{2\alpha,\alpha k+2}(-1-\pi^2/l^2)t^{2\alpha} \tag{35}$$

$$\overline{u} = (r-1)\sum_{k=0}^{\infty}(-2)^k t^{\alpha k+1} E^{k+1}_{2\alpha,\alpha k+2}(-1-\pi^2/l^2)t^{2\alpha} \tag{36}$$

Equation (35) and (36) are the upper and lower solutions of (17) which is obtained from the particular integral of the equations (33) and (34) respectively. Also it can be noted that n^{th} wave profile of a damped fractional wave equation will have $(n-1)$ equally spaced nodes in a given interval which is similar to a wave profile of a undamped fractional wave equation. Though the solution by Laplace Transform Method is of the form of an infinite series,it can be written in a closed form in some cases. The graph is plotted by truncating the infinite seres to a finite number of terms and the values of the parameters $x = 1/2$ and $l = \pi$ are taken as a constant and varying both t and r, the obtained fuzzy results are depicted in figure 1.

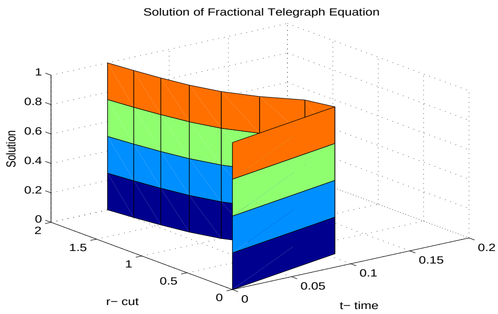

Figure 1.

Solution of Ordinary Telegraph equation when $\alpha = 1$

Consider when $\alpha = 1$, the equation (17) turn into the ordinary telegraph equation. Let we analyze the ordinary telegraph equation with the same initial condition. i.e

$$\frac{\partial^2 u}{\partial t^2} + 2\frac{\partial u}{\partial t} + u = \frac{\partial^2 u}{\partial x^2}, \tag{37}$$

with the initial condition $u(x,0) = 0, u_t(x,0) = (r-1, 1-r)sin(\pi x/l)$ and boundary condition $u(0,t) = 0, u(l,t) = 0$. Taking the Laplace Transform of both sides of Telegraph equation (37), we have

$$s^2U - s^{2\alpha-1}u(x,0) - s^{2\alpha-2}u_t(x,0) + 2s^{\alpha}U - 2s^{\alpha-1}u(x,0) + U = \frac{\partial^2}{\partial x^2}U, \tag{38}$$

By using fuzzy initial condition, the classical form of (38) gives the following two ordinary differential equations as follows:

$$\frac{\partial^2}{\partial x^2}\underline{U}(x,s,r) - [s^2 + 2s + 1]\underline{U}(x,s,r) = (r-1)sin(pix/l), \tag{39}$$

$$\frac{\partial^2}{\partial x^2}\underline{U}(x,s,r) - [s^2 + 2s + 1]\underline{U}(x,s,r) = (r-1)sin(pix/l), \tag{40}$$

by using the Boundary conditions, the solution can be represented as follows;

$$\underline{U} = \frac{(1-r)sin(\pi x/l)}{\pi^2/l^2 + \beta^2} \tag{41}$$

$$\underline{U} = \frac{(r-1)sin(\pi x/l)}{\pi^2/l^2 + \beta^2} \tag{42}$$

where $\beta = p^{\alpha} + 1$. Consequently, taking inverse of Laplace on the both sides of equation (41) and (42) we have

$$\underline{u} = (1-r)e^{-t}\frac{sin(\pi x/l)sin(\pi t/l)}{\pi/l} \tag{43}$$

$$\overline{u} = (r-1)e^{-t}\frac{sin(\pi x/l)sin(\pi t/l)}{\pi/l} \tag{44}$$

Equation (43) and (44) are the upper and lower solutions of (37) which is obtained from the particular integral of the equations (41) and (42) respectively. The graph of upper and lower bounds of the fuzzy solutions are obtained for different values of $r = 0.3, 0.5$ and 0.7 is shown in the figure 2. **Note:** When $r = 1$, the fuzzy initial condition converted into crisp

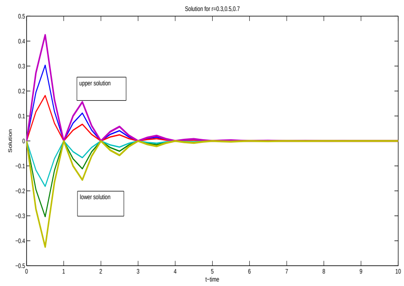

Figure 2.

initial condition. It is interesting to note that lower and upper bounds of the fuzzy solutions are same for $r = 1$. The surface plot for the crisp ordinary telegraph equation is depicted in the following figure 3.

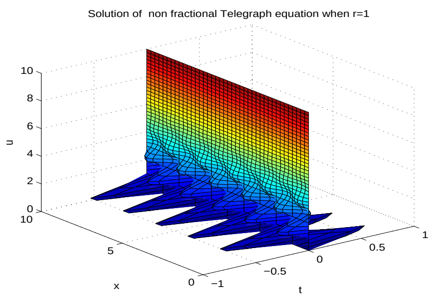

Figure 3.

3. Conclusion

The fractional order theory is used to describe the behavior of many physical systems. In fact, real world processes generally or most likely are fractional order systems. Fractional partial differential system with fuzzy initial condition plays a significant role because, in real life, unfortunately, the exact values of the initial condition is not known exactly. By using Fuzzy function it is possible to get the exact solution. From this we may accomplish that the fractional order system has attractive feature comparing with integer order system. In this paper Solution of Fractional Telegraph equations with fuzzy initial condition is investigated by using the Fuzzy Laplace transform method. To show the applicability of the method, example is presented and it is compared with the integer order system.

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