Int. J. Math. And Appl., 6(1–A)(2018), 163–170
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Intuitionistic Fuzzy-(ρ,σ)-Semi Pre Compact Spaces

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Abstract: In the present paper, we define and study variety of intuitionistic fuzzy (or IF-) compact spaces: IF-alpha compact space, IF-semi compact space, IF-pre compact space and IF-semi pre compact spaces in sense of Sostak. Further we investigate their significant properties and relationship among them.

MSC: 54A40.

Keywords: Intuitionistic fuzzy sets, intuitionistic fuzzy topological spaces, intuitionistic fuzzy continuous map, intuitionistic gradation of openness, intuitionistic fuzzy compactness.

1. Introduction

In [10], Zadeh introduced the concepts of fuzzy sets and Chang in [2], introduced fuzzy topological spaces. Then it was noted that fuzziness in the concept of openness of a fuzzy set is absent. Sostak [9] introduced the concept of gradation of openness of fuzzy sets on a universe X as a mapping $\tau : I^X \to I$ satisfying the following conditions:

(1). $\tau(0) = \tau(1) = 1$;

(2). $\tau(A \cap B) \geq \tau(A) \wedge \tau(B)$ for any $A, B \in I^X$;

(3). $\tau(\cup_{i\in J} A) \geq \cap_{i\in J} \tau(A_i)$, for any $\{A_i : i \in J\} \subseteq I^X$.

The pair (X, τ) is called a Sostak fuzzy topological space. Using this idea of gradation of openness and considering all fuzzy sets having gradation of openness greater than a fixed number ρ, Chattopadhyay et.al. [3] obtained a fuzzy topology τ_ρ in the sense of Chang, called ρ-level fuzzy topology.

Atanassov [1] introduced intuitionistic fuzzy sets and Coker [4] defined intuitionistic fuzzy topological spaces parallel to Chang's fuzzy topological spaces. In 1996, Coker and Dimirci [5] introduced intuitionistic fuzzy topological spaces in Sostak's sense. Later Ramadan et.al. [8] put forward the concept of compactness in these intuitionistic fuzzy topological spaces.

In this paper, we define IF-alpha compact space, IF-semi compact space, IF-pre compact space and IF-semi pre compact spaces in Sostak intuitionistic fuzzy topological spaces. Further we investigate the properties of intuitionistic fuzzy semi pre compact spaces and the relation among variety of IF-compact spaces.

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2. Preliminaries

In this section, we first define the intuitionistic fuzzy topological spaces in sense of Sostak.

Let X be a nonempty set. Let $I \equiv [0,1]$ be the closed unit interval of real line and let $I_0 \equiv (0,1]$; $I_1 \equiv [0,1)$. An entity A is called an intuitionistic fuzzy set (or IF-set in short) of X denoted as $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$, where $\mu_A(x)$ and $\nu_A(x)$ are the degree of membership and degree of non-membership respectively of x in A, such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Let ξ^X be the collection of all intuitionistic fuzzy sets on X. IF-sets $0 \equiv \{< x, 0, 1 >\}$ and $1 \equiv \{< x, 1, 0 >\}$ are called the null IF-set and whole IF-set respectively for each $x \in X$. The complement A^c of an IF-set A is given by $A^c \equiv \{< x, \nu_A(x), \mu_A(x) >\}$ for each $x \in X$.

Let a and b be two real numbers in $[0,1]$ satisfying the condition $a + b \leq 1$. Then the pair $< a, b >$ is called an intuitionistic fuzzy pair (or IF-pair in short) (see [5]).

An IF-set $\lambda \equiv \{< A, \mu_\lambda(A), \nu_\lambda(A) >: A \in \xi^X\}$ on ξ^X defines a collection of IF-pairs $< \mu_\lambda(A), \nu_\lambda(A) >$ satisfying the condition $\mu_\lambda(A) + \nu_\lambda(A) \leq 1$ for each $A \in \xi^X$. This collection is called an intuitionistic fuzzy family (or IF-family in short) on X denoted as $\lambda = \{< \mu_\lambda, \nu_\lambda >\} \equiv < \mu_\lambda(A), \nu_\lambda(A) >$, $A \in \xi^X$.

An intuitionistic fuzzy topology in sense of Sostak (So-IF-topology in short) on a non-empty set X is an IF-family τ on X satisfying the following axioms:

(1). $\tau(0) = \tau(1) = 1$;

(2). $\tau(A \cap B) \geq \tau(A) \wedge \tau(B)$ for any $A, B \in \xi^X$;

(3). $\tau(\cup_{i\in J} A) \geq \cap_{i\in J} \tau(A_i)$, for any $\{A_i : i \in J\} \subseteq \xi^X$.

The pair (X, τ) is called an intuitionistic fuzzy topological space in Sostak's sense (So-IF-topological spaces in short). For any IF-set $A \in \xi^X$, the number $\mu_\tau(A)$ is called the degree of openness and $\nu_\tau(A)$ is called the degree of non-openness.

Now for each IF-set $A = \{< x, \mu_A(x), \nu_A(x) >\} \in \xi^X$, we define a map $\tau : \xi^X \to I \times I$ as follows:

$$\tau(A) =< \mu_\tau(A), \nu_\tau(A) >= \begin{cases} < 1, 0 >, & \text{if } A = 0 \\ < \inf_{x\in X}(\mu_A(x)), \sup_{x\in X}(\nu_A(x)) >, & \text{if } A \neq 0 \end{cases}$$

for each $A \in \xi^X$. Then τ is a So-IF-topology on X denoted as $\tau(A) =< \mu_\tau(A), \nu_\tau(A) >$ for each $A \in \xi^X$.

Let (X, τ) be a So-IF-topological space, then for a given pair (ρ, σ) of reals such that $\rho \in I_0 = (0,1]$, $\sigma \in I_1 = [0,1)$ and $\rho + \sigma \leq 1$, the family $\tau_{\rho,\sigma}$ defined as $\tau_{\rho,\sigma} \equiv \{A \in \xi^X : \tau(A) \geq < \rho, \sigma >\}$ is actually an IF-topological space in sense of Coker [4] and is called the (ρ, σ)-level IF-topology on X. In this case IF-sets belonging to $\tau_{\rho,\sigma}$ are called IF-(ρ, σ)-open sets and their complements are called IF-(ρ, σ)-closed sets.

Let (X, τ) be a So-IF-topological spaces and $A \in \xi^X$ be an IF-set on X. Then for a given $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$, the interior and closure of A with respect to $\tau_{\rho,\sigma}$ are denoted as $Int_{\rho,\sigma}(A)$ and $Cl_{\rho,\sigma}(A)$ respectively. Thus

$$Int_{\rho,\sigma}(A) = \cup\{G \in \xi^X : G \subseteq A, G \in \tau_{\rho,\sigma}\}$$

$$Cl_{\rho,\sigma}(A) = \cap\{K \in \xi^X : A \subseteq K, K^c \in \tau_{\rho,\sigma}\}$$

3. Various Types of IF-(ρ, σ)-Compact Spaces

We first discuss different types of IF-(ρ, σ)-open sets namely IF-(ρ, σ)-alpha open set, IF-(ρ, σ)-semi open set, IF-(ρ, σ)-pre open set and IF-(ρ, σ)-semi pre open set. Then we define the various types of IF-(ρ, σ)-compact spaces and relation among them (see [7]).

Definition 3.1. *Let (X, τ) be a So-IF-topological space. Then for a given $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$, an IF-set A is said to be an*

(1). IF-(ρ, σ)-alpha open set if $A \subseteq Int_{\rho,\sigma}(Cl_{\rho,\sigma}(Int_{\rho,\sigma}(A)))$

(2). IF-(ρ, σ)-semi open set if $A \subseteq Cl_{\rho,\sigma}(Int_{\rho,\sigma}(A))$

(3). IF-(ρ, σ)-pre open set if $A \subseteq Int_{\rho,\sigma}(Cl_{\rho,\sigma}(A))$

(4). IF-(ρ, σ)-semi pre open set if $A \subseteq Cl_{\rho,\sigma}(Int_{\rho,\sigma}(Cl_{\rho,\sigma}(A)))$.

Remark 3.2. *For a given $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$*

(1). Every IF-(ρ, σ)-open (resp. IF-(ρ, σ)-closed) set is IF-(ρ, σ)-alpha open (resp. IF-(ρ, σ)-alpha closed) set.

(2). Every IF-(ρ, σ)-alpha open (resp. IF-(ρ, σ)-alpha closed) set is an IF-(ρ, σ)-semi open (resp. IF-(ρ, σ)-semi closed) set and an IF-(ρ, σ)-pre open (resp. IF-(ρ, σ)-pre closed) set.

(3). Every IF-(ρ, σ)-semi open (resp. IF-(ρ, σ)-semi closed) set and an IF-(ρ, σ)-pre open (resp. IF-(ρ, σ)-pre closed) set is an IF-(ρ, σ)-semi pre open (resp. IF-(ρ, σ)-semi pre closed) set.

But converse of (1), (2), (3) may not be true in general (see [6], [7]).

Definition 3.3. *Let (X, τ) be a So-IF-topological space. A family $W = \{G_i : i \in J\}$ of IF-(ρ, σ)-open sets is called an IF-(ρ, σ)-open cover of X iff $\cup_{i \in J} G_i = 1$, where $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$. A finite subfamily of W, which is also an IF-(ρ, σ)-open cover of A, is called a open subcover of $W = \{G_i : i \in J\}$.*

Definition 3.4. *Let (X, τ) be a So-IF-topological space. A family $W = \{G_i : i \in J\}$ of IF-(ρ, σ)-alpha open sets is called an IF-(ρ, σ)-alpha open cover of X iff $\cup_{i \in J} G_i = 1$, where $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$. A finite subfamily of W, which is also an IF-(ρ, σ)-alpha open cover of A, is called an alpha open subcover of $W = \{G_i : i \in J\}$.*

Definition 3.5. *Let (X, τ) be a So-IF-topological space. A family $W = \{G_i : i \in J\}$ of IF-(ρ, σ)-semi open sets is called an IF-(ρ, σ)-semi open cover of X iff $\cup_{i \in J} G_i = 1$, where $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$. A finite subfamily of W, which is also an IF-(ρ, σ)-semi open cover of A, is called a semi open subcover of $W = \{G_i : i \in J\}$.*

Definition 3.6. *Let (X, τ) be a So-IF-topological space. A family $W = \{G_i : i \in J\}$ of IF-(ρ, σ)-pre open sets is called an IF-(ρ, σ)-pre open cover of X iff $\cup_{i \in J} G_i = 1$, where $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$. A finite subfamily of W, which is also an IF-(ρ, σ)-pre open cover of A, is called a pre open subcover of $W = \{G_i : i \in J\}$.*

Definition 3.7. *Let (X, τ) be a So-IF-topological space. A family $W = \{G_i : i \in J\}$ of IF-(ρ, σ)-semi pre open sets is called an IF-(ρ, σ)-semi pre open cover of X iff $\cup_{i \in J} G_i = 1$, where $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$. A finite subfamily of W, which is also an IF-(ρ, σ)-semi pre open cover of A, is called a semi pre open subcover of $W = \{G_i : i \in J\}$.*

Remark 3.8. *It is clear that for a given $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$*

(1). Every IF-(ρ, σ)-open cover is an IF-(ρ, σ)-alpha open (IF-(ρ, σ)-semi open, IF-(ρ, σ)-pre open and IF-(ρ, σ)-semi pre open) cover.

(2). Every IF-(ρ, σ)-alpha open cover is an IF-(ρ, σ)-semi open cover and an IF-(ρ, σ)-pre open cover.

(3). Every IF-(ρ, σ)-semi open cover and every IF-(ρ, σ)-pre open cover is an IF-(ρ, σ)-semi pre open cover.

But converse of (1), (2) and (3) may not true in general.

Definition 3.9. *Let (X, τ) be a So-IF-topological space. Then for a given $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$, X is called an*

(1). IF-(ρ, σ)-compact space iff every IF-(ρ, σ)-open cover of X has a finite subcover.

(2). IF-$(\rho, , \sigma)$-alpha compact space iff every IF-(ρ, σ)-alpha open cover of X has a finite subcover.

(3). IF-$(\rho, , \sigma)$-semi compact space iff every IF-(ρ, σ)-semi open cover of X has a finite subcover.

(4). IF-$(\rho, , \sigma)$-pre compact space iff every IF-(ρ, σ)-pre open cover of X has a finite subcover.

Proposition 3.10. *We observe that for a given $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$*

(1) Every IF-(ρ, σ)-semi compact space is an IF-(ρ, σ)-alpha compact space and IF-(ρ, σ)-compact space.

(2) Every IF-(ρ, σ)-pre compact space is an IF-(ρ, σ)-alpha compact space and IF-(ρ, σ)-compact space.

(3) Every IF-(ρ, σ)-alpha compact space is an IF-(ρ, σ)-compact space.

Proof. It follows from Remark 3.2. □

4. IF-(ρ, σ)-Semi Pre Compact Space

In this section, we define IF-(ρ, σ)-semi pre compact space and investigate its characteristic properties.

Definition 4.1. *Let (X, τ) be a So-IF-topological space. Then for a given $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$, space X is said to be an IF-(ρ, σ)-semi pre compact space iff every IF-(ρ, σ)-semi pre open cover of X has a finite subcover.*

Example 4.2. *Let X be a nonempty set and consider the IF-sets $\{A_n : n \in N\}$ defined as $A_n = \{< x, 1-1/n, 1/n >: n \in N\}$ for each $x \in X$. Now we define a So-IF-topology $\tau : \xi^X \to I \times I$ as follows:*

$$\tau(F) = \begin{cases} < 1, 0 >, & \text{if } F = 0, 1 \\ < 1/n, 1/2n >, & \text{if } F = A_n, \\ < 0, 1 >, & \text{otherwise} \end{cases}$$

Let $\alpha = 0.2, \beta = 0.8$. We see that each member of collection $\{A_n : n \in N\}$ is an IF-semi pre open set because $A_n \subseteq Cl_{\rho,\sigma}(Int_{\rho,\sigma}(Cl_{\rho,\sigma}(A_n))) = 1, \forall n \in N$ and also $\cup_{n \in N} A_n = 1$. Thus the collection $\{A_n : n \in N\}$ is an IF-semi pre open cover of X. But no finite subset of $\{A_n : n \in N\}$ covers X. Hence X is not an IF-semi pre compact space.

Definition 4.3. *Let (X, τ) be a So-IF-topological space and A be an IF-set in X. Then for a given $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$*

(1). a family $W = \{G_i : i \in J\}$ of IF-(ρ, σ)-semi pre open sets is called an IF-(ρ, σ)-semi pre open cover of A iff $A \subseteq \cup_{i \in J} G_i$. A finite subfamily of W, which is also an IF-(ρ, σ)-semi pre open cover of A is called a semi pre open subcover of $W = \{G_i : i \in J\}$ and

(2). IF-set A is called an IF-(ρ, σ)-semi pre compact iff every IF-(ρ, σ)-semi pre open cover of X has a finite subcover.

Definition 4.4. *A family $W = \{G_i : i \in J\}$ of IF-(ρ, σ)-closed sets in X has finite intersection property iff the intersection of members of each finite subfamily of W is nonempty.*

Now we investigate the properties of IF-(ρ, σ)-semi pre compact spaces as follows.

Proposition 4.5. *Every IF-(ρ, σ)-semi pre compact space is an IF-(ρ, σ)-compact space.*

Proof. Let (X, τ) be an IF-(ρ, σ)-semi pre compact space and let a family $W = \{G_i : i \in J\} \subseteq \xi^X$ be an IF-(ρ, σ)-open cover of X such that $\cup_{i\in J} G_i = 1$. Since every IF-(ρ, σ)-open cover is an IF-(ρ, σ)-semi pre open cover. Therefore the collection $W = \{G_i : i \in J\}$ is an IF-(ρ, σ)-semi pre open cover of X. Now X is an IF-(ρ, σ)-semi pre compact space, sothat there exists a finite subset J_0 of J such that $\cup_{i\in J_o} G_i = 1$. Hence X has a finite subcover which belong to W. Thus X is an IF-(ρ, σ)-compact space. □

Proposition 4.6. *Every IF-(ρ, σ)-semi pre compact space is an IF-(ρ, σ)-semi compact space.*

Proof. It follows from Remark 3.2(3). □

Proposition 4.7. *Every IF-(ρ, σ)-semi pre compact space is an IF-(ρ, σ)-pre compact space.*

Proof. It is immediate from Remark 3.2(3). □

Remark 4.8. *For a given $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$, an IF-(ρ, σ)-compact (respectively IF-(ρ, σ)-alpha compact, IF-(ρ, σ)-semi compact, IF-(ρ, σ)-pre compact) space need not be an IF-(ρ, σ)-semi pre compact space.*

Proof. We can easily explain this by using Remark 3.2. □

Theorem 4.9. *A So-IF-topological space (X, τ) is an IF-(ρ, σ)-semi pre compact space if and only if every collection $\{G_i : i \in J\}$ of IF-(ρ, σ)-semi pre closed sets, where $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$ having finite intersection property has a nonempty intersection.*

Proof. Let (X, τ) be an IF-(ρ, σ)-semi pre compact space, where $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$. Let $W = \{G_i : i \in J\}$ be a collection of IF-(ρ, σ)-semi pre closed sets having finite intersection property. To show that collection W has a nonempty intersection i.e. $\cap_{i\in J} G_i \neq 0$. Suppose $\cap_{i\in J} G_i = 0$, then $\cup_{i\in J} G_i^c = 1$. Since for each $i \in J$, each G_i is an IF-(ρ, σ)-semi pre closed set, sothat its complement G_i^c is an IF-(ρ, σ)-semi pre open set. Further X is an IF-(ρ, σ)-semi pre compact space and every IF-(ρ, σ)-semi pre open cover of X has a finite subcover. Thus there exists a finite sub-collection $\{G_i : i \in J_0\}$, where $J_0 \subseteq J$ such that $\cup_{i\in J_0} G_i^c = 1$. It implies $\cap_{i\in J_0} G_i = 0$, which is a contradiction. Hence $\cap_{i\in J} G_i \neq 0$.
Conversely; Let a family of IF-(ρ, σ)-semi pre closed sets in X with the finite intersection property has a nonempty intersection, then we shall show that X is an IF-(ρ, σ)-semi pre compact space. Let $W = \{G_i : i \in J\}$ be a family of IF-(ρ, σ)-semi pre open sets such that $\cup_{i\in J} G_i = 1$. Now if $\cup_{i\in J_0} G_i \neq 1$ for every finite subset J_0 of J, then $\cap_{i\in J_0} G_i^c \neq 0$ and the family $\{G_i^c : i \in J\}$ has finite intersection property. Hence from given condition, we have $\cap_{i\in J} G_i^c \neq 0$, sothat $\cup_{i\in J} G_i \neq 1$, which is a contradiction. Hence X is an IF-(ρ, σ)-semi pre compact space. □

Proposition 4.10. *Every IF-(ρ, σ)-semi pre closed subset of an IF-(ρ, σ)-semi pre compact space is an IF-(ρ, σ)-compact space.*

Proof. Let (X, τ) be an IF-(ρ, σ)-semi pre compact space for a given $\rho \in I_0, \sigma \in I_1$ such that $\rho+\sigma \leq 1$ and $A \in \xi^X$ be an IF-(ρ, σ)-semi pre closed subset of X. Then we shall prove that A is an IF-(ρ, σ)-compact space. Suppose $W = \{G_i : i \in J\} \subseteq \xi^X$ be an IF-(ρ, σ)-open cover of A, sothat $A \subseteq \cup_{i\in J} G_i$. Since every IF-$(\rho, \sigma)$-open cover is an IF-$(\rho, \sigma)$-semi pre open cover, thus $W = \{G_i : i \in J\}$ is an IF-(ρ, σ)-semi pre open cover of A. Since A is an IF-(ρ, σ)-semi pre closed subset of X, A^c is an IF-(ρ, σ)-semi pre open subset of X. Therefore the collection $\{G_i : i \in J\} \cup A^c$ is an IF-(ρ, σ)-semi pre open cover of X. Now X is an IF-(ρ, σ)-semi pre compact space, there exists a finite subset J_0 of J such that $\cup[\{G_i : i \in J_0\} \cup A^c] = 1$.

Therefore X has a finite subcovers $\{G_i : i \in J_0\} \cup A^c$ and $\{G_i : i \in J_0\}$ is a finite subcover of A. Hence for IF-(ρ, σ)-open cover W of A has a finite subcover such that $A \subseteq \cup_{i\in J_0} G_i$. Thus A is an IF-(ρ, σ)-compact space. □

Proposition 4.11. *Every IF-(ρ, σ)-semi pre closed subset of an IF-(ρ, σ)-semi pre compact space is an IF-(ρ, σ)-semi pre compact.*

Proof. Let (X, τ) be an IF-(ρ, σ)-semi pre compact space and $A \in \xi^X$ be an IF-(ρ, σ)-semi pre closed subset of X for a given $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$. Then we shall prove that A is an IF-(ρ, σ)-semi pre compact space. Suppose $W = \{G_i : i \in J\} \subseteq \xi^X$ be an IF-(ρ, σ)-open cover of A, then $A \subseteq \cup_{i\in J} G_i$. Since every IF-$(\rho, \sigma)$-open cover is an IF-$(\rho, \sigma)$-semi pre open cover. Hence $W = \{G_i : i \in J\}$ is an IF-(ρ, σ)-semi pre open cover of A and each $G_i, \forall i \in J$ is an IF-(ρ, σ)-semi pre open set. Since A is an IF-(ρ, σ)-semi pre closed subset of X, then A^c is an IF-(ρ, σ)-semi pre open subset of X. Therefore the collection $\{G_i : i \in J\} \cup A^c$ is an IF-(ρ, σ)-semi pre open cover of X, which is an IF-(ρ, σ)-semi pre compact space. It follows that there exists a finite subset J_0 of J such that $\cup[\{G_i : i \in J_0\} \cup A^c] = 1$. Therefore X has a finite subcovers $\{G_i : i \in J_0\} \cup A^c$ and $\{G_i : i \in J_0\}$ is a finite subcover of A. Hence IF-(ρ, σ)-semi pre open cover W of A has a finite subcover such that $A \subseteq \cup_{i\in J_0} G_i$. Thus A is an IF-(ρ, σ)-semi pre compact space. □

Corollary 4.12. *Every IF-(ρ, σ)-closed subset of an IF-(ρ, σ)-semi pre compact space is an IF-(ρ, σ)-compact.*

Proof. It follows from Proposition 4.5, because every IF-(ρ, σ)-closed set is an IF-(ρ, σ)-semi pre closed set. □

Remark 4.13. *We observe that for a given $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$, an IF-(ρ, σ)-semi pre closed subset of an IF-(ρ, σ)-compact space need not be IF-(ρ, σ)-compact.*

Proof. We can explain this by using Remark 3.1. □

Proposition 4.14. *Let (X, τ) be a So-IF-topological space. IF A and B are two IF-(ρ, σ)-semi pre compact subsets of X for a given $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$, then $A \cup B$ is also an IF-(ρ, σ)-semi pre compact.*

Proof. Suppose a collection $W = \{G_i : i \in J\}$ is an IF-(ρ, σ)-semi pre open cover of $A \cup B$, then $A \cup B \subseteq \cup_{i\in J} G_i$. It follows that $A \subseteq \cup_{i\in J} G_i$ and $B \subseteq \cup_{i\in J} G_i$ (because $A \subseteq A \cup B$ and $B \subseteq A \cup B$). Hence W is an IF-(ρ, σ)-semi pre open cover of A as well as B. Since A is an IF-(ρ, σ)-semi pre compact subset of X, then there exists a finite sub-collection $\{G_i : i \in J_0 \text{ and } J_0 \subseteq J\}$, which covers A such that $A \subseteq \cup_{i\in J_0} G_i$. Similarly IF-$(\rho, \sigma)$-semi pre open cover W of B has a finite subcover $\{G_i : i \in J_1 \text{ and } J_1 \subseteq J\}$ such that $B \subseteq \cup_{i\in J_1} G_i$. Let $J_2 = max\{J_0, J_1\} \subseteq J$, then $A \cup B \subseteq \cup_{i\in J_2} G_i$. Hence IF-$(\rho, \sigma)$-semi pre open cover W of $A \cup B$ has a finite subcover. Thus $A \cup B$ is an IF-(ρ, σ)-semi pre compact. □

Proposition 4.15. *Let (X, τ) be a So-IF-topological space. IF A and B are two IF-(ρ, σ)-semi pre compact subsets of X for a given $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$, then $A \cap B$ need not be IF-(ρ, σ)-semi pre compact.*

Proof. It is easy to prove because the intersection of two IF-(ρ, σ)-semi pre open sets need not be an IF-(ρ, σ)-semi pre open set. □

Definition 4.16. *Let (X, τ) and (Y, δ) be two So-IF-topological spaces, where τ and δ are (ρ, σ)-level IF-topologies on X and Y respectively. Then a map $f : X \to Y$ is said to be an*

(1). IF-(ρ, σ)-continuous iff $\tau(f^{-1}(B)) \geq \delta(B)$ for each $B \in \xi^X$ such that $\delta(B) \geq <\rho, \sigma>$ and

(2). IF-(ρ, σ)-semi pre continuous map if $f^{-1}(B)$ is an IF-(ρ, σ)-semi pre open set in X for each $B \in \xi^Y$ such that $\delta(B) \geq <\rho, \sigma>$.

It is easily observe that every IF-(ρ, σ)-continuous map is an IF-(ρ, σ)-semi pre continuous map.

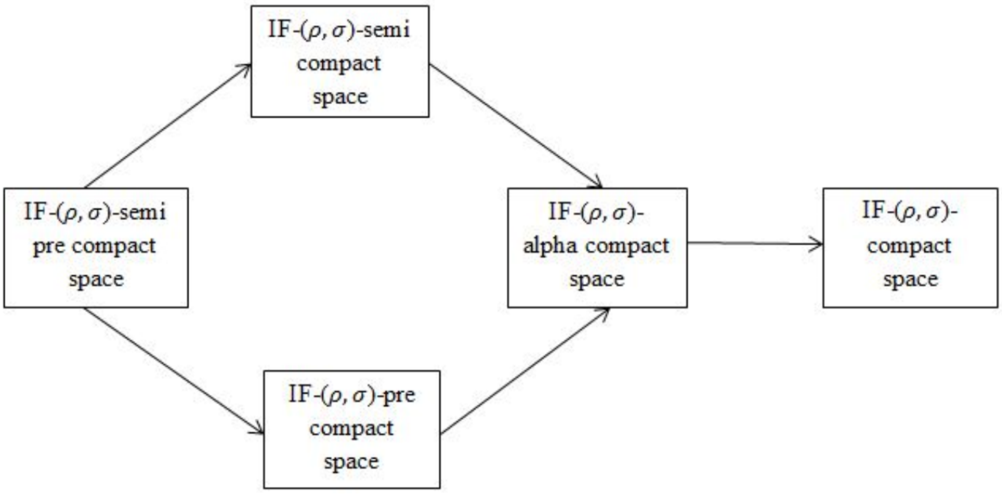
Proposition 4.17. *The IF-(ρ, σ)-semi pre continuous image of an IF-(ρ, σ)-semi pre compact space is an IF-(ρ, σ)-compact space.*

Proof. Let (X, τ) be an IF-(ρ, σ)-semi pre compact space and (Y, δ) be any So-IF-topological space. Let $f : (X, \tau) \to (Y, \delta)$ be an IF-(ρ, σ)-semi pre continuous map. Then we shall prove that (Y, δ) is an IF-(ρ, σ)-compact space. Let $W = \{G_i : i \in J\}$ be an IF-(ρ, σ)-open cover of Y such that $\cup_{i\in J} G_i = 1$, then $\{f^{-1}(G_i) : i \in J\}$ is an IF-(ρ, σ)-semi pre open cover of X. Since X is an IF-(ρ, σ)-semi pre compact space, then there exists a finite subcover $\{f^{-1}(G_i) : i \in J_o\}$, where $J_0 \subseteq J$ such that $\cup_{i\in J_0} f^{-1}(G_i) = 1$. Then $f(\cup_{i\in J_0} f^{-1}(G_i)) = f(1)$. It implies $\cup_{i\in J_0} f(f^{-1}(G_i)) = f(1)$. Therefore $\cup_{i\in J_0} G_i = 1$. Hence for IF-(ρ, σ)-open cover of Y, there exists a finite subcover $\{G_i : i \in J_0$, which covering Y. Thus Y is an IF-(ρ, σ)-compact space. □

Proposition 4.18. *The IF-(ρ, σ)-continuous image of an IF-(ρ, σ)-semi pre compact space is an IF-(ρ, σ)-compact space.*

Proof. It is immediate from Proposition 4.10 because every IF-(ρ, σ)-continuous map is an IF-(ρ, σ)-semi pre continuous map. □

Remark 4.19. *The following diagram explain the relationship among different types of IF-(ρ, σ)-compact spaces.*



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