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Topological Indices of Vitamin A

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Abstract: Graph theory has provided chemists with a variety of useful tools, such as topological indices. A topological index $Top(G)$ of a graph G is a number with the property that for every graph H isomorphic to G, $Top(H) = Top(G)$. In this paper, we compute ABC index, ABC_4 index, Randić connectivity index, Sum connectivity index, GA index , GA_5 index, First Zagreb index, Second Zagreb index, First Multiple Zagreb index, Second Multiple Zagreb index, Augmented Zagreb index, Harmonic index and Hyper Zagreb index of Vitamin A.

MSC: 05C12, 05C90.

Keywords: ABC index, ABC_4 index, Randić connectivity index, Sum connectivity index, GA index , GA_5 index, First Zagreb index, Second Zagreb index, First Multiple Zagreb index, Second Multiple Zagreb index, Augmented Zagreb index, Harmonic index, Hyper Zagreb index and Vitamin A.

1. Introduction

Vitamin A, also called retinol, a fat-soluble alcohol, most abundant in fatty fish and especially in fish-liver oils. Vitamin A is also found in milk fat, eggs, and liver. Vitamin A is not present in plants, but many vegetables and fruits contain one or more of a class of pigments that can be converted to vitamin A in the body. It's molecular formula is $C_{20}H_{30}O$. In 1912, an English biochemist called Frederick Gowland Hopkins found unknown factors present in milk that were not fats, proteins or carbohydrates, but were required to aid growth in rats. Hopkins was later awarded the Nobel Prize (in 1929) for this discovery. In 1917, Elmer McCollum from the University of WisconsinMadison along with Lafayette Mendel and Thomas Burr Osborne from Yale University discovered one of these substances while researching the role of dietary fats. In 1918, these accessory factors were described as fat soluble and in 1920, they were referred to as vitamin A.

Topological indices are the molecular descriptors that describe the structures of chemical compounds and they help us to predict certain physico-chemical properties like boiling point, enthalpy of vaporization, stability, etc. Molecules and molecular compounds are often modeled by molecular graph. A molecular graph is a representation of the structural formula of a chemical compound in terms of graph theory, whose vertices correspond to the atoms of the compound and edges correspond to chemical bonds. Note that hydrogen atoms are often omitted. All molecular graphs considered in this paper are finite, connected, loopless and without multiple edges. Let $G = (V, E)$ be a graph with vertex set V and edge set E . The degree of a vertex $u \in E(G)$ is denoted by d_u and is the number of vertices that are adjacent to u. The edge connecting the vertices

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u and v is denoted by uv.

In this paper, we determine the topological indices like Atom-bond connectivity index, Fourth Atom-bond connectivity index, Sum connectivity index, Randić connectivity index, Geometric- arithmetic connectivity index and Fifth Geometric-arithmetic connectivity index, First Zagreb index, Second Zagreb index, First Multiple Zagreb index, Second Multiple Zagreb index, Augmented Zagreb index, Harmonic index and Hyper Zagreb index of Vitamin A.

The Atom-bond connectivity index, ABC index is one of the degree based molecular descripter, which was introduced by Estrada et al. [8] in late $1990's$ and it can be used for modelling thermodynamic properties of organic chemical compounds, it is also used as a tool for explaining the stability of branched alkanes [9]. Some upper bounds for the atom-bond connectivity index of graphs can be found in [4], The atom-bond connectivity index of chemical bicyclic graphs, connected graphs can be seen in [5, 34]. For further results on ABC index of trees see the papers [12, 21, 33, 35] and the references cited there in.

Definition 1.1. *Let $G{=}(V,E)$ be a molecular graph and d_u is the degree of the vertex u, then ABC index of G is defined as,* $ABC(G){=}\sum\limits_{uv\in E}\sqrt{\dfrac{d_u+d_v-2}{d_ud_v}}$.

The fourth atom bond connectivity index, $ABC_4(G)$ index was introduced by M. Ghorbani et al. [15] in 2010. Further studies on $ABC_4(G)$ index can be found in [10, 11].

Definition 1.2. *Let G be a graph, then its fourth ABC index is defined as,* $ABC_4(G){=}\sum\limits_{uv\in E(G)}\sqrt{\dfrac{S_u+S_v-2}{S_uS_v}}$, *where S_u is sum of the degrees of all neighbours of vertex u in G. In other words,* $S_u=\sum\limits_{uv\in E(G)} d_v$, *Similarly for S_v.*

The first and oldest degree based topological index is Randić index [25] denoted by $\chi(G)$ and was introduced by Milan Randić in 1975. It provides a quantitative assessment of branching of molecules.

Definition 1.3. *For the graph G Randić index is defined as,* $\chi(G){=}\sum\limits_{uv\in E(G)}\dfrac{1}{\sqrt{d_ud_v}}$.

Sum connectivity index belongs to a family of Randić like indices and it was introduced by Zhou and N. Trinajstić [37]. Further studies on Sum connectivity index can be found in [38, 39].

Definition 1.4. *For a simple connected graph G, its sum connectivity index $S(G)$ is defined as,* $S(G)=\sum\limits_{uv\in E(G)}\dfrac{1}{\sqrt{d_u+d_v}}$.

The Geometric-arithmetic index, $GA(G)$ index of a graph G was introduced by D. Vukičević et.al [29]. Further studies on GA index can be found in [3, 6, 32].

Definition 1.5. *Let G be a graph and $e=uv$ be an edge of G then,* $GA(G)=\sum\limits_{e=uv\in E(G)}\dfrac{2\sqrt{d_ud_v}}{d_u+d_v}$.

The fifth Geometric-arithmetic index, $GA_5(G)$ was introduced by A.Graovac et al [16] in 2011.

Definition 1.6. *For a Graph G, the fifth Geometric-arithmetic index is defined as* $GA_5(G)=\sum\limits_{uv\in E(G)}\dfrac{2\sqrt{S_uS_v}}{S_u+S_v}$, *where S_u is the sum of the degrees of all neighbors of the vertex u in G, similarly S_v.*

A pair of molecular descriptors (or topological index), known as the First Zagreb index $M_1(G)$ and Second Zagreb index $M_2(G)$, first appeared in the topological formula for the total π-energy of conjugated molecules that has been derived in 1972 by I. Gutman and N.Trinajstic[17]. Soon after these indices have been used as branching indices. Later the Zagreb indices found applications in QSPR and QSAR studies. Zagreb indices are included in a number of programs used for the routine computation of topological indices POLLY, DRAGON, CERIUS, TAM, DISSI. $M_1(G)$ and $M_2(G)$ were recognize as measures of the branching of the carbonc atom molecular skeleton [20], and since then these are frequently used for structureproperty modeling. Details on the chemical applications of the two Zagreb indices can be found in the books [27, 28]. Further studies on Zagreb indices can be found in [2, 18, 37–39].

Definition 1.7. *For a simple connected graph G, the first and second Zagreb indices were defined as follows*

$$M_1(G) = \sum_{e=uv \in E(G)} (d_u + d_v), \qquad M_2(G) = \sum_{e=uv \in E(G)} d_u d_v,$$

where d_v denotes the degree (number of first neighbors) of vertex v in G.

In 2012, M. Ghorbani and N. Azimi [14] defined the Multiple Zagreb topological indices of a graph G, based on degree of vertices of G.

Definition 1.8. *For a simple connected graph G, the first and second multiple Zagreb indices were defined as follows*

$$PM_1(G) = \prod_{e=uv \in E(G)} (d_u + d_v), \qquad PM_2(G) = \prod_{e=uv \in E(G)} d_u d_v.$$

Properties of the first and second Multiple Zagreb indices may be found in [7, 19].

The Augmented Zagreb index was introduced by Furtula et al [13]. This graph invariant has proven to be a valuable predictive index in the study of the heat of formation in octanes and heptanes, is a novel topological index in chemical graph theory, whose prediction power is better than atom-bond connectivity index. Some basic investigation implied that AZI index has better correlation properties and structural sensitivity among the very well established degree based topological indices.

Definition 1.9. *Let $G = (V, E)$ be a graph and d_u be the degree of a vertex u, then augmented Zagreb index is denoted by $AZI(G)$ and is defined as $AZI(G) = \sum_{uv \in E} \left[\frac{d_u d_v}{d_u + d_v - 2}\right]^3$. Further studies can be found in [23] and the references cited there in.*

The Harmonic index was introduced by Zhong [36]. It has been found that the harmonic index correlates well with the Randić index and with the π-electron energy of benzenoid hydrocarbons.

Definition 1.10. *Let $G = (V, E)$ be a graph and d_u be the degree of a vertex u then Harmonic index is defined as $H(G) = \sum_{e=uv \in E(G)} \frac{2}{d_u + d_v}$. Further studies on $H(G)$ can be found in [31, 38].*

G.H. Shirdel et.al [26] introduced a new distance-based of Zagreb indices of a graph G named Hyper-Zagreb Index.

Definition 1.11. *The hyper Zagreb index is defined as, $HM(G) = \sum_{e=uv \in E(G)} (d_u + d_v)^2$.*

2. Main results

Theorem 2.1. *The Atom bond connectivity index of Vitamin A is given by, $ABC(C_{20}H_{30}O) = 15 \cdot 3932$.*

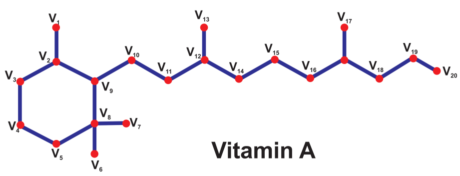

Figure 1.

Proof. Consider a Vitamin A $C_{20}H_{30}O$. Let $m_{i,j}$ denotes edges connecting the vertices of degrees d_i and d_j. Two-dimensional structure of Vitamin A (as shown in the Figure−1) contains edges of the type $m_{1,2}$, $m_{1,3}$, $m_{1,4}$, $m_{2,2}$, $m_{2,3}$, $m_{2,4}$, $m_{3,3}$ and $m_{3,4}$. From the figure, the number edges of these types are as follows. $|m_{1,2}| = 1$, $|m_{1,3}| = 3$, $|m_{1,4}| = 2$, $|m_{2,2}| = 6$, $|m_{2,3}| = 6$, $|m_{2,4}| = 1$, $|m_{3,3}| = 1$ and $|m_{3,4}| = 1$. Therefore The atom-bond connectivity index of Vitamin A

$$ABC(C_{20}H_{30}O) = \sum_{uv\in E} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}.$$

$$\begin{aligned} ABC(C_{20}H_{30}O) = {} & |m_{1,2}|\sqrt{\frac{1+2-2}{1\cdot 2}} + |m_{1,3}|\sqrt{\frac{1+3-2}{1\cdot 3}} + |m_{1,4}|\sqrt{\frac{1+4-2}{1\cdot 4}} + |m_{2,2}|\sqrt{\frac{2+2-2}{2\cdot 2}} \\ & + |m_{2,3}|\sqrt{\frac{2+3-2}{2\cdot 3}} + |m_{2,4}|\sqrt{\frac{2+4-2}{2\cdot 4}} + |m_{3,3}|\sqrt{\frac{3+3-2}{3\cdot 3}} + |m_{3,4}|\sqrt{\frac{3+4-2}{3\cdot 4}} \end{aligned}$$

$$ABC(C_{20}H_{30}O) = 15 \cdot 3932.$$

□

Theorem 2.2. *The fourth atom bond connectivity index of Vitamin A is* $ABC_4(C_{20}H_{30}O) = 12 \cdot 0805$.

Proof. Let $e_{i,j}$ denotes the edges of Vitamin A with $i = S_u$ and $j = S_v$. It is easy to see that the summation of degrees of edge endpoints of Vitamin A have twelve edge types $e_{2,3}$, $e_{3,5}$, $e_{3,6}$, $e_{4,5}$, $e_{4,6}$, $e_{4,7}$, $e_{5,5}$, $e_{5,6}$, $e_{5,9}$, $e_{6,7}$, $e_{6,9}$,$e_{7,9}$ as shown in the following Figure 1. Clearly from the figure −1 $|e_{2,3}| = 1$, $|e_{3,5}| = 3$, $|e_{3,6}| = 1$, $|e_{4,5}| = 3$, $|e_{4,6}| = 1$, $|e_{4,7}| = 2$, $|e_{5,5}| = 5$, $|e_{5,6}| = 1$, $|e_{5,9}| = 1$, $|e_{6,7}| = 1$, $|e_{6,9}| = 1$, $|e_{7,9}| = 1$. The fourth atom-bond connectivity index of Vitamin A

$$ABC_4(C_{20}H_{30}O) = \sum_{uv\in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}$$

$$\begin{aligned} ABC_4(C_{20}H_{30}O) = {} & |e_{2,3}|\left(\sqrt{\frac{2+3-2}{2\cdot 3}}\right) + |e_{3,5}|\left(\frac{\sqrt{3+5-2}}{3\cdot 5}\right) + |e_{3,6}|\left(\sqrt{\frac{3+6-2}{3\cdot 6}}\right) + |e_{4,5}|\left(\sqrt{\frac{4+5-2}{4\cdot 5}}\right) \\ & + |e_{4,6}|\left(\sqrt{\frac{4+6-2}{4\cdot 6}}\right) + |e_{4,7}|\left(\sqrt{\frac{4+7-2}{4\cdot 7}}\right) + |e_{5,5}|\left(\sqrt{\frac{5+5-2}{5\cdot 5}}\right) + |e_{5,6}|\left(\sqrt{\frac{5+6-2}{5\cdot 6}}\right) \\ & + |e_{5,9}|\left(\sqrt{\frac{5+9-2}{5\cdot 6}}\right) + |e_{6,7}|\left(\sqrt{\frac{6+7-2}{6\cdot 7}}\right) + |e_{6,9}|\left(\sqrt{\frac{6+9-2}{6\cdot 9}}\right) + |e_{7,9}|\left(\sqrt{\frac{7+9-2}{7\cdot 9}}\right) \end{aligned}$$

$$ABC_4(C_{20}H_{30}O) = 12 \cdot 0805$$

□

Theorem 2.3. *The Randić connectivity index of Vitamin A is* $\chi(C_{20}H_{30}O) = 9 \cdot 8642$.

Proof. Consider Randić connectivity index of Vitamin A

$$\begin{aligned} \chi(C_{20}H_{30}O) = {} & \sum_{e=uv\in E(G)} \frac{1}{\sqrt{d_u d_v}} \\ & = |m_{1,2}|\left(\frac{1}{\sqrt{1\cdot 2}}\right) + |m_{1,3}|\left(\frac{1}{\sqrt{1\cdot 3}}\right) + |m_{1,4}|\left(\frac{1}{\sqrt{1\cdot 4}}\right) + +|m_{2,2}|\left(\frac{1}{\sqrt{2\cdot 2}}\right) \\ & + |m_{2,3}|\left(\frac{1}{\sqrt{2\cdot 3}}\right) + |m_{2,4}|\left(\frac{1}{\sqrt{2\cdot 4}}\right) + |m_{3,3}|\left(\frac{1}{\sqrt{3\cdot 3}}\right) + +|m_{3,4}|\left(\frac{1}{\sqrt{3\cdot 4}}\right) \end{aligned}$$

$$\therefore \chi(C_{20}H_{30}O) = 9 \cdot 8642.$$

□

Theorem 2.4. *The sum connectivity index of Vitamin A is* $S(C_{20}H_{30}O) = 9 \cdot 8495$.

Proof. Consider the sum connectivity index of Vitamin A

$$S(C_{20}H_{30}O) = \sum_{e=uv\in E(G)} \frac{1}{\sqrt{d_u + d_v}}$$

$$= |m_{1,2}|\Big(\frac{1}{\sqrt{1+2}}\Big) + |m_{1,3}|\Big(\frac{1}{\sqrt{1+3}}\Big) + |m_{1,4}|\Big(\frac{1}{\sqrt{1+4}}\Big) + |m_{2,2}|\Big(\frac{1}{\sqrt{2+2}}\Big)$$
$$+ |m_{2,3}|\Big(\frac{1}{\sqrt{2+3}}\Big) + |m_{2,4}|\Big(\frac{1}{\sqrt{2+4}}\Big) + |m_{3,3}|\Big(\frac{1}{\sqrt{3+3}}\Big) + |m_{3,4}|\Big(\frac{1}{\sqrt{3+4}}\Big)$$
$$\therefore S(C_{20}H_{30}O) = 9 \cdot 8495.$$

□

Theorem 2.5. *The Geometric-Arithmetic index of Vitamin A is $GA(C_{20}H_{30}O) = 25 \cdot 9522$.*

Proof. Consider the Geometric-Arithmetic index of Vitamin A

$$GA(C_{20}H_{30}O) = \sum_{e=uv\in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}$$
$$= |m_{1,2}|\Big(\frac{2\sqrt{1\cdot 2}}{1+2}\Big) + |m_{1,3}|\Big(\frac{2\sqrt{1\cdot 3}}{1+3}\Big) + |m_{1,4}|\Big(\frac{2\sqrt{1\cdot 4}}{1+4}\Big) + |m_{2,2}|\Big(\frac{2\sqrt{2\cdot 2}}{2+2}\Big)$$
$$+ |m_{2,3}|\Big(\frac{2\sqrt{2\cdot 3}}{2+3}\Big) + |m_{2,4}|\Big(\frac{2\sqrt{2\cdot 4}}{2+4}\Big) + |m_{3,3}|\Big(\frac{2\sqrt{3\cdot 3}}{3+3}\Big) + |m_{3,4}|\Big(\frac{2\sqrt{3\cdot 4}}{3+4}\Big)$$
$$GA(C_{20}H_{30}O) = 25 \cdot 9522.$$

□

Theorem 2.6. *The fifth Geometric-Arithemetic index of Vitamin A is $GA_5(C_{20}H_{30}O) = 20 \cdot 6359$.*

Proof. Consider the fifth Geometric-Arithemetic index of Vitamin A

$$GA_5(C_{20}H_{30}O) = \sum_{uv\in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v}$$
$$= |e_{2,3}|\Big(\frac{2\sqrt{2\cdot 3}}{2+3}\Big) + |e_{3,5}|\Big(\frac{2\sqrt{3\cdot 5}}{3+5}\Big) + |e_{3,6}|\Big(\frac{2\sqrt{3\cdot 6}}{3+6}\Big) + |e_{4,5}|\Big(\frac{2\sqrt{4\cdot 5}}{4+5}\Big)$$
$$+ |e_{4,6}|\Big(\frac{2\sqrt{4\cdot 6}}{4+6}\Big) + |e_{4,7}|\Big(\frac{2\sqrt{4\cdot 7}}{4+7}\Big) + |e_{5,5}|\Big(\frac{2\sqrt{5\cdot 5}}{5+5}\Big) + |e_{5,6}|\Big(\frac{2\sqrt{5\cdot 6}}{5+6}\Big)$$
$$+ |e_{5,9}|\Big(\frac{2\sqrt{5\cdot 9}}{5+9}\Big) + |e_{6,7}|\Big(\frac{2\sqrt{6\cdot 7}}{6+7}\Big) + |e_{6,9}|\Big(\frac{2\sqrt{6\cdot 9}}{6+9}\Big) + |e_{7,9}|\Big(\frac{2\sqrt{7\cdot 9}}{7+9}\Big)$$
$$\therefore GA_5(C_{20}H_{30}O) = 20 \cdot 6359.$$

□

Theorem 2.7. *The First Zagreb index of Vitamin A is $M_1(C_{20}H_{30}O) = 98$.*

Proof. Consider First Zagreb index of Vitamin A is

$$M_1(C_{20}H_{30}O) = \sum_{e=uv\in E(G)} (d_u + d_v)$$
$$=| m_{1,2} | (1+2)+ | m_{1,3} | (1+3)+ | m_{1,4} | (1+4)+ | m_{2,2} | (2+2)$$
$$+ | m_{2,3} | (2+3)+ | m_{2,4} | (2+4)+ | m_{3,3} | (3+3)+ | m_{3,4} | (3+4)$$
$$\therefore M_1(C_{20}H_{30}O) = 98.$$

□

Theorem 2.8. *The Second Zagreb index of Vitamin A is $M_2(C_{20}H_{30}O) = 108$.*

Proof. The Second Zagreb index of Vitamin A is

$$M_2(C_{20}H_{30}O) = \sum_{e=uv\in E(G)} d_u d_v$$

$$=| m_{1,2} | (1 \cdot 2)+ | m_{1,3} | (1 \cdot 3)+ | m_{1,4} | (1 \cdot 4)+ | m_{2,2} | (2 \cdot 2)$$
$$+ | m_{2,3} | (2 \cdot 3)+ | m_{2,4} | (2 \cdot 4)+ | m_{3,3} | (3 \cdot 3)+ | m_{3,4} | (3 \cdot 4)$$
$$\therefore M_2(C_{20}H_{30}O) = 108.$$

□

Theorem 2.9. *The First multiple Zagreb index of Vitamin A is* $PM_1(C_{20}H_{30}O) = 774144 \times 10^{13}$.

Proof. The First multiple Zagreb index of Vitamin A is

$$PM_1(C_{20}H_{30}O) = \prod_{e=uv\in E(G)} (d_u + d_v)$$
$$= \prod_{e=uv\in m_{1,2}} (d_u + d_v) \prod_{e=uv\in m_{1,3}} (d_u + d_v) \prod_{e=uv\in m_{1,4}} (d_u + d_v) \prod_{e=uv\in m_{2,2}} (d_u + d_v)$$
$$\prod_{e=uv\in m_{2,3}} (d_u + d_v) \prod_{e=uv\in m_{2,4}} (d_u + d_v) \prod_{e=uv\in m_{3,3}} (d_u + d_v) \prod_{e=uv\in m_{3,4}} (d_u + d_v)$$
$$= 3^1 \times 4^3 \times 5^2 \times 4^6 \times 5^6 \times 6^1 \times 6^1 \times 7^1$$
$$\therefore PM_1(C_{20}H_{30}O) = 774144 \times 10^{13}.$$

□

Theorem 2.10. *The Second multiple Zagreb index of Vitamin A is* $PM_2(C_{20}H_{30}O) = 1426576072 \times 10^{14}$.

Proof. The Second multiple Zagreb index of Vitamin A is

$$PM_2(C_{20}H_{30}O) = \prod_{e=uv\in E(G)} d_u d_v$$
$$= \prod_{e=uv\in m_{1,2}} (d_u d_v) \prod_{e=uv\in m_{1,3}} (d_u d_v) \prod_{e=uv\in m_{1,4}} (d_u d_v) \prod_{e=uv\in m_{2,2}} (d_u d_v)$$
$$\prod_{e=uv\in m_{2,3}} (d_u d_v) \prod_{e=uv\in m_{2,4}} (d_u d_v) \prod_{e=uv\in m_{3,3}} (d_u d_v) \prod_{e=uv\in m_{3,4}} (d_u d_v)$$
$$= 2^1 \times 3^3 \times 4^2 \times 4^6 \times 6^6 \times 8^1 \times 9^1 \times 12^1$$
$$PM_2(C_{20}H_{30}O) = 1426576072 \times 10^{14}.$$

□

Theorem 2.11. *The Augmented Zagreb index of Vitamin A is* $AZI(C_{20}H_{30}O) = 152 \cdot 0804$.

Proof. The augmented Zagreb index of Vitamin A is

$$AZI(C_{20}H_{30}O) = \sum_{e=uv\in E(G)} \left[\frac{d_u d_v}{d_u + d_v - 2}\right]^3$$
$$=| m_{1,2} | \left(\frac{1 \cdot 2}{1+2-2}\right)^3 + | m_{1,3} | \left(\frac{1 \cdot 3}{1+3-2}\right)^3 + | m_{1,4} | \left(\frac{1 \cdot 4}{1+4-2}\right)^3 + | m_{2,2} | \left(\frac{2 \cdot 2}{2+2-2}\right)^3$$
$$+ | m_{2,3} | \left(\frac{2 \cdot 3}{2+3-2}\right)^3 + | m_{2,4} | \left(\frac{2 \cdot 4}{2+4-2}\right)^3 + | m_{3,3} | \left(\frac{3 \cdot 3}{3+3-2}\right)^3 + | m_{3,4} | \left(\frac{3 \cdot 4}{3+4-2}\right)^3$$
$$\therefore AZI(C_{20}H_{30}O) = 152 \cdot 0804.$$

□

Theorem 2.12. *The harmonic index of Vitamin A is* $H(C_{20}H_{30}O) = 9 \cdot 3190$.

Proof. The harmonic index of Vitamin A is

$$H(C_{20}H_{30}O) = \sum_{e=uv\in E(G)} \frac{2}{d_u+d_v}$$
$$=| m_{1,2} | \left(\frac{2}{1+2}\right)+| m_{1,3} | \left(\frac{2}{1+3}\right)+| m_{1,4} | \left(\frac{2}{1+4}\right)+| m_{2,2} | \left(\frac{2}{2+2}\right)$$
$$+| m_{2,3} | \left(\frac{2}{2+3}\right)+| m_{2,4} | \left(\frac{2}{2+4}\right)+| m_{3,3} | \left(\frac{2}{3+3}\right)+| m_{3,4} | \left(\frac{2}{3+4}\right)$$
$$\therefore H(C_{20}H_{30}O) = 9\cdot 3190.$$

□

Theorem 2.13. *The hyper Zagreb index of Vitamin A is $HM(C_{20}H_{30}O) = 474$.*

Proof. The hyper Zagreb index of Vitamin A is

$$HM(C_{20}H_{30}O) = \sum_{e=uv\in E(G)} (d_u+d_v)^2$$
$$=| m_{1,2} | (1+2)^2+| m_{1,3} | (1+3)^2+| m_{1,4} | (1+4)^2+| m_{2,2} | (2+2)^2$$
$$+| m_{2,3} | (2+3)^2+| m_{2,4} | (2+4)^2+| m_{3,3} | (3+3)^2+| m_{3,4} | (3+4)^2$$
$$\therefore H(C_{20}H_{30}O) = 230.$$

□

3. Conclusion

ABC index, ABC_4 index, Randić connectivity index, Sum connectivity index, GA index and GA_5 index, First Zagreb index, Second Zagreb index, First Multiple Zagreb index, Second Multiple Zagreb index, Augmented Zagreb index, Harmonic index and Hyper Zagreb index of Vitamin A was computed.

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