Int. J. Math. And Appl., 9(2)(2021), 253–257
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

Equable Right Angle Triangle, Special Number for a Right Angle Triangle

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Abstract: In this paper I will prove conditions for a right angle triangle to be Equable right angle triangle, I shall establish some properties with proofs, extending the idea for semi equable triangles.

Keywords: Equable Right Angle Triangle, Special Number, Special Angle, Hypotenuse.

1. Introduction

A Right Angle Triangle [1] is a triangle in which one angle is a right angle (i.e, 90-degree angle). The side opposite the right angle is called the hypotenuse (longest side). Equable right angle [2] triangles are special type of right angle triangle, in this paper I will show a relation between the sides of equable right angle triangle and that every equable right angle triangle is unique and it arises from a general idea of semi equable triangle whose definition will be discussed. The unique aspect of this article is every property of equable right angle triangles is expressed by the special angle. I will also discuss the existence of threshold hypotenuse for an equable right angle triangle.

2. Preliminaries

In this section I will introduce some definitions which will serve as core of this article.

- Equable Right Angle Triangle: Equable Right Angle Triangles are right angle triangles whose area is numerically equal to their perimeter.
- Special Angle: It is an angle between the hypotenuse of right angle triangle and the base of right angle triangle for which the right angle triangle becomes equable right angle triangle.
- Let $K = \sin x + \cos x + 1$, this term will be ubiquitous in equable geometry, lets us call this Special number.
- Threshold hypotenuse: It is the minimum value of the hypotenuse below which it is impossible to construct equable right angle triangle.
- Semi Equable Triangle: It is a triangle whose at least one of the angles is special and it is not a right angle triangle.

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3. Main Results

Theorem 3.1. *Every equable right angle triangle obeys a relation given below* $4 = a + b - c$

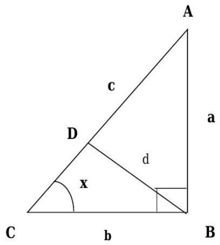

Figure 1.

Where a, b, c *are the sides of equable right angle as shown with* $AB = a$, $BC = b$, $CA = c$, $DB = d$, $\angle ACB = x$, $\angle ABC = 90°$, $\angle ADB = 90°$.

Proof. Consider an equable right angle triangle ABC as shown above (Figure 1), Using the definition of equable right angle triangle

$$\text{Area of triangle } ABC = \text{Perimeter of triangle ABC}$$

1.

$$\frac{1}{2}ab = a + b + c \tag{1}$$

$$\frac{1}{2}c^2 \sin x \cos x = c(\sin x + \cos x + 1)$$

$$\frac{1}{2}c \sin x \cos x = K$$

$$\text{where } K = \sin x + \cos x + 1, \forall \;\; x \in \left(0, \frac{\pi}{2}\right) \tag{2}$$

2.

$$C = \frac{2K}{\sin x \cos x} \tag{3}$$

Consider $K = \sin x + \cos x + 1 \Rightarrow K - 1 = \sin x + \cos x$. Squaring on both sides

$$K^2 + 1 - 2K = 1 + 2\sin x \cos x \tag{4}$$

3. $K^2 - 2K = 2\sin x \cos x$. Using (3) in (4), we get

$$\begin{aligned} c &= \frac{4K}{K^2 - 2K} \\ c &= \frac{4}{K - 2} \\ c &= \frac{4}{\sin x + \cos x - 1} \\ c &= \frac{4}{\frac{a}{c} + \frac{b}{c} - 1} \\ c &= \frac{4c}{a + b - c} \end{aligned} \tag{5}$$

$$a + b - c = 4 \;\; \text{(this relation looks like Euler characteristics } 2 = V + F - E)$$

□

Result 3.2. *Area of equable right angle triangle* $A = \dfrac{4K}{K-2}$.

Proof. Area of right angle triangle $= \dfrac{1}{2}ab$. $A = \frac{1}{2}c^2 \sin x \cos x$ using trigonometry. But $\sin x \cos x = \dfrac{2K}{c}$ using (3) $\Rightarrow A = cK$, but $c = \frac{4}{K-2}$ using (5) $\Rightarrow A = \frac{4K}{K-2}$. By definition of equable right angle triangle, perimeter $P = \frac{4K}{K-2}$. □

Result 3.3. *The altitude of equable right angle triangle* $DB = d = 2K$.

Proof. The altitude

$$DB = d = \frac{ab}{c} \tag{6}$$

$$d = c \sin x \cos x \quad \text{using trigonometry}$$

$$d = \frac{2ck}{c} \quad \text{using equation (3)}$$

$$d = 2K$$

□

Theorem 3.4. *If equable right angle triangles are similar then they are congruent (Uniqueness).*

Proof. Consider two equable right angle triangle ABC and PQR which are similar whose special number and areas are given by K_1, K_2, A_1, A_2 respectively. Assume two equable right triangles ABC and PQR are similar but not congruent.

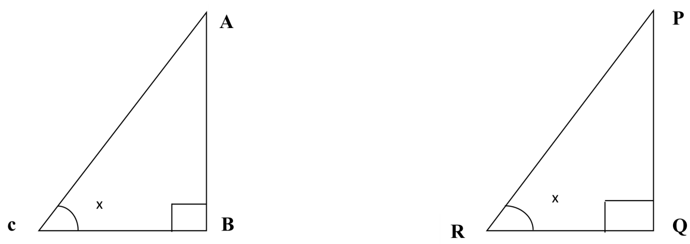

Figure 2.

Since two triangles are similar the angle $ACB = x$ and angle $PRQ = x$ are equal. Special number for triangle ABC and Special number for triangle $PQRK_1 = K_2 = K$ because K_1 and K_2 are function of angle x only. So area of triangle ABC is given by

$$A_1 = \frac{4K}{K-2} \tag{7}$$

Also for triangle, PQR the area is given by

$$A_2 = \frac{4K}{K-2} \tag{8}$$

A contradiction so therefore if equable right angle triangles are similar then they are congruent. □

Theorem 3.5 (Existence of threshold hypotenuse)**.** *There exist a minimum value of hypotenuse below which its impossible to construct equable right angle triangle.*

Proof. Using the identity

$$4 = a + b - c \tag{9}$$

Squaring on both sides and using Pythagoras theorem

$$16 + c^2 + 8c = a^2 + b^2 + 2ab \Rightarrow 16 + 8c = 2ab \tag{10}$$

Substituting for b from (9)

$$8 + 4c = a(4 + c - a) \tag{11}$$

$$8 + 4c = 4a + ac - a^2 \tag{12}$$

Equation (11) is quadratic in a, solving for a using quadratic formula we have

$$\begin{aligned} a &= \frac{4 + c \pm \sqrt{c^2 - 8c - 16}}{2} \\ a &= \frac{4 + c + \sqrt{c^2 - 8c - 16}}{2} \\ b &= \frac{4 + c - \sqrt{c^2 - 8c - 16}}{2} \end{aligned} \tag{13}$$

Since we had two roots one of them is a and the other is b. Setting the radical to zero and solving for c (Because if the terms inside the radical is negative we get complex number for the sides of the triangle which is not allowed)

$$0 = \frac{\sqrt{c^2 - 8c - 16}}{1} \tag{14}$$

$$c = \frac{8 + \sqrt{64 + 64}}{2} \quad \text{using quadratic formula} \tag{15}$$

$$C = 9.6568542494923801952067548968387923142785 \quad \text{Units} \tag{16}$$

This is the required threshold hypotenuse. $\square$

Remark 3.6. *General Triangle with one of the angles being special.*

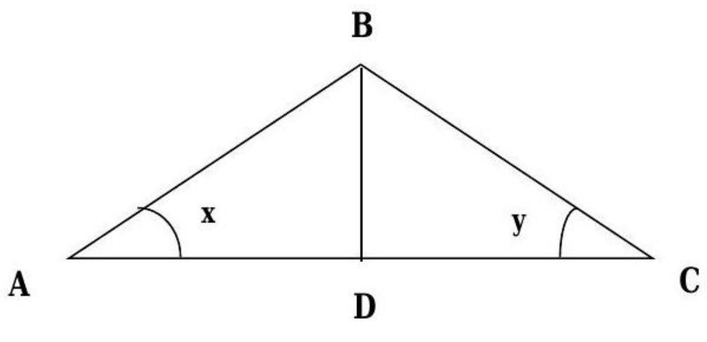

Figure 3.

ABC is a triangle with ABD being equable right angle triangle, angle $BAC = x$. Angle $BCA = y$, since ABD is equable right triangle

$$\begin{aligned} AB &= \frac{4}{k-2} \\ AD &= \frac{4}{k-2}\cos x \\ BD &= \frac{4}{k-2}\sin x \end{aligned}$$

$$BC = \frac{4}{k-2}\frac{\sin x}{k^2 \sin y}$$
$$DC = \frac{4}{k-2}\frac{\sin x}{\sin y}\cos y$$
$$AC = AD + DC = \frac{4}{k-2}\cos x + \frac{4}{k-2}\frac{\sin x}{\sin y}\cos y$$

These type of triangles can be called semi equable triangles because every property of these triangles is function of angles x and y and when $y = 90$ degrees the semi equable triangle becomes equable right angle triangle.

4. Conclusion

It is easier to construct equable right angle triangles using above formulae because the only variable is length of the hypotenuse or special angle. It is clear that there many are many geometric figures like rectangles, triangles, whose properties solely depend on special angle as they are composed of equable right angle triangle and many semi equable triangles whose properties depend on only angles. There is so much of it that it can be considered a separate branch of geometry called Geometry of figures composed of right equable triangle.

References

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