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Study of Viscous Fluid in the Presence of Transverse Magnetic Field via Spherical and Cylindrical Structure

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Abstract

Many investigators have discussed the study of the MHD flow of viscous fluid through different structures. In this analysis we study, the concept of streamlined flow in the presence of a transverse magnetic field has been introduced. The main objective of the paper is to discuss the viscous fluid, which is past a sphere or circular cylinder in the presence of a clear and porous zone. The Brinkman model is passed down for the discussion of viscous fluid flow in porous media. The matching condition for the flow of viscous fluid is suggested by Williams. It has been chosen for both structures such as sphere and circular cylinder. The solutions to the problems have been procured analytically. MATLAB software is used for the graphical study. The graphs of streamlined flow in spherical as well as circular cylinders have been studied. It is observed that with the increase of the Magnetic field on the inside of the cylinder the drag increases sharply similar effect is also observed in a circular cylinder. It is also observed that the increase of Magnetic field stress increases sharply. A result of this study is compared with the result procured by putting in the Ochoa Tapia and Whitaker matching conditions at the interface.

Keywords: MHD Flow; Viscous Fluid; Viscous Flow; Drag Force; Circular Cylinder.
2020 Mathematics Subject Classification: 76A10, 76D09.

1. Introduction

The study of uniform shear flow past embedding body in Stokes flow is of particular interest in many industrial problems, particularly in the chemical industry, lubrication of bearing and hydrology. The mathematical theory of the flow of viscous fluids through a porous medium was established by

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Darcy [11] given the following law:

$$\nabla p = -\frac{\mu}{k} V, \tag{1}$$

Where V is the velocity, μ is the viscosity of the fluid, k is the permeability of the porous medium, and p is the pressure. Darcy's law holds good for the flow of low Reynolds Number and when the diameters of the particles forming the porous medium are vanishingly small. A calculation of viscous force exerted by a flowing fluid in a dense swarm of particles has been studied by Brinkman [8]. He has also studied in another paper on the permeability of media consisting of closed packed porous particles. He has suggested this law by proposing the following equation:

$$\nabla p = -\frac{\mu}{k} V + \mu \nabla^2 V, \tag{2}$$

In the presence of Magnetic field the above equation can be changed in the form of

$$\nabla p = -\frac{\mu}{k} V + \mu \nabla^2 V + \frac{\sigma_e B^2}{\rho} V, \tag{3}$$

Where B is the magnetic field, ρ is the density of the fluids and σ is the electrical conductivity of the fluids. Kaplun has studied low Reynolds number flow past a circular cylinder. Scheidegger has presented the physics of flow through porous media. Joseph and Tao [16] have used the Darcy model [11] to study the flow past a porous sphere, which title is the effect of permeability of slow motion of a porous sphere in a viscous liquid. Singh and Gupta [39] have studied the flow of a viscous fluid past an inhomogeneous porous cylinder. Rudraiah et al have presented Hartmann flow over a permeable bed. Verma and Bhatt [51] has been studied Low Reynolds Number flow past a heterogeneous porous sphere using an asymptotic technique. When a viscous fluid flows over a permeable surface, the no-slip boundary conditions are no longer true on the surface. The velocity inside the permeable material will be different from the true velocity and one must match the two velocities at the surface by some suitable conditions. Williams [53] has analyzed constitutive equations for the flow of an incompressible viscous fluid through a porous medium. He has suggested the following matching conditions at the interface:

$$V_{outside} = \varnothing V_{inside}, \tag{4}$$

$$\frac{\partial V_{outside}}{\partial \eta} = \lambda \frac{\partial V_{inside}}{\partial \eta}. \tag{5}$$

Gupta [14] studied the flow of fluid motion past a porous circular cylinder with an initial pressure gradient. Higdon [15] have presented the calculation of Stokes flow past porous particles. Raptis [26] have studied magneto hydrodynamics Effects on Mass Transfer Flow through a Porous medium. Yu and Kaloni [60] have studied the Cartesian tensor solution of the Brinkman equation. Chandna and Labropulu [10] have analyzed Riabouchinsky flows in Magneto hydrodynamics. Padmavathi et al. [22] has studied Stokes flow past a porous sphere using Brinkman's model. Bhatt and Sacheti [7] studied flow past a porous spherical shell using the Brinkman Model. Ochoa Tapia and Whitaker [21] have studied momentum transfer at the boundary between a porous medium and a homogeneous fluid. Part I: Theoretical development. Shi and Braden [37] have studied the effect of permeability on low Reynolds number flow past a circular porous cylinder.

Srivastava [43] has studied the flow through porous media. Keh [20] have studied the creeping motion of a composite sphere in a spherical cavity. Rudraiah et. al [28] has been studied flow past an impervious sphere embedded in a porous medium based on the non-Darcy model. Srivastava [44] have studied flow past a porous sphere at a small Reynolds number. Kamel and Hamdan [19] have studied Riabouchinsky Flow through Porous media. Rudraiah and Kalal [29] have analyzed the Electrohydrodynamic surface Instabilities: Role of porous Lining at the ablative surface of laser-driven inertial fusion energy target. Srinivasacharya [38] has studied flow past a porous approximate spherical shell. Shafahi and Vafai [33] have presented biofilm-affected characteristics of porous structures. Seth et al. [34] analyzed unsteady MHD convective flow within a parallel plate rotating channel with a thermal source/sink in a porous medium under slip boundary conditions.

Taamneh and Bataineh [46] have studied drag and separation flow past a solid sphere with a porous shell at a moderate Reynolds Number. Verma and Datta [52] have studied the analytical solution of slow flow past a heterogeneous porous sphere with the radial variation of permeability using the Brinkman model. Saad et al. [30] has been studied the slow motion of a porous eccentric spherical particle in cell models. Saxena and Kumar [31] have studied the flow of a viscous fluid through different porous structures embedded in the porous medium. Seth et al. [35] has been studied the unsteady Hydromagnetic natural convection flow of a heat-absorbing fluid within a rotating vertical channel in a porous medium with Hall effects. Shukla and Das [45] have studied the effect of a uniform magnetic field on the motion of a porous sphere in a spherical container. Yadav et al. [54] has been studied the numerical analysis of the MHD flow of viscous fluid between parallel porous bounding walls. Agarwal et al. [1] has been analyzed the investigation of magneto-hydrodynamic flow in a channel with a porous bounding wall. Yadav and Joseph [55] have studied the numerical investigation of Heat transfer along symmetrical porous wedges and the effects of radiation in the presence of variable fluid viscosity on flow. Yadav and Singh [56] have studied the Analytical investigation of Thermal radiation effects of the laminar flow of fluid and heat transfer in a channel with two porous bounding walls. In another paper, Yadav [57] presented the analytical study of the magneto-hydrodynamic flow of incompressible viscous fluid between the parallel porous and rigid bounding wall.

Seth et al. [36] has been studied the Hydromagnetic thin film flow of Casson fluid in a non-Darcy porous medium with Joule dissipation and Navier Partial slip. Singh and Yadav [40] have presented the investigation of heat transfer of non-Newtonian fluid in the presence of a porous wall. Agarwal et al. [2] has been studied the applications of the Optimal Homotopy Asymptotic Method for solving non-linear boundary layer problems. Agarwal et al. [3] also has been studied the numerical investigation of a channel flow with third-grade fluid in the presence of the magnetic field. Agarwal et al. [4] has been studied the solution of the magneto-hydrodynamic flow of fluid and heat transfer problem via the Optimal Homotopy Asymptotic Method. In another paper, Agarwal et al. [5] also studied the numerical investigation of heat transfer of non-Newtonian fluid with variable viscosity along a symmetrical porous wedge. Dixit et al. [12] has been studied the numerical investigation of magneto-hydrodynamic flow with variable fluid viscosity and heat transfer in presence of a symmetrical porous wedge. Yadav et al. [58] has been studied the numerical analysis of the magneto-hydrodynamic flow of non-Newtonian fluid past over a sharp wedge in presence of a thermal boundary layer.

Jabeen et al. [17] analyzed a comparative study of MHD flow analysis in a porous medium by using the differential transformation method and variation iteration method. Pandya et al. [24] has been studied the Mangento-hydrodynamic flow of a viscous fluid in a channel with a porous boundary wall of different Permeabilities. Singh [41] has studied the analytical investigation of Couette flow in a composite porous cylindrical channel of variable permeability. Singh and Verma [42] have studied slow flow past a porous shell of variable permeability with a cavity at the center. Turkyilmazoglu [47] has studied MHD natural convection in saturated porous media with heat generation/absorption and thermal radiation: closed-form solutions. Dixit et al. [13] has been studied the numerical investigation of the magneto-hydrodynamic flow of non-Newtonian fluid with a sharp porous wedge in presence of a thermal boundary layer. Pandey et al. [25] has been studied numerical analysis of variable fluid viscosity passing a symmetrical sharp wedge shape in the presence of a transverse magnetic field. Turkyilmazoglu [48] has studied velocity slip and entropy generation phenomena in thermal transport through metallic porous channels. Yadav et al. [59] has been studied the investigation of Torsional Oscillations and heat transfer of a second-grade fluid bounded by a porous medium in the presence of a heated disk. Turkyilmazoglu [49] has studied magneto-hydrodynamic moving liquid plug within a micro channel: Analytical solutions. Turkyilmazoglu [50] has studied the flow and heat over a rotating disk subject to a uniform horizontal magnetic field.

2. Mathematical Formulation of the Problem

Let us consider the flow of the viscous fluid in the uniform velocity U in a pore medium in which a space inside and outside the body is fully soaked with the viscous fluid. Assuming r^*, θ, and z be a set of polar coordinates system and their origin at the center of the body. Now the surface of the body is $r^* = a$. This problem is presented by dividing the flow area in two zones, Zone I and Zone II. The region inside pore is represented as Zone I and the region the outside it is represented Zone II. The schematic diagram is shown in Fig.1. For the case of a porous cylinder and the same figure can be utilized for understanding the case of a porous sphere, now we assumed the uniform velocity it is represented by U. Let the index i is the subscript of an entity χ_i, $i = 1, 2$ indicate the zone in which the entity is represented. Let u_i^*, v_i^*, w_i^* be the velocity components in the directions of r^*, θ, and z

respectively, in the ith zone, then under this notation, the boundary conditions of the problem for the case of porous sphere can be written as

$$u_2^* \to -U\cos\theta, \quad v_2^* \to U\sin\theta, \quad as \quad r^* \to \infty, \tag{6}$$

and the boundary conditions for the case of a porous circular cylinder can be written as

$$u_2^* \to U_\infty\cos\theta, \quad v_2^* \to U_\infty\sin\theta, \quad as\ r^* \to \infty, \tag{7}$$

where U_∞ is defined the velocity at infinity.

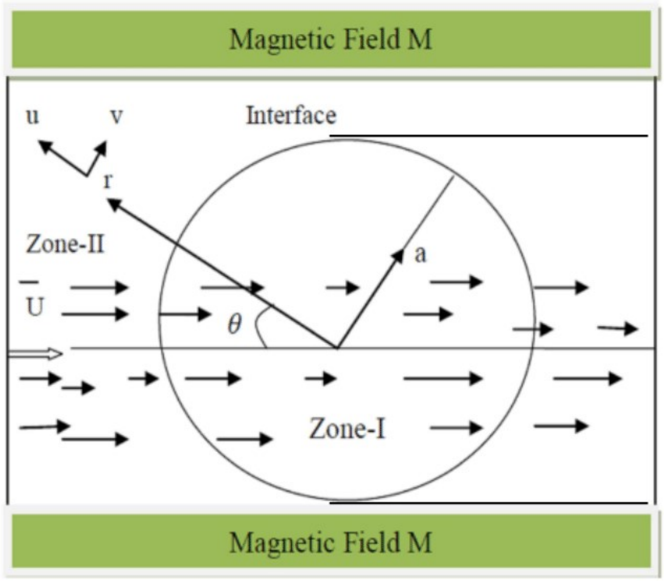

Figure 1: Schematic Diagram of the Problem

There is an axial symmetry in the problem; hence all quantities are independent of $\varnothing$ and $w_i^* = w_2^* = 0$. We take the following matching conditions for the flows in two zones at the surface of the sphere as suggested by Williams [53].

$$\varnothing_1 u_1 = \varnothing_2 u_2 \ \ \& \ \ \varnothing_1 v_1 = \varnothing_2 v_2; \quad at \quad r = 1, \tag{8}$$

$$\left.\begin{aligned} \varnothing_1 \tau^*_{r\theta(1)} &= \varnothing_2 \tau^*_{r\theta(2)} \\ \varnothing_1 \tau^*_{rr(1)} &= \varnothing_2 \tau^*_{rr(2)}; \end{aligned}\right) at \quad r = 1, \tag{9}$$

where $r = \frac{r^*}{a}, u_i^* = \frac{u_i}{U}, v_i^* = \frac{v_i}{U}, \tau^*_{r\theta(i)}$ and $\tau^*_{rr(i)}$ are shearing and normal stresses, respectively, in the ith zone for the case of sphere and U is replaced by U_∞ for the case of a cylinder.

Dimensionless equations of motion (for a spherical system) in the direction of r, θ based on the Brinkman model [6].

$$\frac{\partial P_i}{\partial r} = \frac{\partial^2 u_i}{\partial r^2} + \frac{2}{r}\frac{\partial u_i}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_i}{\partial \theta^2} + \frac{1}{r^2}\cot\theta\ \frac{\partial u_i}{\partial \theta} - \frac{2}{r^2}\ u_i - \frac{2}{r^2}\frac{\partial v_i}{\partial \theta} - \frac{2}{r^2}\ v_i\cot\theta\ - M^2 u_i\ , \tag{10}$$

$$\frac{1}{r}\frac{\partial P_i}{\partial \theta} = \frac{\partial^2 v_i}{\partial r^2} + \frac{2}{r}\frac{\partial v_i}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v_i}{\partial \theta^2} + \frac{1}{r^2}\cot\theta\ \frac{\partial v_i}{\partial \theta} - \frac{1}{r^2}\ v_i cosec^2\theta - \frac{2}{r^2}\frac{\partial u_i}{\partial \theta} - M^2 v_i\ , \tag{11}$$

For the case of a porous circular cylinder, studied by Pop and Cheng [23],

$$\frac{\partial P_i}{\partial r} = \frac{\partial^2 u_i}{\partial r^2} + \frac{1}{r}\frac{\partial u_i}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_i}{\partial \theta^2} - \frac{1}{r^2}\ u_i - \frac{2}{r^2}\frac{\partial v_i}{\partial \theta} - M^2 u_i\ , \tag{12}$$

$$\frac{1}{r}\frac{\partial P_i}{\partial \theta} = \frac{\partial^2 v_i}{\partial r^2} + \frac{2}{r}\frac{\partial v_i}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v_i}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial u_i}{\partial \theta} - \frac{1}{r^2}v_i - M^2 v_i, \tag{13}$$

where $M^2 = \frac{\sigma_e r^2 B^2}{\rho v}$, $P_i = \frac{a p_i}{\mu_i U}$, μ_i is the viscosity of the fluids, P_i is the dimensionless pressure in the fluid, p_i is the pressure in the fluid and k_i is the permeability of the porous medium and U is replaced by U_∞ in the case of a porous circular cylinder.

Stokes stream function ψ_i in spherical polar coordinates is given by

$$u_i = \frac{1}{r^2 \sin\theta}\frac{\partial \psi_i}{\partial \theta}, \qquad v_i = \frac{1}{r\sin\theta}\frac{\partial \psi_i}{\partial r} \tag{14}$$

Eliminating P_i from (10) and (11) by cross differentiation, we get the following differential equations for ψ_i:

$$E^2\left(E^2 - M^2\right)\psi_i = 0, \tag{15}$$

where

$$E^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\right). \tag{16}$$

Boundary conditions (6) in terms of ψ_i becomes

$$\psi_2\left(r,\theta\right) \sim \frac{1}{2} r^2 sin^2\theta \ \ as \ \ r \to \infty \tag{17}$$

and in the case of cylindrical polar coordinates the Stokes stream function is given by

$$u_i = -\frac{1}{r}\frac{\partial \psi_i}{\partial \theta}, \quad v_i = \frac{\partial \psi_i}{\partial r}, \quad i = 1, 2. \tag{18}$$

This form of Velocity components satisfies the equation of continuity. Eliminating P_i from (12) and (13) by cross differentiation, we get the following differential equation for ψ_i:

$$\nabla^2\left(\nabla^2 - M^2\right)\psi_i = 0, \quad i = 1, 2, \tag{19}$$

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}. \tag{20}$$

Boundary condition (7) in terms of ψ_i becomes

$$\psi_2 \sim r\sin\theta \quad as \ \ r \to \infty. \tag{21}$$

3. Solution of the Problem

The solution of the problem has been classified in two cases which are given. First is the Solution of the problem in Spherical coordinate system and second is the solution of cylindrical coordinate system.

3.1 Solution of the problem for spherical case

Boundary condition (17) suggests the following form for the stream function ψ_i:

$$\psi_i = f_i\left(r\right)\sin^2\theta, \tag{22}$$

where $f_i\left(r\right)$ is function of distance, porosity and magnetic field, substituting the value of ψ_i in equation (14), we get the results

$$u_i = \frac{2}{r^2} f_i\left(r\right)\cos\theta, \qquad v_i = \frac{1}{r} f_i'\left(r\right)\sin\theta, \tag{23}$$

where a prime denotes differentiation with respect to r. Substituting (22) in (14), we get the following differential equation for $f_i(r)$:

$$\left(\frac{d^2}{dr^2} - \frac{2}{r^2}\right)\left(\frac{d^2}{dr^2} - \frac{2}{r^2} - M^2\right) f_i\left(r\right) = 0. \tag{24}$$

Equation (24) is a fourth-order differential equation and its solution is given by

$$f_i\left(r\right) = Ar^2 + \frac{C}{r} + B\left[\frac{\sinh\left(rM\right)}{r} - M\cosh\left(rM\right)\right] + N\left[M\sinh\left(rM\right) - \frac{\cosh\left(rM\right)}{r}\right], \tag{25}$$

Where A, C, B, N are fourth constants to be determined from the fourth matching conditions (8) – (9). This solution gives the flow in both the zones with proper choice of constants.
For the flow in the zone I where $r < 1$ and origin occurs, the constants C and N are zero and the expression for $f_1(r)$ can be written as

$$f_1\left(r\right) = Ar^2 + B\left[\frac{\sinh\left(rM\right)}{r} - M\cosh\left(rM\right)\right], \tag{26}$$

For the flow in zone II where $r > 1$ and $f_2\left(r\right) \to \frac{1}{2r^2} \;\; as \;\; r \to \infty$, the constants $A = \frac{1}{2}$ and $B = 0$, so the expression for $f_2(r)$ is given by

$$f_2\left(r\right) = \frac{1}{2}r^2 + \frac{C}{r} + N\left[M\sinh\left(rM\right) - \frac{\cosh\left(rM\right)}{r}\right], \tag{27}$$

Integrating equations (10) and (11) and substituting $u_i, \;\; v_i$ from (23), we get the following expression for P_i:

$$-P_i = \left[f_i'' + 4f_i - \left(M^2 + 2\right) f_i'\right]\cos\theta. \tag{28}$$

The pressure P_i is taken to be zero. The expression for $\tau_{r\theta(i)}, \;\; \tau_{rr(i)}$ in spherical polar coordinates can be written as

$$\tau_{r\theta(i)} = \mu_i \left(\frac{1}{r^*} \frac{\partial u_i^*}{\partial \theta^*} - \frac{v^*}{r^*} + \frac{\partial v_i^*}{\partial r^*} \right), \tag{29}$$

$$\tau_{rr(i)} = -P_i + 2\mu_i \frac{\partial u_i^*}{\partial r^*} \tag{30}$$

Substituting u_i^*, v_i^*, P_i, we get the following expressions to $\tau_{r\theta(i)}, \tau_{rr(i)}$:

$$\tau_{r\theta(i)} = -\mu_i \frac{U}{a} \left(\frac{f_i''}{r} - \frac{2f_i'}{r} + \frac{2f_i}{r^3} \right) \sin\theta, \tag{31}$$

$$\tau_{rr(i)} = -\mu_i \frac{U}{a} \left[f_i''' - \frac{6f_i'}{r^2} + \frac{12f_i}{r^3} - M^2 f_i' \right] \cos\theta. \tag{32}$$

Four conditions (8) – (9) in terms of $f_i(r)$ become

$$f_2(1) = \emptyset f_1(1), \tag{33}$$

$$f_2'(1) = \emptyset f_1'(1), \tag{34}$$

$$f_2''(1) - 2f_2'(1) + 2f_2(1) = \lambda\emptyset \left[f_1''(1) + 2f_1'(1) + 2f_1(1) \right], \tag{35}$$

$$f_2'''(1) - 12f_2(1) - \left(6 + M^2\right) f_2'(1) = \lambda\emptyset \times \left[f_1'''(1) - 12f_1(1) - \left(6 + M^2\right) f_1'(1) \right], \tag{36}$$

where $\emptyset = \frac{\emptyset_1}{\emptyset_2}$ and $\lambda = \frac{\mu_1}{\mu_2}$. Substituting the values of $f_1(r)$ and $f_2(r)$ from (26) and (27) in (33) – (36), we get the following four equations to obtain constants A, B, C, and N:

$$\frac{1}{2} + C + N\left[M\sinh M - \cosh M\right] = \phi\left[A + B \times \{\sinh M - M\cosh M\}\right], \tag{37}$$

$$1 - C - N\left[M\sinh M - \left(M^2 + 1\right)\cosh M\right] = \emptyset\left[2A - B \times \left\{\left(M^2 + 1\right)\sinh M - M\cosh M\right\}\right], \tag{38}$$

$$6C + N\left[M\left(M^2 + 6\right)\sinh M - 3\left(M^2 + 2\right)\cosh M\right] = \lambda\emptyset B\left[\left(3M^2 + 6\right)\sinh M - M\left(M^2 + 6\right)\cosh M\right], \tag{39}$$

$$12 - M^2(1 - C) + 4N\left[3M\sinh M - \left(M^2 + 3\right)\cosh M\right] = \lambda\emptyset\left[-2M^2 A + 4B\left\{\left(M^2 + 3\right)\sinh M - 3M\cosh M\right\}\right]. \tag{40}$$

The case when the Sphere is an impervious solid, $u_2 = 0$, $v_2 = 0$ on the surface of the sphere $r = 1$ and the values of C and N is given by equating the left-hand side of equations (37) and (40) to be zero, the values of C and N are given by

$$C = \frac{3}{2M}\tanh M - \left(\frac{1}{2} + \frac{3}{2M^2}\right); N = -\frac{3}{2\,M^2\cosh M}, \tag{41}$$

3.2 Solution of the problem for cylindrical case

Boundary condition (21) on ψ_2 suggests the following form for the stream functions ψ_i:

$$\psi_i = f_i(r)\sin\theta. \tag{42}$$

Substituting the above expression of ψ_i in (18), we get the following form for velocity components:

$$u_i = -\frac{1}{r} f_i(r)\cos\theta, \quad v_i = f_i'(r)\sin\theta, \tag{43}$$

Where a prime denotes differentiation with respect to r. Integrating equations (12) and (13) and substituting u_i and v_i from equation (43), we get the following expression for the dimensionless pressure P_i:

$$P_i = -\left[f_i''' + \frac{f_i''}{r} + \frac{f_i}{r^3} - \left(\frac{2}{r^2} + M^2\right) f_i'\right] r\cos\theta, \tag{44}$$

The constant in the above expression is taken to be zero. The expression for $\tau_{r\theta(i)}$ and $\tau_{rr(i)}$ in polar coordinate system is

$$\tau_{r\theta(i)} = \left(\frac{1}{r}\frac{\partial u_i}{\partial\theta} + \frac{\partial v_i}{\partial r} - \frac{v_i}{r}\right), \tag{45}$$

$$\tau_{rr(i)} = -P_i + 2\mu_i \frac{\partial u_i}{\partial r}, \tag{46}$$

where $\tau_{r\theta(i)} = \frac{a}{\mu_i U_\infty} \tau_{r\theta}^*$, $\tau_{rr(i)} = \frac{a}{\mu_i U_\infty} \tau_{rr(i)}^*$.

Substituting the expression u_i, v_i, and P_i from (43) and (44) in the above expression, we get

$$\tau_{r\theta(i)} = \left[f_i'' - \frac{1}{r} f_i' + \frac{f_i}{r^2}\right]\sin\theta, \tag{47}$$

$$\tau_{rr(i)} = \left[r\, f_i''' + f_i'' + \frac{3f_i}{r^2} - r\left(\frac{4}{r^2} + M^2\right) f_i'\right]\cos\theta, \tag{48}$$

Now, the four matching conditions (8) – (9) in terms of $f_i(r)$ can be written as

$$f_2(1) = \varnothing f_1(1), \tag{49}$$

$$f_2'(1) = \varnothing f_2'(1), \tag{50}$$

$$f_2''(1) - f_2'(1) + f_2(1) = \lambda\varnothing\left[f_1''(1) - f_1'(1) + f_1(1)\right], \tag{51}$$

$$f_2'''(1) + f_2''(1) + 3f_2(1) - \left(4 + M^2\right) f_2'(1) = \lambda\varnothing \times \left[f_1'''(1) - f_1''(1) + 3f_1(1) - \left(4 + M^2\right) f_1'(1)\right], \tag{52}$$

Where $\varnothing = \frac{\varnothing_1}{\varnothing_2}$ and $\lambda = \frac{\mu_1}{\mu_2}$.

Substituting (42) in (19), we get the following differential equation for $f_i(r)$:

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2}\right)\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2} - M^2\right) f_i = 0, \tag{53}$$

Equation (53) is the differential equation of fourth order and its solution is given by

$$f_i(r) = Ar + \frac{C}{r} + BI_1(rM) + NK_1(rM), \tag{54}$$

Where $I_1(rM)$ and $K_1(rM)$ are modified Bessel's functions of order one of the first and second kind, respectively, and A, B, C, N are integration constants. This solution gives the flow in both the zones with proper choice of constants. For the flow in zone I where $r < 1$ and, origin occurs the constants C and N are zero, and the expression for $f_1(r)$ can be written as

$$f_1(r) = r + BI_1(rM). \tag{55}$$

For the flow in the zone II where $r > 1$ and $f_2(r) \to r$ as $r \to \infty$, the constants $A = 1$, $B = 0$ so the expression $f_2(r)$ is given by

$$f_2(r) = r + \frac{C}{r} + NK_1(rM). \tag{56}$$

Substituting the values of $f_1(r)$ and $f_2(r)$ from (55) and (56) in (49) – (52). We get the following flour equations for calculating the four constants A, B, C, and N:

$$1 + C + NK_1(rM) = \varnothing [A + BI_1(rM)], \tag{57}$$

$$1 - C - N[K_1 M + MK_0 M] = \varnothing \times [A + B \times \{MI_0 M - I_1 M\}], \tag{58}$$

$$4C + N\left[\left(M^2 + 4\right)K_1(rM) + 2MK_0 M\right] = \lambda\varphi^2 B \times \left[\left(M^2 + 4\right)I_1 - 2I_0\right], \tag{59}$$

$$-\left(M^2 + 1\right) + \left(M^2 + 3\right)C = -\left(M^2 + 1\right)A + B \times [3I_1 M - 2MI_0 M], \tag{60}$$

where $I_0 M$ and $K_0 M$ are modified Bessel functions of order zero of the first and second kind, respectively? The case when the cylinder is as impervious solid, $u_2 = 0$, $v_2 = 0$ on the surface of cylinder and values of C and N are given by equating left sides of equations (57) and (58) to zero, calculation gives the following expression for C and N:

$$C = -\frac{MK_0 M + 2K_1 M}{MK_0 M}, \tag{61}$$

$$N = -\frac{2}{MK_0 M}. \tag{62}$$

This case has been discussed by Pop and Cheng [23], and their results are identical with those obtained above.

Now we will discuss the motion with another matching condition and the benefits of the approach. In place of the Williams conditions, we use matching conditions at the interface suggested by Ochoa Tapia and Whitaker [21], according to which the velocities and normal stresses are equal at the interface but there is a jump in the shearing stress proportional to the corresponding velocity. Then, using expressions of velocity and stresses from (23), (26), (27), (31) and (32) we get the following equations for evaluating the constants for the case of a porous sphere:

$$\frac{1}{2} + C + N[M\sinh M - \cosh M] = [A + B\{\sinh M - M\cosh M\}], \tag{63}$$

$$1 - C - N \times \left[M\sinh M - \left(M^2 + 1\right)\cosh M\right] = \left[2A - B \times \left\{\left(M^2 + 1\right)\sinh M - M\cosh M\right\}\right] \tag{64}$$

$$6C + N\left[M\left(M^2+6\right)\sinh M - 3\left(M^2+2\right)\cosh M\right] = B\left[\left(3M^2+6\right)\sinh M - M\left(M^2+6\right)\cosh M\right], \quad (65)$$

$$\begin{aligned}\gamma^2\left[12 - M^2(1-C) + 4N\left\{3M\sinh M\right.\right. & \left.\left. - \left(M^2+3\right)\cosh M\right\}\right] \\ & - \left[-2M^2A + 4B\left\{\left(M^2+3\right)\sinh M - 3M\cosh M\right\}\right] \\ & = \beta D_a^{-1/2}\left[\frac{1}{2} + C + N\left\{M\sinh M - \cosh M\right\}\right],\end{aligned} \quad (66)$$

where $\gamma^2 = \frac{\mu_2}{\mu_1}$ and D_a is the Darcy number. For the case of a cylinder, we use the expressions of velocity and stresses from (43), (47) and (48), and we get the following set of equations:

$$1 + C + NK_1\, M = \left[A + BI_1M\right], \quad (67)$$

$$1 - C - N\left[K_1M + MK_0M\right] = \left[A + B\times\left\{MI_0M - I_1M\right\}\right], \quad (68)$$

$$4C + N\left[\left(M^2+4\right)K_1M + 2MK_0M\right] = B\left[\left(M^2+4\right)I_1M - 2MI_0M\right], \quad (69)$$

$$\gamma^2\left[-\left\{1 + M^2 + \left(3+M^2\right)C\right\}\right] + \left(1+M^2\right)A + B\left[3\,I_1M - 2MI_0M\right] = \beta D_a^{-1/2}\left[1 + C + EK_1M\right], \quad (70)$$

where K_0, K_1, I_0, I_1 are constants, Equations (63) – (66) and (67) – (70) are used to evaluate the values of constants. With the help of these constants the flow is determined completely.

4. Results and Discussions

The drag, D^* on the sphere will be extended by the fluid outside it, hence it is given by

$$D^* = 2\pi a^2\int_0^{\pi}\left[\tau_{rr(2)}\cos\theta - \tau_{r\theta(2)}\sin\theta\right]d\theta. \quad (71)$$

Substituting the values of $\tau_{rr(2)}$ and $\tau_{r\theta(2)}$ from (31) and (32), the expression for the dimensionless drag $D = \frac{D^*}{\pi\mu_2 Ua}$ is given by

$$D = -\frac{4}{3}\left[f_2'''(1) - 8f_2''(1) + 8f_2(1) - \left(M^2+2\right)f_2'(1)\right]. \quad (72)$$

Substituting the values of $f_2(r)$ from (27) in (72), we get

$$D = \frac{2}{3}\left[M^2(1-C) - 2N\left\{\left(M^4+4M^2\right)\cosh M - M^3\sinh M\right\}\right]. \quad (73)$$

Substituting the values of C and N from equations (41) in (73), the drag D_s on an impervious solid is given by

$$D_S = 3M(M-1)\tanh M + 9 + \frac{4}{M^2}. \quad (74)$$

Through Barman [6] has discussed this case, he has not calculated the drag on a solid sphere. Equations (74) show that when a solid sphere is embedded in a porous medium the drag on it is proportional to the velocity of the fluid and approximately proportional to the cube of the radius and inversely proportional to the permeability of the embedding medium.

Figure 2, represents the streamline flow velocity of fluid in Spherical coordinate system in Zone I, which is graph between θ against velocity of fluid and variation of Magnetic Parameter (M) has been shown. It is seen that increase of Magnetic Parameter (M), velocity of fluid decreases sharply in the range ($0 \leq \theta \leq \pi$ and $\pi \leq \theta \leq 2\pi$). It is like as sin wave nature. Figure 3 represents the streamline flow velocity of fluid in spherical coordinate coordinated system in Zone I, which is similar graph as Figure 2, here is small difference, and it is variation of radius of sphere. It is seen that increase of radius of sphere (r), velocity of fluid increases sharply in the range ($0 \leq \theta \leq \pi$ and $\pi \leq \theta \leq 2\pi$).

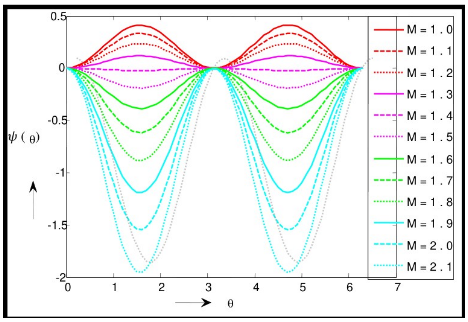

Figure 2: Graph is streamline flow of fluid in spherical coordinate system in Zone I, and variation of Magnetic field (M) at constant value of r = 0.8

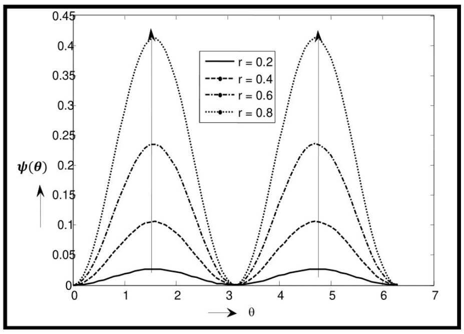

Figure 3: Graph is streamline flow of fluid in spherical coordinate system in Zone I, and variation of radius of sphere (r) at constant value of M = 1

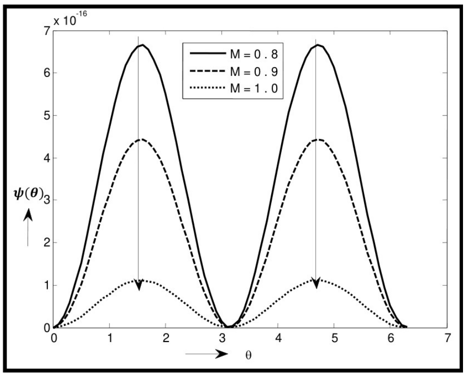

Figure 4: Graph is streamline flow of fluid in spherical coordinate system in Zone II, and variation of Magnetic field (M) at constant value of r = 1

Figures 4, 5 is represents the variation of Magnetic field in Zone II, it is seen that increases of Magnetic Field ($0 \leq M \leq 1$), Velocity of fluid decreases sharply. Figure 6 represents the velocity of fluid in Zone II, it is seen that increase of radius of sphere, velocity of fluid increases sharply. Figures 7 and 8 represents the variation of Magnetic parameter M and variation of radius of cylinder in clear zone (Zone I). it is seen that increase of magnetic parameter M, Velocity of fluid increases sharply in ($0 \leq \theta \leq \pi$) and velocity of fluid decreases sharply in ($\pi \leq \theta \leq 2\pi$). Whereas the reverse variation in the velocity of fluid has been seen with increase of radius of cylinder which is seen in figure 8. These figures represent the sin nature. Figures 9 and 10 represents the variation of Magnetic Parameter M and radius of cylinder in Zone II. It is seen that increase of magnetic parameter M, Velocity of fluid increases sharply in ($0 \leq \theta \leq \pi$) and velocity of fluid decreases sharply in ($\pi \leq \theta \leq 2\pi$). Similar effect has seen with increase of radius of cylinder, which is given in Figure 10. Now out interest is to observe the pattern of shearing stress. For this purpose the expression of shearing stress $\left[\tau_{r\theta(2)}\right]_{r=1}$ on the surface of the cylinder is given by substituting the expression for u_2 and v_2 in terms of $f_2(r)$ from (43) in (45) which can be written as

$$\left[\tau_{r\theta(2)}\right]_{r=1} = f_2''(1) - f_2'(1) + f_2(1)\sin\theta, \tag{75}$$

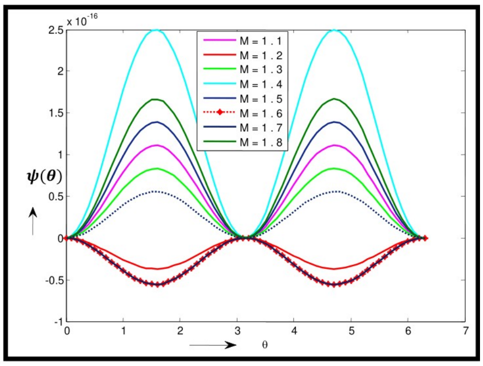

Figure 5: Graph is streamline flow of fluid in spherical coordinate system in Zone II, and variation of Magnetic field (M) at constant value of r = 1

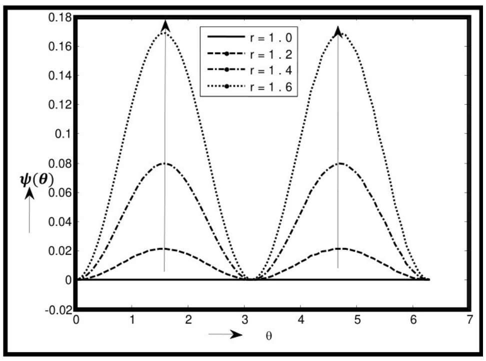

Figure 6: Graph is streamline flow of fluid in spherical coordinate system in Zone II, and variation of radius of sphere (r) at constant value of M = 0.5

Substituting $f_2(r)$ from equation (56) in the above expression, we get

$$\left[\tau_{r\theta(2)}\right]_{r=1} = \left[4C + N\left\{\left(M^2+4\right) K_1 M + 2MK_0 M\right\}\right] \sin\theta. \tag{76}$$

In the case when the cylinder is an impervious solid, we get the following value of shearing stress on it by substituting values of C and M from equations (61) and (62) into (76)

$$\left[\tau_{r\theta(2)}\right]_{r=1} = (1+2M) \sin\theta. \tag{77}$$

$$G = \frac{\left[\tau_{r\theta(2)}\right]_{r=1}}{\sin\theta}, \tag{78}$$

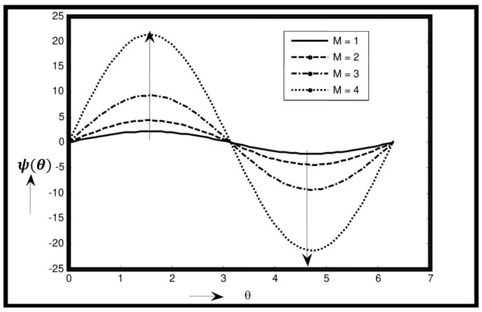

Figure 7: Graph is streamline flow of fluid in cylindrical coordinate system in Zone I, and variation of Magnetic field (M) at constant value of r = 0.8

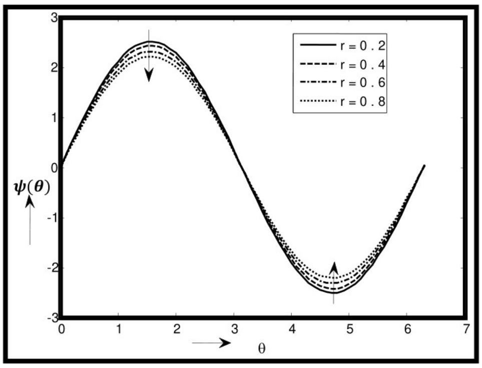

Figure 8: Graph is streamline flow of fluid in cylindrical coordinate system in Zone I, and variation of radius of sphere (r) at constant value of M = 1.0

However, the pattern of the graph changes slightly with Ochoa Tapia and Whitaker matching condition. In this case the effect of σ_1 is almost negligible on the shearing stress but it increases with the increase of σ_2. When the cylinder is an impervious solid, the value of G is given by $(1+2M)$. Now one of the useful features of flow, i.e., drag on the cylinder has been discussed for which the expression of drag on the cylinder is given by

$$D_r = \mu_2 U_\infty \int_0^\pi \left[\tau_{rr(2)} \cos\theta - \tau_{r\theta(2)} \sin\theta\right]_{r=1} d\theta. \tag{79}$$

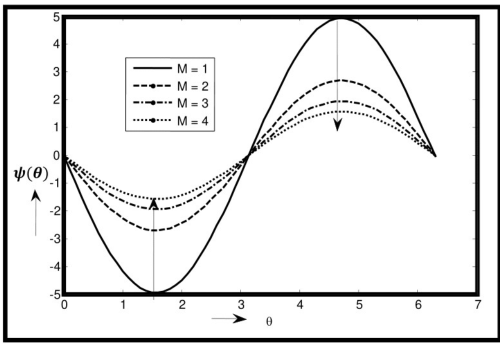

Figure 9: Graph is streamline flow of fluid in cylindrical coordinate system in Zone II, and variation of Magnetic field (M) at constant value of r = 0.8

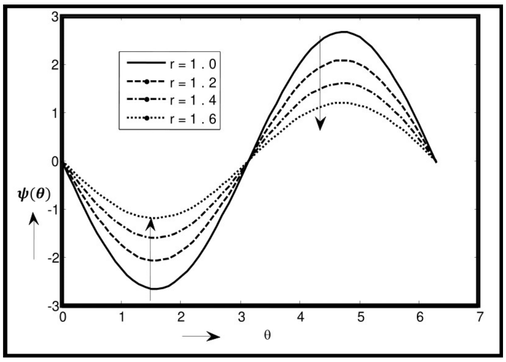

Figure 10: Graph is streamline flow of fluid in cylindrical coordinate system in Zone II, and variation of radius of sphere (r) at constant value of M = 1.5

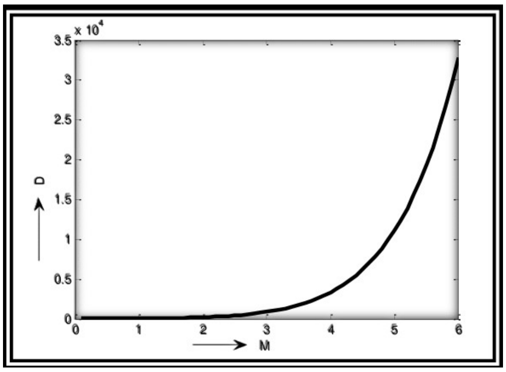

Figure 11: Graph between Drag D and Magnetic field M

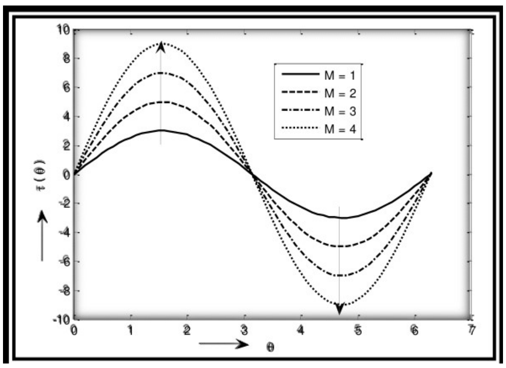

Figure 12: Graph between Stress τ and $\sin\theta$ and variation of Magnetic Field (M)

Substituting $\tau_{rr(2)}, \tau_{r\theta(2)}$ from (47) and (48) the expression (72) gives the dimensionless drag D as

$$D = \frac{D_r}{(\mu_2 \pi U_\infty/2)} = \left[f_2''' (1) + 2 f_2 (1) - \left(3 + M^2\right) f_2'(1) \right]. \tag{80}$$

Substituting $f_2(r)$ from equation (56) in the above expression, we get

$$D = -\left(1 + M^2\right) + C\left(M^2 - 1\right) - N\left[\left(M^2 + 1\right) K_1 M\right]. \tag{81}$$

From Figure 11, it is seen that increase of Magnetic parameter M, Drag D is increase sharply. Figure 12, is graph between θ and stress τ, it is seen that increase of Magnetic Parameter M, Stress increases sharply in ($0 \leq \theta \leq \pi$) and stress decreases sharply in ($\pi \leq \theta \leq 2\pi$).

5. Conclusion

The main objective is to study viscous fluid in the presence of a transverse magnetic field via spherical and cylindrical structures. An analytical procedure is used for the solution of the differential equation. It is observed that, with the enhancement of the magnetic field, the velocity of the streamline flow of fluid decreases sharply in both zones of the sphere as well as the cylinder, whereas the reverse effect is seen with the increase in the radius of the sphere as well as the cylinder. It is also observed that drag and stress increase sharply with an increase in the magnetic parameter M. The spherical flow of fluid occurs in oceans, the atmosphere, glaciers, and each of the other spheres. The circular cylinder application is widely used in real life. There are three common uses of the circular cylinder, water bottles, candles, and deodorant cans. All of these objects reflect the shape of the circular cylinder.

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