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Cordial and Product Cordial Labeling for the Extended Duplicate Graph of Kite Graph

Research Article

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Abstract: In this paper, we prove that the extended duplicate graph of kite graph is cordial, total cordial, product cordial and total product cordial labeling.

MSC: 05C78.

Keywords: Graph labeling, Cordial labeling, Product cordial labeling, Kite graph.

1. Introduction

The field of graph theory plays a vital role in various fields. One of the important areas in graph theory is graph labeling. Labeled graphs serve as useful models for a wide range of applications. After the introduction of graph labeling, various labeling of graphs such as graceful labeling, cordial labeling, prime cordial labeling, magic labeling, anti magic labeling etc., have been studied in over 2100 papers [1]. Several researchers refer to Rosa's [2] work.

One of the most famous and productive labeling of graph theory is cordial labeling. This labeling was introduced by Cahit in the year 1987 [3]. In 1990, Cahit has proved that every tree is cordial; K_n is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; all fans are cordial [3]. The concepts of product cordial labeling and total product labeling was introduced and studied by Sundaram and Somasundaram [5–7]. The concept of duplicate graph was introduced by E.Sampath kumar and he proved many results on it [4]. K.Thirusangu, P.P. Ulaganathan and B. Selvam have proved that the duplicate graph of a path graph P_m is cordial [9]. K.Thirusangu, B. Selvam and P.P. Ulaganathan have proved that the extended duplicate graph of twig graphs is cordial and total cordial [8].

2. Preliminaries

In this section, we give the basic definitions relevant to this paper. Let $G = G(V, E)$ be a finite, simple and undirected graph with p vertices and q edges. By a labeling we mean a one-to-one mapping that carries a set of graph elements onto a set of numbers, called labels (usually the set of integers).

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Definition 2.1 (Kite Graph)**.** *The kite graph is obtained by attaching a path of length m with a cycle of length n and it is denoted as $K_{n,m}$. Kite graphs is also known as the Dragon Graphs or Canoe Paddle Graphs. Clearly it has $m+3$ vertices and $m+3$ edges.*

Illustration 2.2. *Kite Graph*

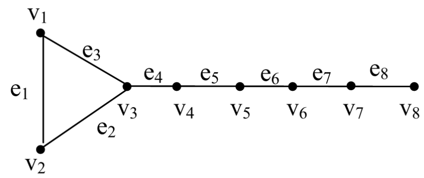

Figure 1. $K_{3,5}$

Definition 2.3 (Duplicate Graph)**.** *Let $G(V,E)$ be a simple graph and the duplicate graph of G is $DG = (V_1, E_1)$, where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and $f : V \to V'$ is bijective (for $v \in V$, we write $f(v) = v'$ for convenience) and the edge set E_1 of DG is defined as the edge ab is in E if and only if both ab' and $a'b$ are edges in E_1.*

Definition 2.4 (Extended Duplicate Graph of Kite Graph)**.** *Let $DG = (V_1, E_1)$ be a duplicate graph of the kite graph $G(V,E)$. Extended duplicate graph of kite graph is obtained by adding the edge v_2v_2' to the duplicate graph. It is denoted by $EDG\ (K_{3,m})$, $m \geq 1$. Clearly it has $2m+6$ vertices and $2m+7$ edges.*

Illustration 2.5. *Extended Duplicate Graph of Kite Graph*

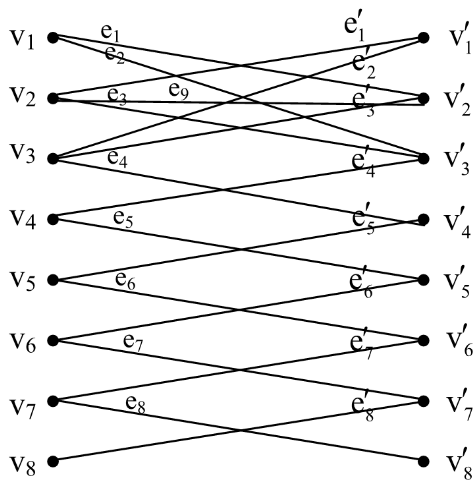

Figure 2. $EDG(K_{3,5})$

Definition 2.6 (Cordial labeling)**.** *A function $f : V \to \{0,1\}$ such that each edge uv receives the label $|f(u) - f(v)|$ is said to be cordial labeling if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most one, the number of edges labeled 0 and the number of edges labeled 1 differ by at most one.*

Definition 2.7 (Total Cordial Labeling)**.** *A function $f : V \to \{0,1\}$ such that each edge uv receives the label $|f(u) - f(v)|$ is said to be total cordial labeling if the number of vertices and edges labeled 0 and the number of vertices and edges labeled 1 differ by at most one.*

Definition 2.8 (Product Cordial Labeling)**.** *A function $f : V \to \{0, 1\}$ such that each edge uv receives the label $f(u) \times f(v)$ is said to be product cordial labeling if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most one, the number of edges labeled 0 and the number of edges labeled 1 differ by at most one.*

Definition 2.9 (Total Product Cordial Labeling)**.** *A function $f : V \to \{0, 1\}$ such that each edge uv receives the label $f(u) \times f(v)$ is said to be total product cordial labeling if the number of vertices and edges labeled 0 and the number of vertices and edges labeled 1 differ by at most one.*

3. Main Results

3.1. Cordial Labeling

In this section, we present an algorithm and prove the existence of cordial labeling for the extended duplicate graph of kite graph $K_{3,m}$, $m \geq 1$.

Algorithm 3.1. *Procedure (Cordial labeling for EDG($K_{3,m}$), $m \geq 1$)*

$V \leftarrow \{v_1, v_2, \ldots, v_{m+3}, v'_1, v'_2, ..., v'_{m+3}\}$

$E \leftarrow \{e_1, e_2, \ldots, e_{m+4}, e'_1, e'_2, ..., e'_{m+3}\}$

if $m = 4n - 3$

for i = 0 to (m-1)/4 ***do***

j = 0 to 1 ***do***

$v_{1+4i+j} \leftarrow 0$

$v_{3+4i+j} \leftarrow 1$

$v'_{1+4i+j} \leftarrow 1$

$v'_{3+4i+j} \leftarrow 0$

end for

else

if $m = 4n - 2$

for i=0 to (m+2)/4 ***do***

$v_{1+4i} \leftarrow 0$

$v'_{1+4i} \leftarrow 1$

end for

for i = 0 to (m-2)/4 ***do***

j = 0 to 1 ***do***

$v_{2+4i} \leftarrow 0$

$v_{3+4i+j} \leftarrow 1$

$v'_{2+4i} \leftarrow 1$

$v'_{3+4i+j} \leftarrow 0$

end for

else

if m = 4n-1

for i = 0 to (m+1)/4 ***do***

j = 0 to 1 ***do***

$v_{1+4i+j} \leftarrow 0$

$v'_{1+4i+j} \leftarrow 1$

end for

for i = 0 to (m-3)/4 ***do***

j = 0 to 1 ***do***

$v_{3+4i+j} \leftarrow 1$

$v'_{3+4i+j} \leftarrow 0$

end for

else

if m = 4n

for i = 0 to m/4 ***do***

j = 0 to 1 ***do***

$v_{1+4i+j} \leftarrow 0$

$v_{3+4i} \leftarrow 1$

$v'_{1+4i+j} \leftarrow 1$

$v'_{3+4i} \leftarrow 0$

end for

for i=0 to (m-4)/4 ***do***

$v_{4+4i} \leftarrow 1$

$v'_{4+4i} \leftarrow 0$

end for

end if

end procedure

Theorem 3.2. *The extended duplicate graph of kite graph $K_{3,m},\ m \geq 1$ is cordial labeling.*

Proof. Let $K_{3,m},\ m \geq 1$ be a kite graph. In order to label the vertices, define a function $f : V \to \{0,\ 1\}$ as given in Algorithm 3.1.

Case(i): If $m = 4n - 3$; $n \in N$, for $0 \leq i \leq (m-1)/4$ and $0 \leq j \leq 1$, the vertices v_{1+4i+j} and v'_{3+4i+j} receive label 0; the vertices v_{3+4i+j} and v'_{1+4i+j} receive label 1. Thus all the $2m + 6$ vertices namely the vertices $v_1,\ v_2,\ v_5,\ v_6,\ldots,v_m,\ v_{m+1}$ receive label 0; the vertices $v_3,\ v_4,\ v_7,\ v_8,\ldots,\ v_{m+2},\ v_{m+3}$ receive label 1; the vertices $v'_1,\ v'_2,\ v'_5,\ v'_6,\ldots,\ v'_m,\ v'_{m+1}$ receive label 1; the vertices $v'_3,\ v'_4,\ v'_7,\ v'_8,\ldots,v'_{m+2},\ v'_{m+3}$ receive label 0.

Case (ii): If $m = 4n - 2$; $n \in N$, for $0 \leq i \leq (m+2)/4$, the vertices v_{1+4i} receive label 0 and the vertices v'_{1+4i} receive label 1; for $0 \leq i \leq (m-2)/4$ and $0 \leq j \leq 1$, the vertices v_{2+4i} and v'_{3+4i+j} receive label 0; the vertices v_{3+4i+j} and v'_{2+4i} receive label 1. Thus all the $2m + 6$ vertices namely the vertices $v_1,\ v_2,\ v_5,\ v_6,\ v_9,\ldots,v_{m-1},\ v_m,\ v_{m+3}$ receive label 0; the vertices $v_3,\ v_4,\ v_7,\ v_8,\ldots,\ v_{m+1},\ v_{m+2}$ receive label 1; the vertices $v'_1,\ v'_2,\ v'_5,\ v'_6,\ v'_9,\ldots,\ v'_{m-1},\ v'_m,\ v'_{m+3}$ receive label 1; the vertices $v'_3,\ v'_4,\ v'_7,\ v'_8,\ldots,v'_{m+1},\ v'_{m+2}$ receive label 0.

Case (iii): If $m = 4n - 1$; $n \in N$, for $0 \leq i \leq (m+1)/4$ and $0 \leq j \leq 1$, the vertices v_{1+4i+j} receive label 0 and the vertices v'_{1+4i+j} receive label 1; for $0 \leq i \leq (m-3)/4$ and $0 \leq j \leq 1$, the vertices v_{3+4i+j} receive label 1 and the vertices v'_{3+4i+j} receive label 0. Thus all the 2m+6 vertices namely the vertices $v_1,\ v_2,\ v_5,\ v_6,\ v_9,\ v_{10}\ldots,\ v_m + 2,\ v_m + 3$ receive label 0; the vertices $v_3,\ v_4,\ v_7,\ v_8,\ \ldots,\ v_m,\ v_m + 1$ receive label 1; the vertices $v'_1,\ v'_2,\ v'_5,\ v'_6,\ v'_9,\ v'_{10},\ldots,\ v'_m + 2,\ v'_m + 3$ receive label 1; the vertices $v'_3,\ v'_4,\ v'_7,\ v'_8,\ldots,\ v'_m,\ v'_m + 1$ receive label 0.

Case (iv): If $m = 4n$; $n \in N$, for $0 \leq i \leq m/4$ and $0 \leq j \leq 1$, the vertices v_{1+4i+j} and v'_{3+4i} receive label 0; the vertices v_{3+4i} and v'_{1+4i+j} receive label 1. Thus all the $2m+6$ vertices namely the vertices v_1, v_2, v_5, v_6, v_9, $v_{10}, \ldots, v_{m+1}$, v_{m+2} receive label 0; the vertices v_3, v_4, v_7, v_8, $v_{11}, \ldots, v_{m-1}$, v_m, v_{m+3} receive label 1; the vertices v'_1, v'_2, v'_5, v'_6, v'_9, $v'_{10}, \ldots,$ v'_{m+1}, v'_{m+2} receive label 1; the vertices v'_3, v'_4, v'_7, v'_8, v'_{11}, $\ldots$, v'_{m-1}, v'_m, v'_{m+3} receive label 0.

Hence in all the cases, the entire $2m+6$ vertices are labeled such that the number of vertices labeled 0 and 1 differ by at most one, which satisfies the required condition. To obtain the labels for edges , we define the induced function $f^* : E \to \{0, 1\}$ such that

$$f^*(v_i v_j) = |f(v_i) - f(v_j)| \text{ where } v_i, \ v_j \in V$$

The induced function yields the label 1 for the edges e_1, e'_1 and e_{m+4}; label 0 for the edges e_2 and e'_2; label 0 for the edges e_{2i+1} and e'_{2i+1} if $0 \leq i \leq (m+1)/2$, m is odd and if $0 \leq i \leq (m+2)/2$, m is even; label 1 for the edges e_{2i+2} and e'_{2i+2} if $0 \leq i \leq (m+1)/2$, m is odd and if $0 \leq i \leq m/2$, m is even. Thus the entire $2m+7$ edges are labeled namely when m is odd, $m+3$ edges e_2, e_3, e_5, e_7, $e_9, \ldots, e_{m+2}$, e'_2, e'_3, e'_5, e'_7, $e'_9, \ldots, e'_{m+2}$ receive label 0 and $m+4$ edges e_1, e_4, e_6, e_8, $e_10, \ldots, e_{m+3}$, e'_1, e'_4, e'_6, e'_8, $e'_10, \ldots, e'_{m+3}$ and e_{m+4} receive label 1 and when m is even, $m+4$ edges e_2, e_3, e_5, e_7, $e_9, \ldots,$ e_{m+3}, e'_2, e'_3, e'_5, e'_7, $e'_9, \ldots, e'_{m+3}$ receive label 0 and $m+3$ edges e_1, e_4, e_6, $e_8, \ldots, e_{m+2}$, e'_1, e'_4, e'_6, $e'_8, \ldots, e'_{m+2}$ and e_{m+4} receive label 1, which differ by one and satisfies the required condition. Hence the extended duplicate graph of kite graph $K_{3,m}$, $m \geq 1$ is cordial labeling. □

Theorem 3.3. *The extended duplicate graph of kite graph $K_{3,m}$, $m \geq 1$ is total cordial.*

Proof. In Theorem 3.2, $(m+3)$ vertices were assigned the label 0, $(m+3)$ vertices were assigned the label 1 and it has been proved that when m is odd, the number of edges labeled 0 is $(m+3)$ and the number of edges labeled 1 is $(m+4)$ and when m is even, the number of edges labeled 0 is $(m+4)$ and the number of edges labeled 1 is $(m+3)$. From this we conclude that, when m is odd, the number of vertices and edges labeled 0 is $(m+3)+(m+3) = 2m+6$ and the number of vertices and edges labeled 1 is $(m+3)+(m+4) = 2m+7$, when m is even, the number of vertices and edges labeled 0 is $(m+3)+(m+4) = 2m+7$ and the number of vertices and edges labeled 1 is $(m+3)+(m+3) = 2m+6$, which differ by atmost one and satisfies the required condition. Hence the extended duplicate graph of kite graph $K_{3,m}$, $m \geq 1$ is total cordial. □

Illustration 3.4. *Cordial labeling for the graphs **EDG(**$K_{3,5}$**)** and **EDG(**$K_{3,6}$**)**.*

Cordial Labeling for the Extended Duplicate Graph of Kite Graph

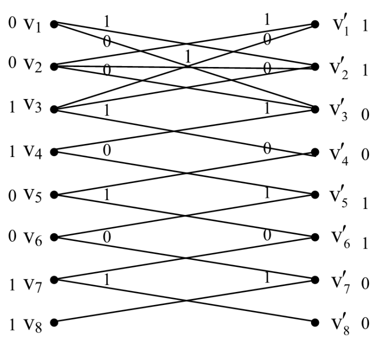

Figure 3. **EDG(**$K_{3,5}$**)**

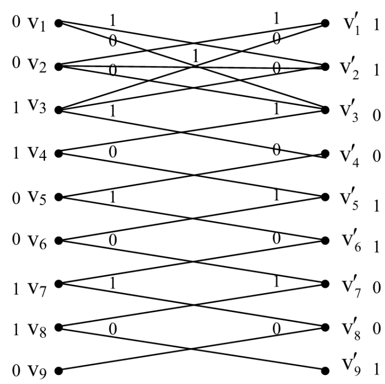

Figure 4. **EDG(**$K_{3,6}$**)**

3.2. Product Cordial Labeling

Algorithm 3.5. *Procedure (Product cordial labeling for EDG($K_{3,m}$), $m \geq 1$)*

$V \leftarrow \{v_1, v_2, \ldots, v_{m+3}, v'_1, v'_2, \ldots, v'_{m+3}\}$

$E \leftarrow \{e_1, e_2, \ldots, e_{m+4}, e'_1, e'_2, \ldots, e'_{m+3}\}$

if $m = 2n - 1$

$v_1 \leftarrow 0,\ v_2 \leftarrow 1$

$v' \leftarrow 1,\ v'_{m+3} \leftarrow 0$

for i = 0 to (m-1)/2 ***do***

$v_{3+2i} \leftarrow 1$

$v_{4+2i} \leftarrow 0$

$v'_{2+2i} \leftarrow 1$

$v'_{3+2i} \leftarrow 0$

end for

else

if m = 2n

$v_1 \leftarrow 0,\ v_2 \leftarrow 1$

$v' \leftarrow 1,\ v_{m+3} \leftarrow 0$

for i=0 to (m-2)/2 ***do***

$v_{3+2i} \leftarrow 1$

$v_{4+2i} \leftarrow 0$

end for

for i = 0 to m/2 ***do***

$v'_{2+2i} \leftarrow 1$

$v'_{3+2i} \leftarrow 0$

end for

end if

end procedure

Theorem 3.6. *The extended duplicate graph of kite graph $K_{3,m}$, $m \geq 1$ is product cordial labeling.*

Proof. Let $K_{3,m}$, $m \geq 1$ be a kite graph. In order to label the vertices, define a function $f : V \to \{0,\ 1\}$ as given in Algorithm 3.5.

Case (i) : If $m = 2n - 1$; $n \in N$, the vertices v_1 and v'_{m+3} receive label 0; the vertices v_2 and v'_1 receive label 1; for $0 \leq i \leq (m-1)/2$, the vertices v_{3+2i} and v'_{2+2i} receive label 1; the vertices v_{4+2i} and v'_{3+2i} receive label 0. Thus all the $2m+6$ vertices namely the vertices $v_1,\ v_4,\ v_6,\ v_8, \ldots,\ v_{m+3}$ receive label 0; the vertices $v_2,\ v_3,\ v_5,\ v_7,\ \ldots,\ v_{m+2}$ receive label 1; the vertices $v'_1,\ v'_2,\ v'_4, v'_6, \ldots, v'_{m+1}$ receive label 1; the vertices $v'_3,\ v'_5,\ v'_7,\ v'_8,\ \ldots, v'_{m+2},\ v'_{m+3}$ receive label 0.

Case (ii) : If $m = 2n$; $n \in N$, the vertices v_1 and v_{m+3} receive label 0; the vertices v_2 and v'_1 receive label 1; for $0 \leq i \leq (m-2)/2$, the vertices v_{3+2i} receive label 1; the vertices v_{4+2i} receive label 0; for $0 \leq i \leq m/2$, the vertices v'_{2+2i} receive label 1; the vertices v'_{3+2i} receive label 0. Thus all the $2m+6$ vertices namely the vertices $v_1,\ v_4,\ v_6,\ v_8,\ v_9, \ldots,\ v_{m+2},\ v_{m+3}$ receive label 0; the vertices $v_2,\ v_3,\ v_5,\ v_7,\ \ldots, v_{m+1}$ receive label 1; the vertices $v'_1, v'_2, v'_4, v'_6, v'_8, \ldots, v'_{m+2}$ receive label 1; the vertices $v'_3, v'_5, v'_7, v'_9, \ldots, v'_{m+3}$ receive label 0.

Hence in both the cases, the entire $2m+6$ vertices are labeled such that the number of vertices labeled 0 and 1 differ by at most one, which satisfies the required condition. To obtain the labels for edges , we define the induced function $f*: E \to \{0,1\}$ such that

$$f*(v_iv_j) = f(v_i) \times f(v_j) \text{ where } v_i,\ v_j \in V$$

The induced function yields the label 1 for the edges e'_1, e'_2 and e_{m+4}; label 0 for the edges e_1, e_2, e_{m+3} and e'_{m+3}; if m is odd , for $0 \le i \le (m+1)/2$ the edges e_{2i+1} receive label 0 and the edges e'_{2i+1} receive label 1 and for $0 \le i \le (m-1)/2$, $m \ge 2$ the edges e_{2i+2} receive label 1 and the edges e'_{2i+2} receive label 0; if m is even, for $0 \le i \le m/2$ the edges e_{2i+1} and the edges e'_{2i+2} receive label 0; the edges e_{2i+2} and the edges e'_{2i+1} receive label 1.

Thus the entire $2m+7$ edges are labeled namely when m is odd, $m+4$ edges $e_1, e_2, e_3, e_5,\ e_7, e_8, \ldots,\ e_{m+2}, e_{m+3}, e'_4, e'_6, e'_8, \ldots, e'_{m+3}$ receive label 0 and $m+3$ edges $e_4, e_6, \ldots, e_{m+1}, e'_1, e'_2, e'_3, e'_5, e'_7, \ldots, e'_{m+2}$ and e_{m+4} receive label 1 and when m is even, $m+4$ edges $e_1,\ e_2,\ e_3,\ e_5,\ e_7,\ e_9,\ \ldots, e_{m+3},\ e'_4, e'_6, e'_8, e'_9, \ldots, e'_{m+2}, e'_{m+3}$ receive label 0 and $m+3$ edges $e_4, e_6, e_8, \ldots, e_{m+2}, e'_1, e'_2, e'_3, e'_5, e'_7, \ldots, e'_{m+1}$ and e_{m+4} receive label 1, which differ by one and satisfies the required condition. Hence the extended duplicate graph of kite graph $K_{3,m}$, $m \ge 1$ is product cordial labeling. □

Theorem 3.7. *The extended duplicate graph of kite graph $K_{3,m}$, $m \ge 1$ is total product cordial.*

Proof. In Theorem 3.6, $(m+3)$ vertices were assigned the label 0, $(m+3)$ vertices were assigned the label 1 and it has been proved that the number of edges labeled 0 is $(m+4)$ and the number of edges labeled 1 is $(m+3)$. From this we conclude that, the number of vertices and edges labeled 0 is $(m+3)+(m+4) = 2m+7$, the number of vertices and edges labeled 1 is $(m+3)+(m+3) = 2m+6$, which differ by atmost one. Hence the extended duplicate graph of kite graph $K_{3,m}$, $m \ge 1$ is total product cordial. □

Illustration 3.8. *Product cordial labeling for the graphs **EDG**($K_{3,5}$) and **EDG**($K_{3,6}$)*

Product Cordial Labeling for the Extended Duplicate Graph of Kite Graph

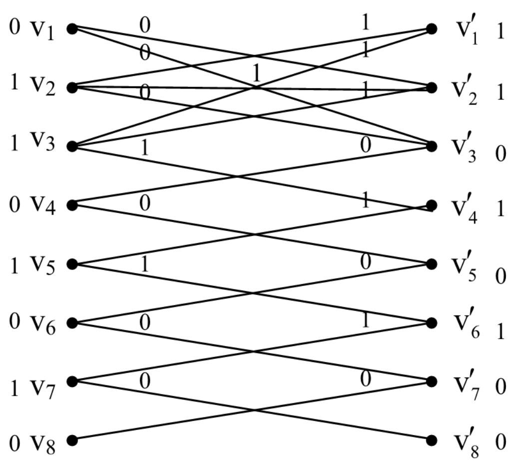

Figure 5. EDG($K_{3,5}$)

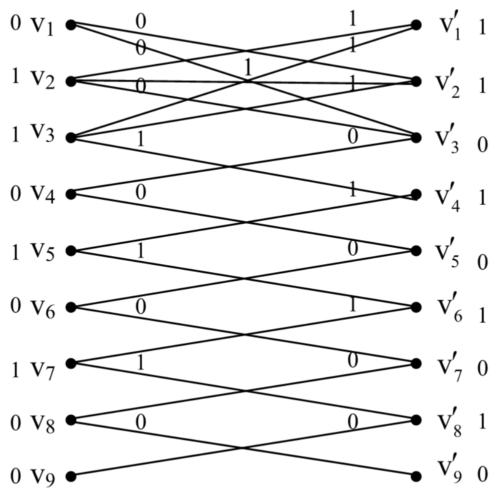

Figure 6. EDG($K_{3,6}$)

4. Conclusion

In this paper, we presented algorithms and prove that the extended duplicate graph of kite graph $K_{3,m}$, $m \geq 1$ is cordial, total cordial, product cordial and total product cordial labeling.

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