International Journal of Mathematics And its Applications
Volume 4, Issue 4 (2016), 101–106. (Special Issue)
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

The b-Chromatic Number of Central Graph of Some Special Graphs

Research Article*

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Abstract: Let $G = (V, E)$ be an undirected and loopless graph. The b- chromatic number of a graph G is the largest integer k such that G admits a proper k- coloring in which every color class contains atleast one vertex adjacent to some vertex in all the other color classes .In this paper we investigate the b-chromatic number of central graph of cycle, path, snake, wheel, helm and denoted as C(C_n), C(P_n), C(T_n), C(W_n) and C(H_n) respectively.

Keywords: Proper coloring, Chromatic number, b-Coloring, b-Chromatic number, central graph, cycle, path, snake, wheel and helm.

1. Introduction

Let G = (V, E) be an undirected graph with loopless and multiple edges. A coloring of vertices of graph G is a mapping c: V (G) → {1, 2. . . . . . k} for every vertex. A coloring is said to be proper if any two adjacent vertices of a graph have different colors. The chromatic number χ(G) of a graph G is the smallest integer k which admits a proper coloring[4]. A particular color which is assigned to a certain set of vertices is called color class. A proper k-coloring c of a graph G is a b-coloring if for every color class c_i, there is a vertex with color i which has atleast one neighbor in every other adjacent color classes. The b-chromatic number χ_b(G) of a graph G is the largest integer k such that G admits a proper k-coloring in which every color class contains at least one vertex adjacent to some vertex in all the other color classes. The central graph of G, is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G. It is denoted by C(G). Irving and Manlove[2] introduced the concept of coloring in 1999, and determined the b-chromatic number ofNP-hard problem. In this paper we found the b-chromatic number of central graph of some special graphs.

Definition 1.1. *A cycle is a circuit in which no vertex except the first (which is also the last) appears more than once. A cycle with n vertices is denoted by C_n.*

Definition 1.2. *A path is a sequence of consecutive edges in a graph and the length of the path is the number of edges traversed. A path with n vertices is denoted as P_n.*

Definition 1.3. *The wheel graph W_n is a graph with n vertices (n≥4), formed by connecting a single vertex to all vertices of an (n-1) cycle.*

* *Proceedings : National Conference on Recent Trends in Applied Mathematics held on 22 & 23.07.2016, organized by Department of Mathematics, St. Joseph's College of Arts & Science, Manjakuppam, Cuddalore (Tamil Nadu), India.*

Definition 1.4. *The helm H_n is a graph obtained from a wheel by attaching a pendant vertex at each vertex of the n-cycle.*

Definition 1.5. *A triangular cactus is a connected graph all of whose blocks are triangles. A triangular snake is a triangular cactus whose block-cut point graph is a path. Equivalently it is obtained from a path $P = v_1,\ v_2,.v_{n+1}$ by joining v_i and v_{i+1} to a new vertex $u_1,u_2.........u_n$. A triangular snake has 2n+1 vertices and 3n edges, where n is the number of blocks in the triangular snake. It is denoted by Tn.*

Definition 1.6. *The central graph of G, is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G. It is denoted by C(G).*

2. The b-chromatic Number of Central Graph of Some Special Graphs

In this paper we determined the b-chromatic number of central graph of some special graphs.

Theorem 2.1. *For n>3, the b-chromatic number of central graph of cycle is $\lceil n/2\rceil$+1 i.e $\chi_b(C(C_n)) = \lceil n/2\rceil+1\ \forall\ n \geq 3$.*

Proof. Let C_n be a cycle of length n with the vertices $v_1,v_2,......v_n$ and edges $e_1,e_2,......e_n$, where ei = v_iv_{i+1} for $1 \leq$ i $\leq$ n-1. By the definition of central graph, $C(C_n)$ is obtained by subdividing each edge $e_i = v_iv_{i+1}$, $1 \leq$ i $\leq$ n-1 of $C_n$exactly once by newly added vertex c_i and subdividing v_nv_1 by C_n join v_i and $v_j$for $1 \leq$ i, j $\leq$ n and for i $\neq$ j. Let V_1= $\{v_1,v_2,......v_n\}$ and $V_2 = \{c_1,c_2,......c_n\}$. Then $V(C(C_n)) = v_1 \cup v_2$. Let us define a mapping Φ : V$\rightarrow$c such that $\Phi(v_i)$ = c_i for all i. And assign b-coloring to the graph $C(C_n)$ by the following procedure. Assign color '1' to the central vertices c_i. i.e color class 1 contains $\{c_1,c_2,...c_n\}$ and assign colors 2,3,....n+1 to the remaining vertices of the graph. Since all the non-adjacent vertices becomes adjacent after subdividing the edges. Hence we assign n+1colors to n vertices. Therefore, $\chi_b(C(C_n)) = \lceil n/2\rceil$ +1$\forall$ n > 3 □

The below example illustrate the procedure discussed in the above theorem.

Example 2.2.

1. Consider the central graph of $C(C_5)$. Let us assign colors to the vertex. The color classes of $C(C_5)$ are, $c_1= \{1\} = \{c_1,c_2,c_3,c_4,c_5\}$, $c_2= \{2\} = \{v_1,v_5\}$, $c_3= \{3\} = \{v_2,v_3\}$, $c_4= \{4\} = \{v_4\}$. Therefore, the b-chromatic number of central graph of C_5= 4. i.e. $\chi_b(C(C_5)) = 4$

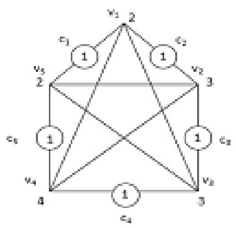


Fig – C5

2. Consider the central graph of $C(C_7)$. Let us assign colors to the vertex.

The color classes of $C(C_7)$ are, $c_1= \{1\} = \{c_1,c_2,c_3,c_4,c_5\}$, $c_2= \{2\} = \{v_1,v_7\}$, $c_3= \{3\} = \{v_2,v_3\}$, $c_4= \{4\} = \{v_4,v_5\}$, $c_5= \{5\} = \{v_6\}$. Therefore, the b-chromatic number of central graph of C_7= 5. i.e. $\chi_b(C(C_7)) = 5$

Theorem 2.3. *For n$\geq$3, the b-chromatic number of central graph of path is$\lceil n/2\rceil$+ i.e $\chi_b(C(P_n)) = \lceil n/2\rceil+1$*

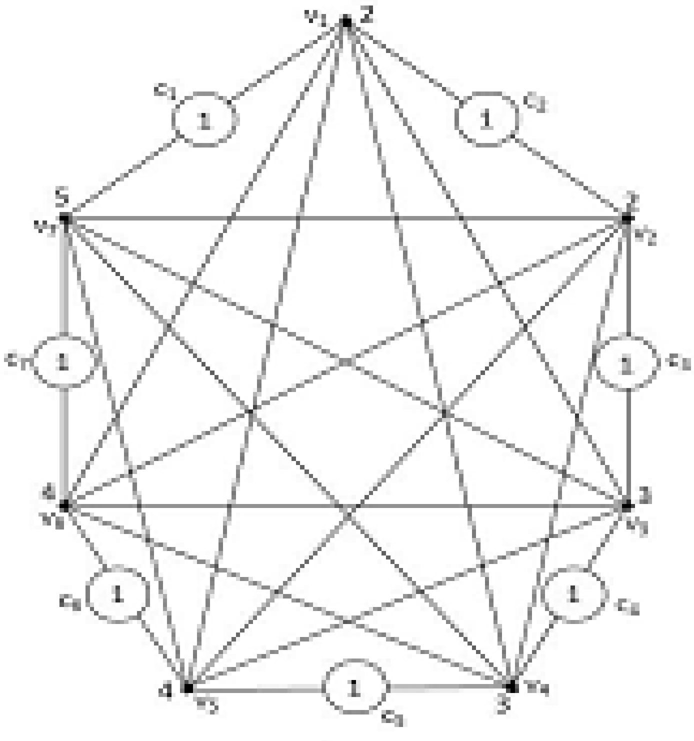


Fig – C_7

Proof. Let $P_n$be a path graph of length n-1 with n vertices. Let V= {v_1,v_2,. . . ..v_n} be the vertices of $P_n$and e_1,e_2. . . ..e_n be the edges of P_n such that e_i = v_iv_{i+1}. By the definition of central graph, C(P_n) is obtained by subdividing each edge e_i = v_iv_{i+1}, 1 ≤ i ≤ n-1 of $P_n$exactly once by newly added vertex c_i and subdividing $v_nv_1$ofP_n by joining v_i and $v_j$for 1 ≤ i, j ≤ n and for i ≠ j. Let us define a mapping Φ: V→c such that Φ(v_i) = C_i for all i. Let us now assign b-coloring to the graph C(P_n) as follows. Assign color '1' to the central vertices c_i. i.e color class 1 contains {c_1,c_2,. . . c_n} and assign colors (2,3,. . . .n) to the remaining vertices of the graph. For any vertex v_i and v_{i+1} which are non-adjacent in the central graph for 1 ≤ i ≤ n-1. Therefore, $\chi_b(C(P_n)) = \lceil n/2 \rceil + 1 \; \forall \; n \geq 3$. □

The following example illustrate the procedure discussed in the above theorem.

Example 2.4.

1. Consider the central graph of C(P_4). Let us assign colors to the vertex. The color classes of C(P_4) are, c_1= {3} = {c_1,c_2,c_3}, c_2= {2} = {v_3,v_4}, c_3= {1} = {v_1,v_2}. Therefore, the b-chromatic number of central graph of P_4= 3. i.e. $\chi_b(C(P_4))$ = 3

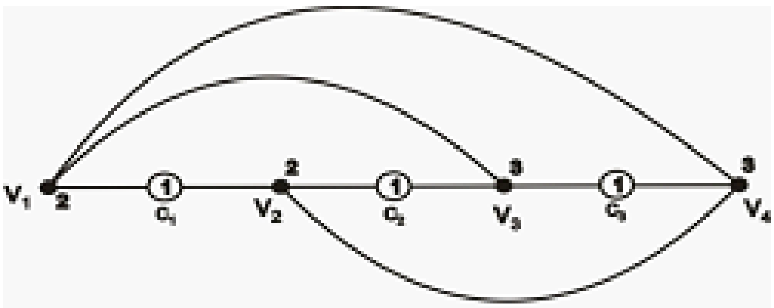
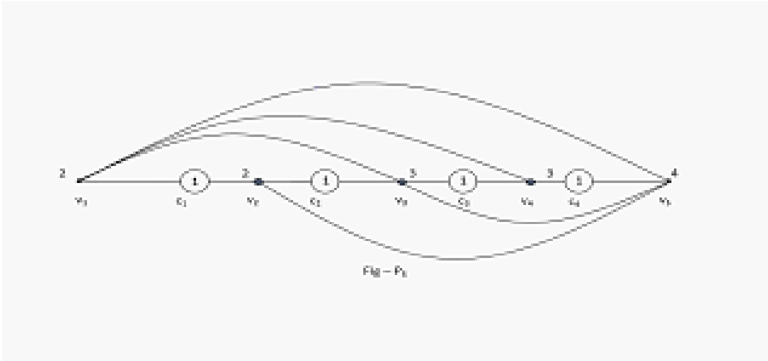


Fig - P_4

2. Consider the central graph of C(P_5). Let us assign colors to the vertex.



The color classes of $C(P_5)$ are, $c_1=\{1\}=\{c_1,c_2,c_3,c_4\}$,$c_2=\{2\}=\{v_1,v_2\}$, $c_3=\{3\}=\{v_4,v_3\}$, $c_4=\{4\}=\{v_5\}$. Therefore, the b-chromatic number of central graph of $P_5=4$. i.e. $\chi_b(C(P_5))=4$

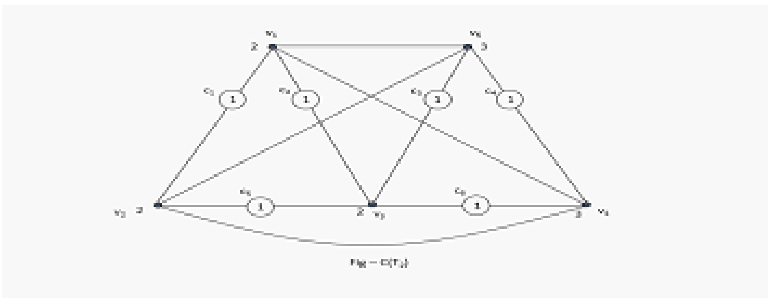
Theorem 2.5. *The b-chromatic number of the central graph of the snake graph is given by $\chi_b(C(T_n)$ =(n+1)$\forall$ n $\geq$ 2.*

Proof. Let T_n be a snake graph with 2n+1 vertices and 3n edges. Let $V(T_n)=\{v_1,v_2,\ldots.v_{2n+1}\}$ and $E(T_n)=\{e_1,e_2,\ldots\ldots.e_{3n}\}$. By the definition of central graph, $C(T_n)$ is obtained by subdividing each edge $e_i=v_iv_{i+1}$, $1\leq i\leq n-1$ of $T_n$exactly once by newly added vertex c_i and join v_i and $v_j$for $1\leq i,j\leq n$ and for $i\neq j$. Let us define a mapping $\Phi: V\rightarrow c$ such that $\Phi(v_i)=c_i$ for all i. Now let us assign b-coloring to the graph $C(T_n)$ as follows. Assign color '1' to the central vertices c_i.i.e color class 1 contains $\{c_1,c_2,\ldots c_n\}$ and assign colors (2,3,....n+1) to the remaining vertices of the graph. Hence we assign (n+ 1) color to (2n+1) vertices. Therefore, $\chi_b(C(T_n)=n+1$ ☐

The below example illustrate the procedure discussed in the above theorem.

Example 2.6.

1. Consider the central graph of $C(T_2)$. Let us assign colors to the vertex. The color classes of $C(T_2)$ are, $c_1=\{1\}=\{c_1,c_2,c_3,c_4\}$, $c_2=\{2\}=\{v_1,v_2,v_3\}$, $c_3=\{3\}=\{v_4,v_5\}$. Therefore, the b-chromatic number of central graph of $T_2=3$. i.e. $\chi_b(C(T_2))=3$



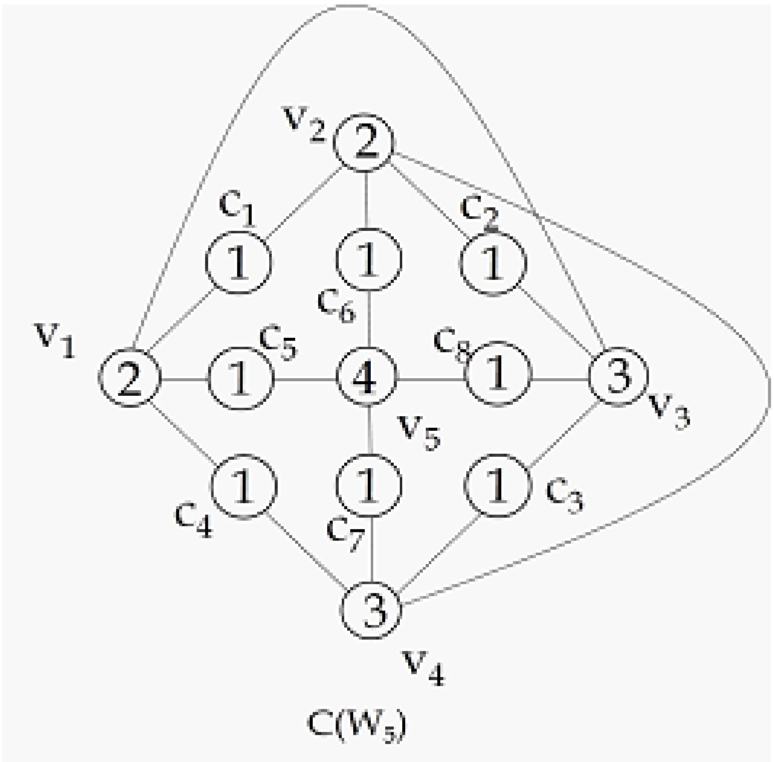
Theorem 2.7. *For n $\geq$ 5, the b-chromatic number of central graph of the wheel graph is $\chi_b(C(W_n))=\lceil n/2\rceil+1\forall$ n.*

Proof. Let $W_n$be a wheel graph with n+ 1 vertex. Let $V(W_n)=\{v_1,v_2,\ldots.v_n\}$ and $E(W_n)=\{e_1,e_2,\ldots\ldots.e_n\}$ where v_{n+1} is the hub. By the definition of central graph, $C(W_n)$ is obtained by subdividing each edge $e_i=v_iv_{i+1}$, $1\leq i\leq n-1$ of $W_n$exactly once by newly added vertex c_i and subdividing v_nv_1 by W_n join v_i and $v_j$for $1\leq i,j\leq n$ and for $i\neq j$. Let us define a mapping Φ: $V\rightarrow c$ such that $\Phi(v_i)=c_i$ for all i. Now assign b-coloring to the graph $C(W_n)$ as follows. Assign color '1' to the central vertices c_i. i.e color class 1 contains $\{c_1,c_2,\ldots c_n\}$ and assign colors (2,3,....n) to the remaining vertices of the outer cycle c_{n-1} of W_n. Next let us assign color n+1 to the hub. Therefore, $\chi_b(C(W_n))=\lceil n/2\rceil+1\ \forall\ n\geq 5$. ☐

The following below example illustrate the procedure discussed in the above theorem.

Example 2.8.

1. Consider the central graph of $C(W_5)$. Let us assign colors to the vertex. The color classes of $C(W_5)$ are, $c_1 = \{1\} = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8\}$, $c_2 = \{2\} = \{v_1, v_2\}$, $c_3 = \{3\} = \{v_3, v_4\}$, $c_4 = \{4\} = \{v_5\}$. Therefore, the b-chromatic number of central graph of $W_5 = 4$. i.e. $\chi_b(C(W_5)) = 4$



2. Consider the central graph of $C(W_7)$. Let us assign colors to the vertex. The color classes of $C(W_7)$ are, $c_1 = \{1\} = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12}\}$, $c_2 = \{2\} = \{v_1, v_6\}$, $c_3 = \{3\} = \{v_2, v_3\}$, $c_4 = \{4\} = \{v_4, v_5\}$, $c_5 = \{v_7\}$. Therefore, the b-chromatic number of central graph of $W_7 = 5$. i.e. $\chi_b(C(W_7)) = 5$

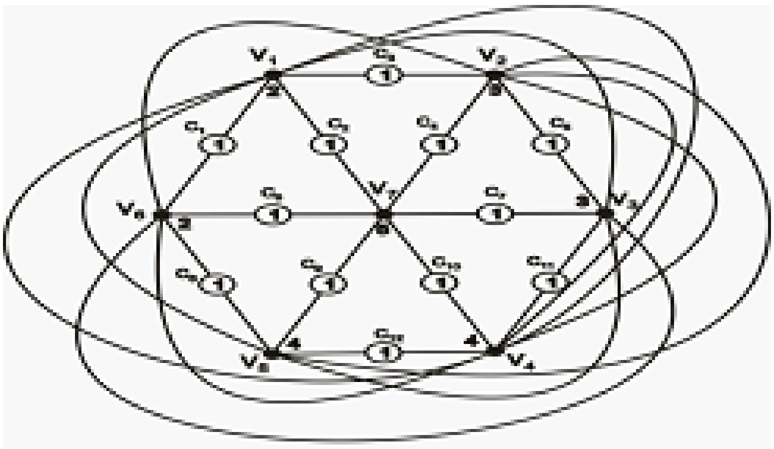


Fig - C(W7)

Theorem 2.9. *For $n \geq 3$, the b-chromatic number of central graph of helm graph is $\chi_b(C(H_n)) = n+2$.*

Proof. Let H_n be a helm graph. Let $V(C_n) = \{v_1, v_2, \ldots. v_n\}$ and $E(C_n) = \{e_1, e_2, \ldots\ldots. e_n\}$. By the definition of central graph, $C(H_n)$ is obtained by subdividing each edge $e_i = v_i v_{i+1}$, $1 \leq i \leq n-1$ of H_n exactly once by newly added vertex c_i and subdividing $v_n v_1$ by H_n join v_i and v_j for $1 \leq i, j \leq n$ and for $i \neq j$. Let us define a mapping $\Phi : V \rightarrow c$ such that $\Phi(v_i) = c_i$ for all i. Now assign b-coloring to the graph $C(H_n)$ as follows. For proper coloring we need n distinct colors. Assign color '1' to the central vertices c_i and 2,3..n to the remaining vertices of the graph. Assign c_i to the pendant vertices of H_n. Hence every color class is adjacent to each other and it satisfies the b-coloring condition. Therefore, $\chi_b(C(H_n)) = n+1$ for $n \geq 3$. □

The below example illustrate the procedure discussed in the above theorem.

Example 2.10.

1. Consider the central graph of $C(H_3)$. Let us assign colors to the vertex. The color classes of $C(H_3)$ are, $c_1 = \{1\} = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9\}$, $c_2 = \{2\} = \{v_1, v_6\}$, $c_3 = \{3\} = \{v_2, v_4\}$, $c_4 = \{4\} = \{v_3, v_5\}$, $c_5 = \{5\} = \{v_7\}$. Therefore, the b-chromatic number of central graph of H_3=5. i.e. $\chi_b(C(H_3)) = 5$.

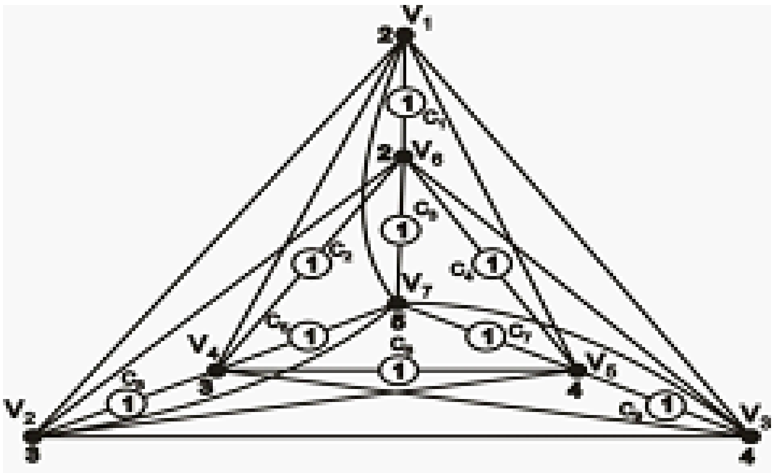


Fig - C(H_3)

3. Conclusion

In this paper we established the b-chromatic number of central graph of some special graphs. This work can be extended to identify the various graphs.

References

[1] Duraisamy Vijayalakshmi, Kandasamy Thilagavathi and Narayanan Roopesh, b-chromatic Number of M[C_n], M[P_n], M[$F_{1,n}$] and M[W_n], 1(2011), 85-88.

[2] R.W.Irving and D.F.Manlove, *Theb-chromatic number of a graph*, Discrete Applied Mathematics, 91(1999), 127-141.

[3] M.Jakovac and S.Klavzar, *The b-chromatic number of cubic graphs*, (2010).

[4] N.Parvathi and R.Nagarathinam, *B-chromatic number of line graph of wheel graph*, 6(2015).

[5] S.K.Vaidya and Rakhimol V.Isaac, *The b-chromatic number of some path related graphs*, 4(1)(2014), 7-12.

[6] D.B.West, *Introduction to Graph theory*, 2/e, Prentice- hall of India, New Delhi, (2003).